

# OPERATIONAL ANALYSIS IN SYSTEM DESIGN OF SUBMARINES DURING THE EARLY PHASES

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## SUMMARY

This paper presents a new method for operational analysis (OA) as a tool in simulation based design (SBD) for Naval Integrated Complex Systems (NICS), here applied to the submarine domain. An operational analysis model is developed and described. The first step of the design process is to identify and collect the needs from the customer and stakeholders, from which requirements can be deduced and designed in an organized way, i.e. requirement elucidation. It is important to evaluate the benefits or penalties of each requirement on the design as early as possible during initial design. Thus the OA-model must be able to evaluate requirements aggregated in synthesised ships such as initial concepts, i.e. Play-Cards, as representations of a submarine concept in the functions domain where the first set of requirements are designed, and establish their Measure of Capability (MoC) and Measure of Effectiveness (MoE). The work has resulted in an OA-model for submarine design that can be used during the development and for evaluation during the life cycle of a submarine system. The purpose of integrating OA in the design process is to explore the design space and evaluate not only technical solutions and cost but also the system effect in the early phases and thereby find and describe a suitable design room. This will generate a more rapid knowledge growth compared to the classic basic ship design procedures which focus on technical performance and cost. It is expected that we not only reach a higher level of knowledge about the design object but also achieve higher precision in the compliance to needs and deduced and designed requirements by the use of an OA-model as an integrated tool during initial design. This approach also invites customer participation within the framework of integrated project teams.

## 1. INTRODUCTION

The purpose of this study is to develop and validate a new modular operational analysis (OA) procedure for submarine system effectiveness predictions in the early phases of submarine design and during the life-cycle of a submarine as a system. As there are differences in the international definitions of these phases, the nominal Swedish design process and related definitions are used in this paper following Nordin (2009) [25], see Figure 1.

|                  |                   |                    |                  |               |      |
|------------------|-------------------|--------------------|------------------|---------------|------|
| Studies          | Conceptual design | Preliminary design | System design    | Detail design |      |
| Definition stage |                   |                    |                  | Building      | Test |
|                  |                   |                    | Production stage |               |      |

Figure 1: The Swedish procurement phases

The early phases include the Studies and Conceptual phases. During the first phase, Studies, the focus is to identify and collect the needs from the customer and stakeholders from which requirements can be explored, deduced and designed in an organized way. That means clarifying initial relevant requirements aggregated in initial concepts, i.e. representations of feasible and balanced submarines defined as Play-Cards in the functions domain described by Nordin (2009) [25] and further developed by Nordin (2013) [27].

This follows the requirement elucidation approach in general as described by Andrews (2011) [3]. In the second phase, the Conceptual design phase, evaluated and selected technologies, initial requirements, and Play-Cards are further developed to full submarine concepts during concept design in the systems domain, i.e. where selected systems solutions are designed and packed in

submarine concepts. The Systems Engineering approach following Blanchard & Fabrycky (2006) [5] is an interdisciplinary field of processes and methods, with the purpose of providing a holistic view of technical systems. According to Rodgers et al (2012) [33] the Model Based System Engineering (MBSE) approach advocates the use of dynamic system models that evolve with increasing accuracy and fidelity through the project phases, and encourages the use of electronic media and tools.

In Naval architecture, with its historical heritage of physically large and complex systems as defined by Andrews (2012) [4], and especially Naval Integrated Complex Systems (NICS), i.e. submarines and naval ships according to Nordin (2009) [25], the holistic approach has been well suited for the development, design and building of few units and usually without prior prototypes.

By introducing an OA-model a more integrated simulation based design method within the framework of NICS, such as submarines, will be achieved. The OA-model will complement both technical and economic models in a coherent design model in the search for best designs and thereby support the complete system design process in a more coherent way compared to classic design. This OA-model is based upon the idea that it is possible to identify technical parametric dependencies for the design object and related system effectiveness elements, i.e. measure of capability and measure of effectiveness in the functional domain as well as in the system and the installation domains for submarines. When such parametric relationships are successfully deduced, developed and implemented in the modular OA-model in the form of technical parameters, the next

step is to explore the procedure and measure systems effectiveness for Play-Cards (PCs) in the functions domain and concepts in the systems and installations domains. During the generation of effectiveness parameters, the systems capabilities are measured and calculated and their relation to the technical design parameters are traced.

The OA-model shall also be able to act as a stimulator in design for the complete conceptual design procedure during the early phases as described by Nordin (2013 and 2014b) [27/29]. It is essential to integrate not only technical and economic models but also OA in the systems analysis (SA) toolbox in search for best designs. The interactive nature of this new model invites customer participation to OA, as a thorough validation in a more mathematical sense is hard to achieve. This OA approach applied to a life-cycle availability analysis is also possible, but is not further discussed in this paper as it focuses on OA during initial design.

With the introduction of an OA-model in the functions domain, and also in the systems and installations domains, a more rapid knowledge growth will be facilitated in the design process compared to classic basic design procedures, e.g. with their narrow concept exploration, as more system specific knowledge will be available earlier, see Figure 2. This example that follows the nominal Swedish design process given in Figure 1, shows the real cost outcome (solid curve) for the procurement per phase for the Swedish submarine Type A17 project, which consisted of four submarines, and the principal curves (dashed and dotted) for knowledge growth, committed cost and ease of change.

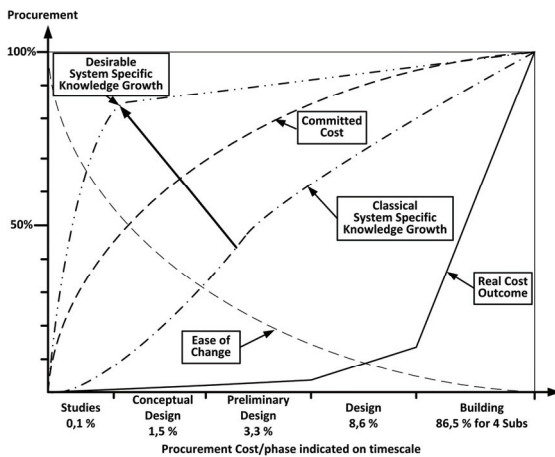


Figure 2: Principal knowledge growth in relation to influence on the design, committed cost and real cost outcomes/phase for the Swedish A17 project, Nordin (2009)[25]

## 2. METHOD FOR THE DEVELOPMENT OF THE OA-MODEL

The approach is based on the generic description of submarines, technical relationships between functional,

system and installation design properties and their performance for some known Swedish and international submarine classes presented by Nordin (2013 and 2014a-b) [27/28/29]. The method contains eight logical steps. Their detailed definition will follow in the relevant sections.

1. Identify international operational and tactical submarine behaviours and observe and separate combat procedures based on different technical solutions and national preferences.
2. Interview commanding officers, observe and analyse actual combat procedures and behaviours based on a set of tactical situations.
3. Develop a set of tactical rules for submarine warfare, especially related to underwater and anti-submarine operations.
4. Differentiate between data and observations i.e. information from:
  - a. Design parameters (DP), Design relations (DR) and technical Measure of Performance (MoP) i.e. Data Elements (DE).
  - b. Measured results in the OA-model i.e. measured elements (ME).
  - c. Calculated aggregated information and results based on DEs and MEs i.e. Calculation Elements (CE).
5. Develop an event and Monte Carlo based simulation model for OA.
6. Identify and extract information, i.e. relevant DEs and MEs, and calculate CEs and form an expression for systems effectiveness as a result of a mission, and form a quantitative MoE.
7. Identify and extract information, i.e. relevant DEs and MEs, and calculate CEs and form an expression for systems capabilities as a footprint of the submarines capabilities based on the result of a mission, and form a quantitative MoC.
8. Validate the model by letting a number of participating operational teams compare a number of tactical situations and document their acceptance.

The detailed result from step 1-3 is usually governed by national secrets act but the general model for tactical decision is described in the relevant section.

## 3. STATE OF ART: OPERATIONAL ANALYSIS OF SUBMARINES

### 3.1 HISTORICAL INTRODUCTION

It is generally accepted that OA was military and operationally introduced by the British government and its armed forces during the Second World War, especially as a means to win the Battle of the Atlantic against the German submarines and their Wolf packs, as reported in Morse & Kimball (1951) [21] and Waddington (1973) [35]. In those days it was called Operations Research (OR) due to its nature of applying

scientific methods in collecting and processing data from the field and in the search for best solutions to different military operational problems, e.g. anti-submarine problems related to the tactical and operational use of weapons and equipment. Thus OR not only helped out in winning the Battle of the Atlantic, it also contributed considerably to the understanding of the nature of submarine and anti-submarine warfare.

After the war, OR became an established methodology in not only military procurement and design but also as an integrated part of industrial management according to Churchman et al (1957) [8]. However, OA was not generally integrated in the design process. After the Second World War a sharp focus was directed towards various model types:

- Manual games theory;
- Analytical models (deterministic models);
- Monte Carlo models (random models);
- Combined analytical and Monte Carlo models (simulation).

As computers were introduced, OA/SA methods grew exponentially during the 1960-80s. During the 1980s, when graphics software matured, complete simulations could be implemented so that random event-driven processes and activities could be used. This enabled the development of analytical and Monte Carlo-based models with graphical display. Game theory now made its entry into computer-based simulations. Game referees were replaced by an embodied simulation engine within which all rules were computerised.

### 3.2 AN EARLY APPLICATION OF OA-MODELS FOR SWEDISH SUBMARINE DESIGN

The Swedish submarine development began to use OA-methods during the final stage of the A11B submarine project during the early 1960s, according to Nordin (2009) [25]. The analysis applied and the models used were based on an engineering approach as given by Gadefelt (1957) [11] in an internal Swedish Defence Material Administration (FMV) report. He systematically describes the various main features impact on a submarine's tactical performance. Gadefelt used the submarine's outer contour displacement in his discussion and based the arguments on the following main features:

- Mobility characteristics, determined by maximum speed and manoeuvrability;
- Endurance, determined by operating range, time at sea, energy storage size and the crew's physical working and living conditions;
- Weapons, type i.e. quantity and quality;
- Information, equipment for surveillance and communication including command and fire control systems;

- Signatures, ability to evade detection, silent speed, degaussing, likelihood of passage through the minefield, etc.

These properties were to return later in the more detailed studies of the OA-models from the 1960s and onwards. The big breakthrough for the use of OA in Swedish submarine design occurred in the beginning of the A14 submarine project during the 1960s. The Swedish Research Institute of National Defence (FOA, later The Swedish Defence Research Agency, FOI) together with The Royal Naval Material Administration (KMF, later The Swedish Defence Material Administration, FMV) developed and The Naval Staff used several of these OA-models for directing submarine design and operations.

The models were used to a substantial degree to find the best solutions and alternatives, including both air dependent propulsion (ADP), i.e. diesel-electric, and air independent propulsion (AIP), e.g. the Stirling system, for the submarine project A14 and its ability to sink ships in an invasion fleet. This extensive use of OA-models for design also generated criticism and discussions. The criticism focused specifically on "the one single model" with only one case scenario forming the basis of analysis for a future submarine system. Most of the criticism came from the national and industrial design teams and was directed toward the following limitations in the OA-model family:

- Too simplified measure of effectiveness– the result–which was the number of sunken ships based on statistics for attack and hit probability;
- Only the offensive attack mission with torpedo against an invasion was analysed;
- Few and standardised tactical scenarios were used;
- Rigid deterministic tactical and combat technical decision models;
- Limited and blunt impact of environmental and technical parameters on the model;
- Lack of simulated human influence on the simulation and its results.

This criticism was focused on tactical relevance including measure of effectiveness. One disadvantage of this discussion was that it cemented the absence of traceable elements that could be linked to the technical design itself and related costs. Before the development of the submarine project A17, the OA-models were therefore modified, to further develop and evaluate the use of alternative submarine missions, such as mine laying missions. System performance was still defined as the ability to sink ships during a given period of time, and so the major limitations of the OA-models still remained. The OA-models were used together with technical data from the parametric models TC 112A (Submarine project A14) and TC 117A (Submarine project A17) which at this time were used by Kockums Yard, as reported in Nordin (2014a) [28]. Within FOA an

extensive modernisation work was conducted during the 1980s, to improve both systems and OA-methods.

When the submarine project UB90 began in the early 1980s, later project A19 Gotland, an extensive modification work of the OA-models was already under way. The result was the OA-model SUB.SIM. This OA-model, in addition to the elimination of some of the earlier limitations, introduced the first graphical interface where the analyst could follow the submarine path on a digital plot.

Altogether, the development of OA-models for submarine systems, from the experiences during the 1960-1980s with the OA-model family to the SUB.SIM model, led to development of models that to a limited degree could be:

- Descriptive;
- Explanatory;
- Explorative.

Faced with the development of the submarine project UB2000 and the challenge of new submarine possibilities, an R&D-project was placed to Chalmers University of Technology (CTH) in 1987, at the Department of Underwater Technology, to develop new design methods on behalf of the FMV Submarine Department. This became a joint effort from 1990 and onwards by the Naval Staff, FMV, FOA and CTH. Development focused on a Simulation Based Design (SBD) approach, in which the submarine was seen as a complete system of systems following the definition of Ackoff (1971) [1] and where technical, economical and operational elements influenced the design parameters, as reported in Nordin (2009) [25].

When, in October 1988, the Swedish submarine project UB2000 was initiated, the options for design and operation were so many that a comprehensive definition of work was needed, according to Nordin et al. (1990a) [23]. This definition was developed jointly by the Naval Staff, FMV and the detachment based at Chalmers. In order to make use of OA experience, FMV, Chalmers, and FOA performed a specific feasibility study together, including development of a deterministic OA-model—Ubat, with the purpose to analyse, document and develop submarine OA-models for design and operation.

The feasibility study, Nordin (1990a) [23] and referred to by Nordin (2009) [25], clearly showed that the submarine system over a relatively long time had been evaluated almost exclusively on the basis of its ability to sink enemy ships, i.e. naval ships, specialised ships for invasion, and amphibious warfare. More directly, the evaluation basis and thus system effectiveness calculation were based on maximizing the number of sunken enemy invasion battalions per invested monetary unit. Surveillance & Reconnaissance, Intelligence, Special operations, and Anti-submarine warfare missions

were not considered. The motive for this was the belief that the armed combat under all circumstances will be the dominating factor for submarine design.

A result of the feasibility study was the observation that the traceability between system performance results and the studied submarine technical design parameters was missing in the former OA-models but also in the newer OA-models. In addition a number now well-known weaknesses and constraints were identified:

- Limited mission profiles for one or two type missions;
- Only attack and mine laying missions against an invasion analysed;
- Limited number of scenarios for interaction with the design objects;
- Lack of simulated human impact on the simulation due to deterministic tactical-combat technical decision models providing rigid tactical and combat procedural behaviours;
- Limited environmental modelling;
- Limited impact from the design objects technical characteristic parameters;
- Limited possibilities for an operator influenced interaction during the simulation.

An observation in the study was that the armed forces requirement office still persisted in defining only detailed performance requirements, rather than to complement these with system effectiveness requirements from OA simulations. The concept of using technical performance parameters like speed, endurance and hit probability for ships/submarines and weapons is easily accessible and something with which both industry and Governmental representatives are familiar with. However, the more abstract system effect – the predicted result of actually using the submarine for its purpose, is closer to the customers' needs. Even if technical performance gives the foundation for the system effect, it can also be reached by different sets of technical performance. With OA-models correctly exploited, and not only for simulation and analysis, i.e. for evaluation, OA-models can also be used for exploration, learning and to get a deeper understanding of realities.

### 3.3 STATE OF ART FOR OA-MODELS IN SWEDISH SUBMARINE DESIGN

Today most scientists, engineers, and economists use classic elements of OA in contemporary computer based analysis and simulation models. Since the introduction of computer based models with high resolution graphics, where results are presented in a communicative form, a wider array of requirements has emerged. The model is not only aiming at final results. It should also give the analyst, including design teams, an opportunity to explore different aspects of the problem. To do this the model must be flexible and adaptable to relevant areas of

interest. The purpose of modern OA is to provide objective fact based results and information, as a quantitative basis for decisions. However, all decision-makers will add their judgement and experience to that basis of information according to Wagner et al (1999) [36]. Following their presentation, symbolically, the relationship of OA and the decision-maker's judgement may be represented as in figure 3.

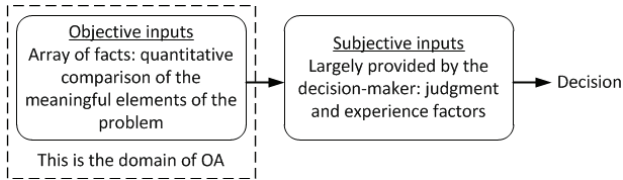


Figure 3: Domains of OA and decision-makers, described by Wagner et al. (1999) [36].

Following the example of Wagner et al (1999) [36] we can outline the general approach of the OA-method in the way described below.

- A. Formulation of the problem
  1. Identification of the objectives of the operation's decision-maker
  2. Identification of reasonable alternative courses of action
  3. Identification of the variables that impact the courses of action
  4. Definition of a measure of effectiveness (MoE), i.e. a quantitative yardstick providing an ordering of the alternative courses of action that is consistent within the objective.
- B. Analysis of the problem
  1. Construct a model of the operation by analytical formulas and/or Monte Carlo simulation that is faithful to reality and amenable to analysis
  2. Evaluate in terms of the MoE, outcomes of the alternative courses of action by exercising the model and by theoretical analysis
  3. Conduct operational trials or observation of "real world" operations to obtain data needed in paragraph 1 and 2 above.
- C. Communication of the results, orally and in writing.
- D. Analyst assistance to implementation of the decision.

It is hard to find anything in the open literature about military OA-models, especially about submarine warfare. However, bits and pieces can be found after the Second World War e.g. Morse and Kimball (1951) [21], Kuenne (1965) [19], Gripstad (1969) [13], Zehna (1971) [37], Waddington (1973) [35] and Wagner et al (1999) [36]. Modern accounts come from Frits et al (2002) [9], Hootman (2003) [14], Hootman & Whitcomb (2005) [15] and Nordin (2009) [25]. For more straight forward technical performance prediction of submarines, the

following sources could be of interest; Burcher & Rydill (1994) [7], Allmendinger (ed. 1999) [2], Van der Nat (1999) [22], Kormilitsin & Khalizev (2001) [18].

#### 4. DEVELOPMENT OF AN OPERATIONAL ANALYSIS SIMULATION MODEL FOR SUBMARINES

Based on the experience gained, which to date is not well documented in open literature, a comprehensive OA-model for integration in the design process were developed and tested.

##### 4.1 REQUIREMENTS ON AN OA-MODEL FOR SUBMARINES

The general requirements on an OA-model for use in a more integrated way in the design process was identified by the author in the feasibility study from 1990a [23] and further developed as reported in Nordin (2009) [25]. These requirements are that the new model should be:

- Descriptive;
- Explanatory;
- Explorative;
- Integrated;
- Flexible;
- Adaptable.

This feasibility study and the report has directly or implicitly highlighted the need for a type of OA-models with the following characteristics:

- Generic structure of missions, scenarios and technical systems;
- Flexible so that both long simulations of complete missions as well as short simulations with intense duels can be performed;
- Adaptable to changes in the course of the analysis, e.g. the adding of new mission types and measure of effectiveness for these in a modular way;
- Introduction of dynamic tactics and combat procedures, and event driven dynamic scenarios and missions;
- Physical descriptions of the 3-dimensional operational environment;
- Parametric descriptions of technical systems;
- Traceability between outcomes (results) and technical solution/system/function;
- Descriptions of human operator's influence on the technical systems' outcomes;
- Ability to lock parameters so that more strict, clear and reduced simulations can be carried out, especially as regards to progressive verification and validation;
- Use of graphical user interface (GUI).

It was identified early in the development process, Nordin (1990b, and in 2009) [24/25], that not only the customer, the stakeholder and the end user but all parties, including the design team, must know the rationale behind the stated needs (i.e. Why).

Thus the development can be dependent on a more general level of knowledge, from the strategic appreciation down to the mission statements, i.e. the different mission profiles based on the mission objectives defined by What, Where and How:

- **WHAT:** What roles and tasks in the different mission types shall the system perform? See Table 1.
- **WHERE:** Where shall the system operate, in which environment?
- **HOW:** How shall/can the tasks be solved? Expressed in mission profiles, within a reasonable technical performance envelop.

A mission profile, e.g. for a Surveillance & Reconnaissance (SR) mission, is illustrated in Figure 4. The submarine starts its mission in a base (Phase A) and sail out to the open sea (Phase B). From there the submarine transits (Phase C) to the operational area (area of interest) for Phase F.

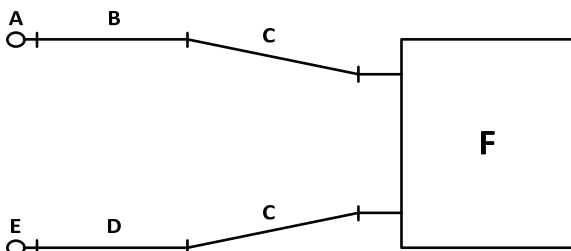


Figure 4: Principal sketch of a mission profile for mission with the phase sequence A-B-C-F-C-D-E.

In the operations area, Phase F, the submarine performs the SR-mission, i.e. Phase F1, see Figure 5 and Table 1 for reference, for the duration of its mission time,  $T_M$  hours. After that, the submarine heads for its base and the sequences of phases are reversed; sails into the base area (Phase D), and finally the submarine returns to its base (Phase E).

In the example of Figure 4, the submarine is searching the area during the mission time,  $T_M$ , i.e. from  $T_0$  to  $T_1$ , trying to find a specific target, a ship, among all other contacts that passes through the area during the SR-mission. The objective can in this case be one- or twofold i.e. (1) to not only correctly detect and classify as many of the ships as possible that pass through the mission area during the mission time,  $T_M$ , but also (2) to detect, classify and positively identify one or more specific targets. In Figure 5 the grey area represents the mission area and the traffic density in all directions. The Target depicted in Figure 5 represents a mission specific target

that needs to be positively identified within the density of ship traffic.

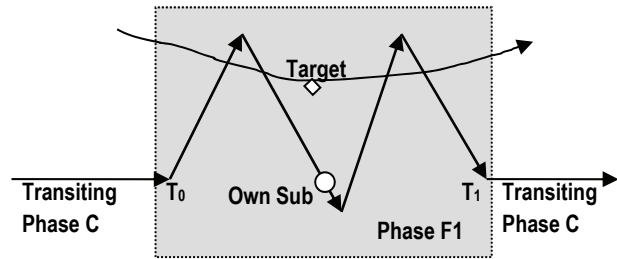


Figure 5: An example of an SR-mission phase F1, including a submarine and one target of interest within the area and time of interest.

#### 4.2 OPERATIONAL ASPECTS OF SUBMARINE OPERATIONS AND CAPABILITIES

The capability for covert operation, beneath the ocean surface, may qualify as the submarine's most characteristic feature, along with the ability to act in a surprising and asymmetric way. These capabilities were the original drivers for the creation and development of submarines. Submarines developed the ability to operate anywhere in the ocean against the sea lines of communications and points of interest during peace time as well as in war. The following presentation is restricted to conventional warfare with SSK/SSG).

The capability of naval forces to project their power by direct and indirect action in different arenas can be described using the basic operational capabilities; command, intelligence, engagement, mobility, protection, and endurance. From a classical naval perspective, these operations are divided into naval operations, maritime peace support operations, and operations other than war:

- Naval operations - Sea Control
  - Securing Command;
  - Exercising Command;
  - Disputing Command or Sea Denial;
- Maritime Peace Support Operations
  - Peace Keeping Operations;
  - Peace Enforcement Operations;
  - Peace Making Operations;
  - Peace Building Operations;
- Operations other than war
  - Humanitarian Support Operations;
  - Civil-Military Cooperation.

The basis for all operational planning is the manoeuvre philosophy. In the multidimensional combat space this means discovering the critical weaknesses of the opponent, subjecting his assets to a rapid and effective intervention, directly or indirectly.

The logic behind the manoeuvre philosophy is based on the principle that one should never confront an enemy head on. The tactic is to find an alternative path or position for reaching the goal from an asymmetrical perspective.

Exposed weaknesses in the opponent's structure are explored and thereafter exploited progressively to achieve a system breakdown of the opponent. This makes the manoeuvre philosophy a more cost-efficient alternative compared to attrition warfare.

Submarines have the ability to stay covert for a long time and by asymmetric behaviour early, forwardly, and with surprise carry out actions against an opponent. These actions may be direct or indirect and can be targeted directly against the opponent's vital points from where the opponent's Centre of gravity can be reached or threatened.

#### 4.3 TACTICAL TASKS AND MISSION TYPES FOR SUBMARINE OPERATIONS

Conventional submarines are fulfilling roles and solve different tasks during various tactical missions. One operation can include several mission types. An example of a representative number of tactical mission types is presented below in table 1. The tactical mission types below put different requirements on the submarine and especially on its combat systems, e.g. weapon, sensor and command systems.

It is therefore important that any evaluation of the submarine operations must recognise the capabilities and effectiveness for the different mission types, if one is to find suitable solutions in the design space.

Table 1: Tactical mission types

| Tactical mission types (F)            | SubOA mission | NATO Abbreviation |
|---------------------------------------|---------------|-------------------|
| Surveillance & reconnaissance mission | F1            | SR                |
| Intelligence & Surveillance mission   | F2            | IS                |
| Special Operations Warfare            | F3            | SOW               |
| Underwater Information Warfare        | F4            | UIW               |
| Underwater Work mission               | F5            | UW                |
| Mine Counter Warfare                  | F6            | MCW               |
| Mine Warfare                          | F7            | MW                |
| Anti-Submarine Warfare                | F8            | ASW               |
| Anti-Surface Warfare                  | F9            | ASuW              |
| Anti-Ground Warfare                   | F10           | AGrW              |

A tactical model, implemented through an Artificial Commanding Officer (ACO), must be able to work with a decision-making process and information model and be

able to replicate how the command and control execute general tactical decisions as well as a palette of different decisions on combat procedures related to the different mission types. See the coming section on tactical decision.

#### 4.4 A FUNCTIONAL DESCRIPTION OF A SUBMARINE'S CHARACTERISTICS

Generically a submarine has the following capabilities as reported by Nordin (2009) [25].

The submarine is an integrated complex system with a crew that can dive and operate under the water surface, i.e. it can safely move and carry out manoeuvres during a long period of time.

Its capabilities depend on precise information on where it is and where it is going. Staying underwater and covert, the submarine shall have the ability to detect and classify selected targets and be able to intercept hostile units with its own systems and weapons or to provide support to friendly units.

If the submarine is detected and attacked it shall, with its endurance, its manoeuvrability and its countermeasures have the capability to avoid fatal damage from hostile actions.

On the basis of the characteristic description of conventional submarine systems briefly described above and reported in Nordin (2009) [25], a functional structure can be assembled as indicated in table 2 below.

Table 2: Aggregated system functions structure

| Aggregated system functions structure (SFS) |   |
|---|---|
| System function                             | Functional description  |
| 1. Hull                                     | Provide a vessel and a watertight environment down to maximum diving depth  |
| 2. Hotel                                    | Provide accommodation for the crew  |
| 3. Protection                               | Provide survivability   |
| 4. Safety                                   | Provide safety  |
| 5. Energy                                   | Provide energy  |
| 6. Propulsion                               | Provide propulsion  |
| 7. Manoeuvre                                | Provide manoeuvre   |
| 8. Navigation                               | Provide navigation  |
| 9. Communication                            | Provide communication   |
| 10. Surveillance                            | Provide surveillance and reconnaissance (in different physical fields; optical, acoustic, magnetic, electrical, etc.) |
| 11. Command, C <sup>2</sup>                 | Provide command and control   |
| 12. Action                                  | Provide action  |

A more detailed breakdown of the system functions structure (SFS) was documented by Nordin et al. (2014b) [29].

An OA-model for submarine operations must be capable of modelling the various tactical mission types and at the same time allow different combinations of technical performance of various Play-Cards, concepts or real submarines and associated combat procedures, as demonstrated in section 5, in the example of an OA simulation.

The effectiveness of the evaluation of these Play-Cards, concepts or real submarines from the results of an OA simulation depends on the ability to trace the connection from tactical results to technical performance (MoP) from the system functions, and their related cost, of the submarine that is evaluated.

There are however different technical solutions for different submarine systems. These differences are linked to the choice of technical design for each submarine system and depend on a combination of the following:

- The submarines' performances, such as underwater speed, endurance, signature.
- The submarines information handling; surveillance, communications, command and control systems.
- Submarine effector systems; weapons, ROV, UUV, divers etc.
- The consequences for the design of the whole submarine.

#### 4.5 MISSION PROFILE AND ITS DECOMPOSITION DOWN TO FUNCTIONS

Based on a stated planned mission profile (1), a decomposition of the profile into phases (2), with planned general activities (PGA) (3) and further subdivided in to planned activities (PA) (4) can be done, as illustrated in figure 6. Having identified the PAs, we can now deduce and allocate the relevant planned functions (PF) and thereby identify the planned functional requirements (PFR) based on the mission profile.

But it is only when the design object executes its mission profile and confronts its surrounding environment (scenario) that the event based tactical requirements can be extracted from the results of an event-driven and Monte Carlo based operations analysis simulation. As a result of the events, new event-based tactical decisions are executed, which generate a set of tactical general activities (TGA) and tactical activities (TA). These in turn requires tactical functions (TF) with additional tactical functional requirements (TFR). We can now compile both the planned (PFR) and the event-based tactical functions and their requirements (TFR).

So, from the planned mission profile a functional flow diagram is developed for the entire system. This diagram provides a structure that is populated with functional requirements; both from PFs and TFs, from the mission profiles. We have then obtained the aggregated functional structure of the overall design object and the

requirement matrix for this mission profile according to Nordin (2014c).

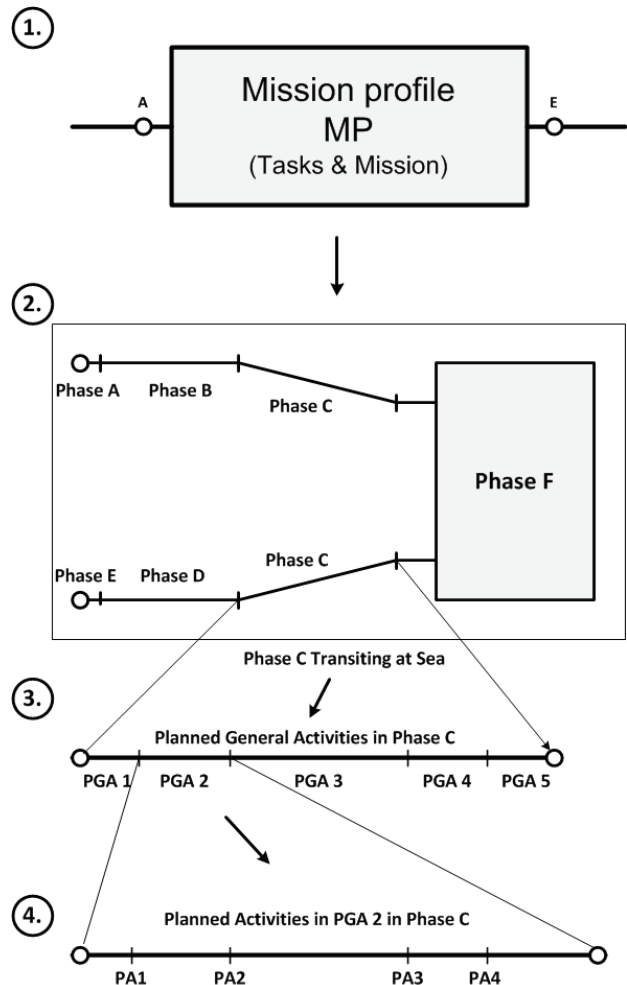


Figure 6: Decomposition of a planned operation profile via phases to planned general activities and activities.

Sensitivity analysis ensures that both the functional structure and its functional requirements are valid for the current conditions. By systematically using this procedure for relevant mission types and profiles, and geographical regions of interest, it was concluded that the operational-tactical-functional requirement space would be identified.

#### 4.6 A SIMULATION MODEL FOR SUBMARINE OPERATIONS

The submarine operations analysis model, SubOA, consists of the following parts:

- An object database of submarines, ships, airplanes and helicopters, operation systems, sensor systems, tactics and weapons, as well as decision-making rules. The database also contains the administrative data that governs and control the simulation;



- A scenario editor for mission and scenario generation;
- A simulation programme for operations analysis and research;
- Results and database management of the elements for the system capability and effectiveness analysis;
- System effectiveness and capability measurement and calculations;
- A test system;
- A report generator.

The SubOA object database (ODB) contains several different assets such as submarines, naval ships including ASW units, amphibious ships and crafts, different kinds of merchant ships, helicopters and maritime patrol aircrafts. The ODB structure makes it possible to assemble an arbitrary actor in a modular way, i.e. an ACO commanding an asset with sensors, weapons, and tactics. Then pose the question “what if...” and assign sensors, weapons, and tactics to an asset and thereby test a new imaginary concept. This can be done for all assets.

Similar flexible and modular procedures apply to geographic areas, missions, and scenarios so that a relevant simulation can be designed and administrative data, such as log directives and seeds for generation of random numbers for the Monte Carlo process, can be set up. The different seeds are also used to recapitulate any simulation later during analysis of the results.

With the scenario editor, a scenario can be designed for a given geographical area of operation including several actors, bases and harbours, bottom based or free-floating sensors, and own and opponents command and control network. The OA-model includes an environmental model which will interact with the different sensor systems involved. The scenario editor is also used to generate the different mission profiles for the different actors.

The simulation is built up around a Monte Carlo based core that addresses simulation order, time steps, and distribution of random numbers to different on-going processes. The simulation core also keeps track of all data going in and out of the simulation, e.g. energy usage for the assets, signatures versus sensor performance, and miss, near hit, and hits with consequential damage.

One essential part of system design is the model for measurement and calculation of the system's effectiveness or the Measure of Effectiveness, (MoE). This is done by using a combination of a physical simulation and event-driven Monte Carlo operational analysis model.

This model can study a submarine's capacity to execute the planned missions in an environment that interacts with the submarine under a set of rules. The submarine's performance and result, i.e. system capability and effectiveness, are measured and calculated. The results are compared and evaluated against the results for other

Play-Cards or concepts. It is possible to run simulations for one Play-Card or concept or a batch of Play-Cards or concepts. A batch can consist of one to several thousands of elementary runs, i.e. one complete run through a mission profile from base to the mission area and back again, see Figure 4 above.

The ODB stores all resulting simulation information for later analysis and recapitulation, including DEs, MEs, and CEs. From the ODB it is possible to go through the simulation and see the difference between the planned mission and the event based simulated mission commanded by ACOs for the different assets. It is also from this ODB that the MoE and MoC retrieve relevant elements for calculation and analysis.

After one elementary run or batch run, it is possible to automatically generate and print out standard reports or retrieve relevant information for a report on a specific topic. In parallel with the simulation model, SubOA, there is also a test module where it is possible to test critical parts before a large batch run, especially regarding the interaction between sensors and signatures and weapons and counter measures.

#### 4.7 A MODEL FOR DETECTING ACTORS IN THE OA-MODEL

One of the more important aspects of the OA-model is its module for detection of the different assets' signatures with relevant sensors. The following signatures are used in the OA-model:

- Radiated noise for different speeds and operational modes;
- Acoustic target echo strength for different aspect angles and frequencies;
- Static and dynamic magnetic signatures;
- Static and dynamic electric signatures;
- Optical and infrared signatures;
- Radar target echo level for different aspect angles and frequencies.

This also includes radiated signatures from active sensors, such as sonar and radar. Based on the Sonar, Radar, and ELFE equations and their different components, introducing Monte Carlo based uncertainties, the model calculates detection ranges, classify contacts, and conduct target motion analysis (TMA) for the decision model. The geographical model supports the sensor-signature model with environmental characteristics and properties relevant for each and every form of signature and sensor.

#### 4.8 A MODEL FOR TACTICAL DECISION AND RULES OF ENGAGEMENT

When the friendly submarine sets to sea it is following a pre-planned mission profile in a given geographic area.

The mission profile will lead the submarine to an operational area, or area of interest.

Once in the area, the submarine has been planned to conduct a certain type of task(s) or mission.

On-board the submarine is an ACO and crew. While going through the mission profile, a number of events will occur due to the fact that the planned submarine mission is interacting with the planned scenario, i.e. events occur that need to be decided upon, see Figure 7 below.

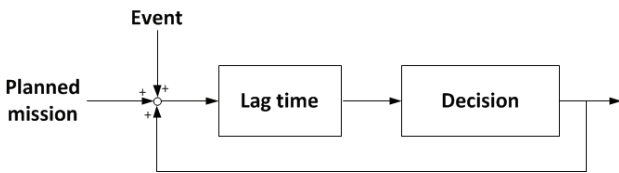


Figure 7: The general decision model. A disturbance in the event based tactical decision machine will generate a response – a decision.

The decision model has two steps. First, based on the type of event, i.e. type of sensor contact, the ACO makes a threat analysis and evaluates the contact against a set of rules and the submarine’s condition.

The ACO can now choose either an offensive or defensive posture or even to disregard the event itself and continue along the pre-planned route. The time to evaluate the event is called “the decision time”.

Second, given an offensive or defensive decision posture, based on the event, the ACO makes a decision on combat procedure based on the following tactical Factors of Influence (FoI):

- Contact list, including classification of contacts;
- Threat evaluation – threat list, including TMA;
- Mission order;
- Rules of Engagement (RoE);
- Own tactical rules and combat procedures;
- Submarine condition
  - Energy status;
  - Signature versus speed;
  - Weapons and countermeasures;
- Environmental factors;
- ACO profile
  - Aggressiveness;
  - Range of wideness in decision.

This decision will affect the complete submarine’s tactical state. Due to the fact that the tactical decision may change depending on, not only a new event, but rather due to changes in FoI, an inner loop within the tactical decision is called for as depicted in Figure 8.

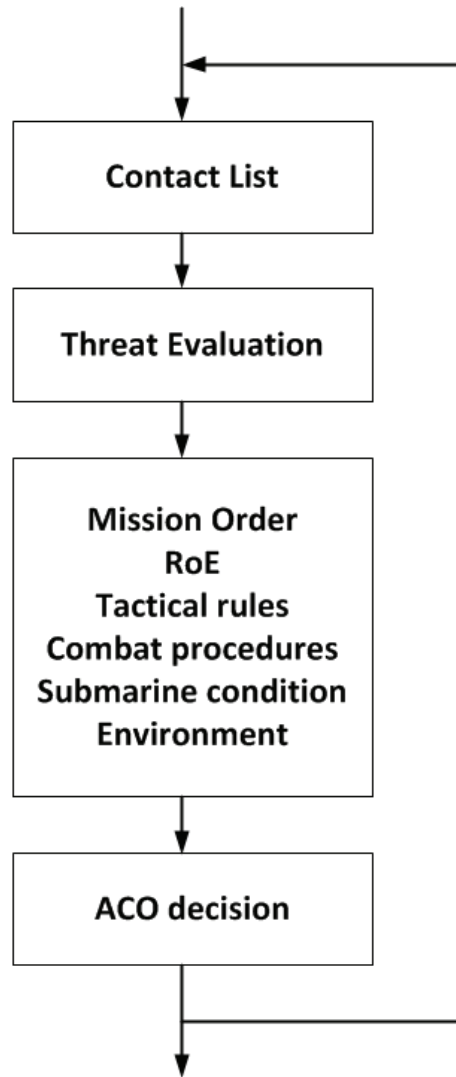


Figure 8: The tactical decision model.

Based on the factors of influence, the ACO decides an appropriate way to act in the specified situation i.e. combat procedures. From this moment, an on-going loop will continue to evaluate any change in FoI that may influence the general decision of tactical behaviour and combat procedures.

When there are no more stimuli from the FoIs, the tactical state is finished and the submarine returns to the pre-planned state. At any time during simulation a human operator can temporarily take over command to test and explore new avenues of interest.

To ensure a diverse behaviour from the ACO, different CO profiles have been observed and systemised into a set of ACO profiles. However, it is also possible to restrict the ACO behaviour to follow the national tactics and combat procedures to the letter. The tactical and combat procedure decision model is designed in a general manner so that it can reflect different national preferences.

#### 4.9 A MODEL FOR HIT AND DAMAGE CALCULATIONS

All movements for actors, including weapons, are simulated dynamically based on their respective manoeuvre characteristics. However, the actual positions of each and every actor are affected by the precision of the sensors and any TMA calculation in progress. As the precision varies, the Monte Carlo approach is introduced to generate variation and uncertainties.

As a result of these uncertainties, a fired weapon is directed against an estimated position, a collision point (CP), according to information from sensors and TMA. For guided weapons, the position estimate will be continuously updated from the controlling platform and the weapon adjusts its course accordingly. Weapons with homing sensors will use them according to weapons rules similar to the platform combat procedures. Weapons rules are defined in the same way as combat procedures and are included in the ACO profile.

The simulation programme calculates the closest point of approach (CPA) between the target and the weapon and determines whether there is a hit, near hit, or complete miss and if the weapon has detonated or not. The simulation program also calculates any damage based on the participants' technical characteristics and properties related to damage when a hit or near hit occurs.

### 5 DEVELOPMENT OF SYSTEM EFFECTIVENESS AND SYSTEM CAPABILITY

#### 5.1 GENERAL ASPECTS OF SYSTEMS EFFECTIVENESS AND CAPABILITY

A normal basis for making a decision is data and predictions based on facts. Such facts should describe the results of the different alternative courses of action that are under consideration. Given different courses of action at least four primary questions should be asked:

1. Which one is the best?
2. How much better is the best one compared to the others?
3. Why was the best one better than the others?
4. How much did the different alternatives cost?

In order to give a rational answer to these questions, the OA must use quantitative measures and assign numerical values to the answers of these questions. Such assignments of values to courses of action must be in agreement with the objective of the analysis and are called Measure of Effectiveness (MoE) according to Green (2001) [12] and Wagner et al (1999) [36]. The properties required of a MoE are summarised as follows:

- a. MoE must be quantitative.

- b. MoE must be measurable or estimable from data and other information available to the analyst.
- c. A significant increase (or decrease) in MoE value must correspond to a significant improvement (or worsening) in achieving the decision-maker's objective.
- d. MoE must reflect both the benefits and the penalties of a given course of action.

Given the ten examples of tactical mission types in Table 1, a MoE can be assigned for each of the mission types, see Table 3.

Table 3: Tactical mission types and related MoE<sub>xx</sub>

| Tactical mission types (Fx)           | SubOA Mission | MoE Abbreviation    |
|---------------------------------------|---------------|---------------------|
| Surveillance & reconnaissance mission | F1            | MoE <sub>SR</sub>   |
| Intelligence & Surveillance mission   | F2            | MoE <sub>IS</sub>   |
| Special Operations Warfare            | F3            | MoE <sub>SOW</sub>  |
| Underwater Information Warfare        | F4            | MoE <sub>UIW</sub>  |
| Underwater Work Mission               | F5            | MoE <sub>UW</sub>   |
| Mine Counter Warfare                  | F6            | MoE <sub>MCW</sub>  |
| Mine Warfare                          | F7            | MoE <sub>MW</sub>   |
| Anti-Submarine Warfare                | F8            | MoE <sub>ASW</sub>  |
| Anti-Surface Warfare                  | F9            | MoE <sub>ASuW</sub> |
| Anti-Ground Warfare                   | F10           | MoE <sub>AGrW</sub> |

It is not only the top decision-maker who needs results from the OA in form of systems effectiveness and systems capability. It is equally important that the design team can continuously evaluate and trace effectiveness of different compromises in the design work, especially regarding results from changing of both requirements and technical solutions.

Therefore it is essential to build a structure that can accommodate both types of decisions. In this paper, MoE is related to the result of a mission, i.e. how well the submarine performed, whereas the measure of capabilities (MoC) is related more to the reasons for the achieved result, i.e. the traceability between results, capabilities, activities, and functions, including dependence on the decisions and execution of tactics and combat procedures that the ACO initiates.

The MoC for a submarine with regards to a specific mission can be viewed as the footprint of the submarine's capabilities. The MoC footprint consists of four main components (MC). The four main components are:

- Action or engagement–capability to produce results (MC<sub>Act</sub>);
- Endurance–capability to operate at sea as a submarine (MC<sub>End</sub>);
- Signatures–capability to stay undetected (MC<sub>Sig</sub>);
- Survivability–capability to survive if detected (MC<sub>Sur</sub>).

These four MCs can be grouped in two pairs and the pairs can be placed orthogonal to each other and generate the MoC footprint as depicted in Figure 9.

The first pair, the two main components action and endurance, competes for the inner volume of the submarine, e.g. volumes for: weapons; command and control systems for the weapons; sensors and communications equipment.

For the action component and equivalent volumes for accommodation, stores, machine controls, machinery, and fuel, etc. for the endurance component.

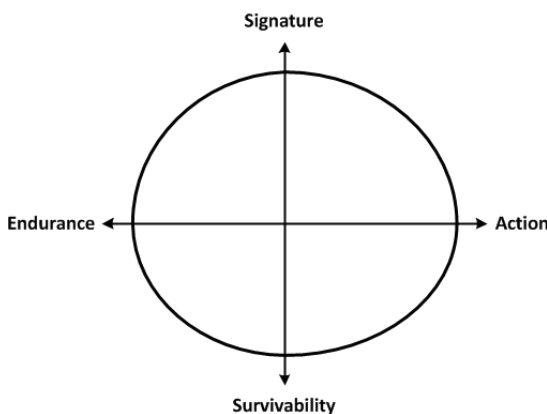


Figure 9: An example of a MoC footprint spanned out by the four MCs organised in two pairs.

In the same way the two remaining MCs, signatures and survivability, compete for the volume of the thickness of the pressure hull e.g. volumes for geometric form, anechoic tiles, degaussing cables and boxes, rafts for single and double mountings for signature components and in the same way volumes for pressure hull, bulkheads, equipment for manoeuvre and counter measures, etc. for the survivability components. The MoE and each MC can be built up with the product of some number  $J$  of calculation elements ( $CE_i$ ) in the general form as described in Equation 1. The same is valid for MoE.

$$MC = \prod_{i=1}^J CE_i \quad (1)$$

where the  $CE_i$  have the general form of;

$$CE_i = f_i(ME_1, \dots, ME_K; DE_1, \dots, DE_L) \quad (2)$$

where the  $ME_x$  and  $DE_x$  are  $K$  and  $L$  selected MEs and DEs, respectively, depending on the relevant CEs.

DEs are built up by design parameters (DPs) and design relations (DRs), e.g. speed-power, speed-operational modes-signatures. The complete list of DEs, MEs, and CEs that the simulation programme SubOA tracks and logs is more comprehensive, so that the analyst can design MoCs, MoEs, MCs, and CEs to correspond to the objectives connected to the different missions included in the relevant OA.

The following hierarchy from design parameters to give Measure of Effectiveness and Capability can be compiled:

- DP, DR, and MoP;
- DE;
- ME;
- CE;
- MC;
- MoE;
- MoC.

## 5.2 DESIGN OF MOE FOR SURVEILLANCE & RECONNAISSANCE MISSIONS – TWO EXAMPLES

During simulation, the submarine goes through the planned mission profile until there is a disturbance, an event, detected by the submarine's sensors. Based on a set of tactical rules, the artificial commanding officer makes a tactical decision on how to act.

The first mission type in Table 3, i.e. the Surveillance & Reconnaissance (SR) mission, this mission type can be described in the following terms (see also Figure 4 and 5 for reference). The submarine departs from base and leaves the coastline and after transiting reaches the operation area, i.e. area of interest. In this example the submarine is searching the area from time  $T_0$  to  $T_1$  trying to find a specific target, a ship, among all other objects in form of contacts that passes through the area during the SR-mission.

The objective can in this case be one- or twofold i.e. (1) to not only correctly detect and classify as many of the ships as possible that pass through the mission area during the mission time  $T_M$ , i.e. from  $T_0$  to  $T_1$ , but also (2) to detect, classify and positively identify one or more specific targets. In Figure 5 the grey area represents the mission area and the traffic density in all directions. The Target depicted in Figure 5 represents the mission specific target that needs to be positively identified.

To be able to identify any weak links in the chain building up the mission specific MoE, it is useful to factorise the chain following Equation 1. Given the objectives above, we can list the MEs of interest:

- Total number of transitors,  $DE_{TR}$ , predefined in the scenario editor;
- How many detected transitors,  $ME_{Dt}$ , that are passing through the area during  $T_M$ , as predefined in the mission editor;
- Number of correctly classified transitors,  $ME_{Cl}$ , during  $T_M$ ;
- Number of correctl TMAs of transitors,  $ME_{Tm}$ , during  $T_M$ ;
- Number of times that the submarine has to hunt and approach (submerged) a detected or classified transitor,  $ME_{Hunt}$ , during  $T_M$ ;

- Number of times that the submarine has enough speed,  $ME_{ESpeed}$ , to approach (submerged) within a distance  $d$  from the ship of interest during  $T_M$ ;
- Number of times that the submarine has enough energy,  $ME_{EEnergy}$ , to approach (submerged) within a distance  $d$  from the ship of interest;
- Number of specific targets that have to be positively identified,  $DE_{ID}$ , during  $T_M$ ;
- Number of specific targets that were positively identified,  $ME_{Identify}$ , during  $T_M$ . In this example not only does the submarine have to take a photo of the ship but also a photo of a readable name of the ship.

From these DEs and MEs given after Equation 2 the necessary MEs and CEs for the  $MoE_{SR}$ , can be given (Equation 1) such that:

$$CE_{Dt} = \frac{ME_{Dt}}{DE_{TR}}$$

$$CE_{Cl} = \frac{ME_{Cl}}{ME_{Dt}}$$

$$CE_{Tm} = \frac{ME_{Tm}}{ME_{Dt}}$$

$$CE_{Sp} = \frac{ME_{ESpeed}}{ME_{Hunt}}$$

$$CE_{En} = \frac{ME_{EEnergy}}{ME_{Hunt}}$$

$$ME_{ESpeed} = \sum_{i=1}^{ME_{Hunt}} g_i$$

Where  $g_i = \begin{cases} 0, & \text{if not enough speed on occasion } i \\ 1, & \text{if enough speed on occasion } i \end{cases}$

$$ME_{EEnergy} = \sum_{i=1}^{ME_{Hunt}} h_i$$

Where  $h_i = \begin{cases} 0, & \text{if not enough energy on occasion } i \\ 1, & \text{if enough energy on occasion } i \end{cases}$

$$CE_{Mb} = \frac{\sum_{i=1}^{ME_{Hunt}} g_i h_i}{ME_{Hunt}}$$

$$CE_{PI} = \frac{ME_{Identify}}{DE_{ID}}$$

The two different MoEs can now be set up:

$$MoE_{SR1} = CE_{Dt} * CE_{Cl}$$

$$MoE_{SR2} = CE_{Dt} * CE_{Cl} * CE_{Tm} * CE_{Mb} * CE_{PI}$$

Alternately, the analyst could choose.

$$MoE_{SR1} = ME_{Cl}$$

$$MoE_{SR2} = ME_{PI}$$

These two MoEs can also act as the two  $MC_{ACT}$  in the two separate MoC footprints.

### 5.3 DESIGN OF MOE FOR ANTI-SUBMARINE WARFARE MISSION – AN EXAMPLE

Following the example of the SR-mission, an equivalent approach can be used for an anti-submarine warfare mission. The objective is to detect, classify and destroy an opponent's submarine in an operational area during the time frame of  $T_M$ . The measures of interest are:

- Total number of opponent submarines,  $DE_{SU}$ , usually only one opponent submarine;
- Number of detected opponent submarines,  $ME_{Dt}$ , that is passing through the area during  $T_M$ ;
- Number of correctly classified opponent submarines,  $ME_{Cl}$ , during  $T_M$ ;
- Number of times that the submarine has to hunt and approach a detected or classified opponent submarine,  $ME_{Hunt}$ ;
- Number of times that the submarine has enough speed,  $ME_{ESpeed}$ , to approach within a distance  $d$  from the opponent submarine of interest;
- Number of times that the submarine has enough energy,  $ME_{EEnergy}$ , to approach within a distance  $d$  from the opponent submarine of interest;
- Number of correctly fired weapons that hit the opponent submarine,  $ME_{Hi}$ ;
- Number of destroyed opponent submarines,  $ME_{Ki}$ .

From these MEs we can put together necessary CEs for the  $MoE_{ASW}$ , including definitions of CEs and MEs from the SR-missions above, such as:

$$CE_{Mb} = \frac{\sum_{i=1}^{ME_{Hunt}} g_i h_i}{ME_{Hunt}}$$

$$CE_{Hi} = \frac{ME_{Hi}}{ME_{Hunt}}$$

$$CE_{Ki} = \frac{ME_{Ki}}{ME_{Hi}}$$

The  $MoE_{ASW}$  can now be set up:

$$MoE_{ASW} = CE_{Dt} * CE_{Cl} * CE_{Tm} * CE_{Mb} * CE_{Hi} * CE_{Ki}$$

Alternately, the analyst could choose e.g. the number of destroyed opponent submarines can be used as a  $MoE_{ASW}$ :

$$MoE_{ASW} = ME_{Ki}$$

The way chosen by the analyst depends on objectives of the analysis and to what degree it is part of a larger analysis. The rationale behind this flexibility and adaptability is to secure the analysts' freedom of designing MoEs and MoCs so that the analysis can be explorative as well as educative. It also gives the analyst the possibility to discriminate unwanted effects and pursue different aspects in a sensitivity analysis.

#### 5.4 DESIGN OF MOC – AN EXAMPLE

In a similar way the three remaining MCs can be built by combinations of DEs, MEs, and CEs as depicted in Figure 10. Whereas  $MC_{Act}$  is a measure of the result of a specific mission,  $MC_{End}$ ,  $MC_{Sig}$  and  $MC_{Sur}$  are complementary measures in support of the mission.

$MC_{End}$  is dependent on the following elements; operational endurance, i.e. time at sea, and tactical endurance, i.e. time in submerged condition without charging batteries in the AIP-mode. Functional reliability for the complete system and subsystems is highly relevant.

$MC_{Sig}$  is dependent on the following elements: radiated noise at different speeds and routines, i.e. normal, quiet, and ultra-quiet routines; target echo strength; static and dynamic magnetic signatures, static and dynamic electric signatures; Radar signature; IR signature.

$MC_{Sur}$  is dependent on the following elements; detection of incoming weapons, counter manoeuvre, countermeasures; and ultimately the vulnerability of the submarine.

The final choice of DEs, MEs, and CEs for the definition of MoEs is the privilege of the different national navies and is usually surrounded by national security regulations.

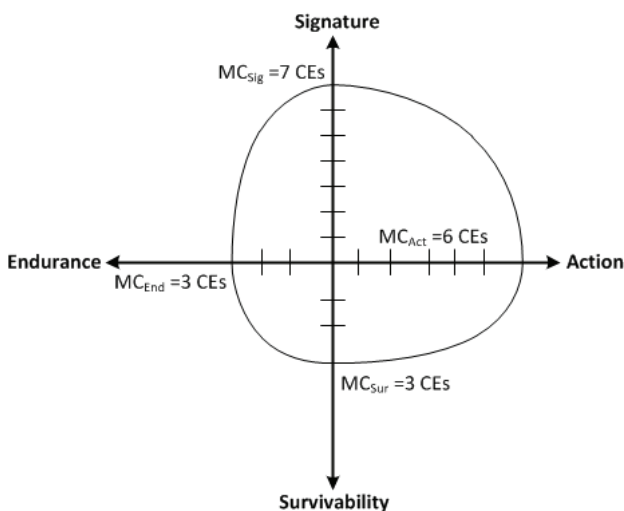


Figure 10: Example MoC footprint with the four MCs and the related number of CEs.

During simulation, the program can also log other MEs and CEs of interests according to the analysts' preference, especially complementary measures in relation to the technical performance, such as sonar performance:

- ME for the number of detected contacts;
- ME for detection ranges of contacts;
- CE for average detection range;
- CE for standard deviation to the average detection ranges.

A similar approach can be used for other systems of interest where there exists a relation between the technical design and the tactical performance and result under given conditions.

#### 5.5 CONVERGENCE OF A SIMULATION – WHEN TO STOP

The simulation will be an on-going process until there are no more events, i.e. the submarine has been sunk or that the submarine is back at its base. During the OA-simulation, the OA-model measures data and stores them in a database for later calculations. The MoC/MoE is plotted for each mission, as is exemplified in Figure 11, with the dimensionless MoE values (crosses) versus run number. The running mean MoE value is drawn with a solid line.

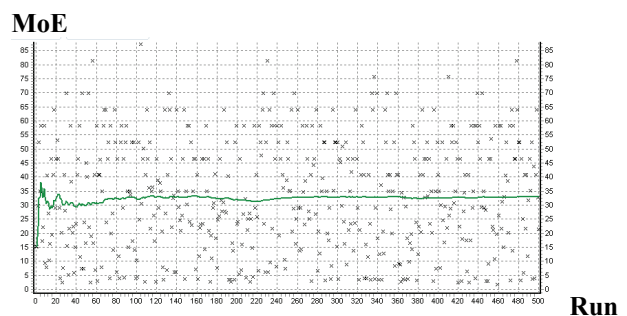


Figure 11: Example of convergence for dimensionless MoE versus run number. Crosses depict the outcome of individual runs, and the solid curve is their running mean.

Depending on complexity, a simulation can contain between 100 to 5000 runs until the running mean MoC/MoE of interest has converged to a stable value. Convergence is taken to occur when the running mean has stabilised, i.e. typically sequential values differ by less than 1%. This final stabilised running mean is taken as the MoC/MoE of the mission. Different tolerances are possible to set depending on the requirement on precision of the analysis.

#### 5.6 DESIGN OF AN OVERALL MOC & MOE FOR ONE MISSION TYPE

As one simulation only consists of one scenario for given circumstances, such as one set of environmental data, it is necessary to add some diversity to this so that a

sensitivity analysis for the different variables can be performed. In the case of the SR-mission several different conditions, and hence simulations, have to be addressed.

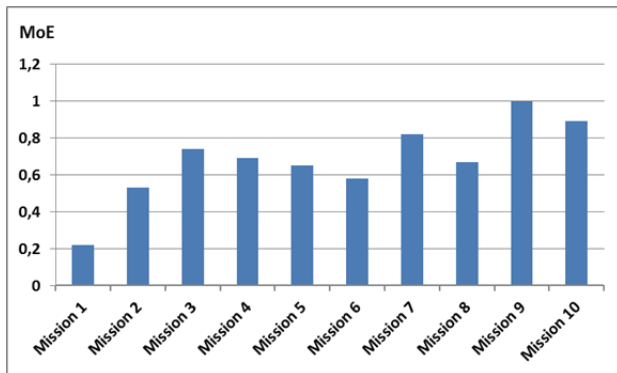


Figure 12: Normalised Measure of Effectiveness results for a submarine based on ten different SR-missions in different conditions (geographic areas, environments, etc.).

Given the objective of finding as many of the transitors that actually passed the given mission area during the mission time, the different simulations can be normalised against the actual numbers of transitors for the mission with the highest result, i.e. mission 9 in the example of Figure 12. This result can vary between zero and one and gives one overall measure of capability and one overall measure of effectiveness for that SR-mission depending on the analyst's choice of CEs for the analysis. The overall measure of capability (OMoC) and measure of effectiveness (OMoE) can then be calculated as the mean for one mission type:

$$OMoC_{Fx} = \frac{1}{M} \sum_{i=1}^M MoC_{Fx,i} \quad (3a)$$

with the standard deviation

$$S = \sqrt{\frac{1}{M-1} \sum_{i=1}^M (MoC_{Fx,i} - OMoC_{Fx})^2} \quad (3b)$$

where M is the number of MoCs, and

$$OMoE_{Fx} = \frac{1}{M} \sum_{i=1}^M MoE_{Fx,i} \quad (4a)$$

with the standard deviation

$$S = \sqrt{\frac{1}{M-1} \sum_{i=1}^M (MoE_{Fx,i} - OMoE_{Fx})^2} \quad (4b)$$

where M is the number of MoEs and Fx is the mission type (F1, F2, ..., F10).

## 6. SIMULATION EXAMPLE

Assume that we are interested in evaluating ten different concepts with different sets of technical performance, Measure of Performance (MoP). The submarine concepts shall also reflect different levels of technology ranging from 1970 to 2010 and 2020.

### 6.1 THE SCENARIO – THE GOTLAND RAID

Assume that a submarine covertly leaves its base in the south-eastern part of Sweden and sails submerged to its patrolling area east of Gotland in the Baltic Sea. The mission is planned for a two week patrol, after which the submarine returns to its base. Assume further that the scenario takes place in modern time in the Baltic. The submarine conducting its patrol is suddenly encounters a north-bound escorted raiding party directed against the island of Gotland. Also assume that the Rules of Engagement permit the use of weapons. As it is a raid, no previous patrols or overt activities, such as intensified air-based ASW patrols, have taken place. In this example there are no opponent submarines present in the scenario.

The raiding group consists of:

- 4 units P1154 Neustrashimy ships;
- 4 units medium sized RoRo ships;
- 4 units small sized RoRo ships;
- 4 units Ka-27 Helix helicopters;
- 2 units IL-38M MPA.

For this simulation example, data for the raiding party has been provided from reference literature, i.e. *Jane's Fighting Ships* and *Jane's Underwater Warfare Systems* from (2009-2010) [34].

### 6.2 THE SUBMARINE CONCEPTS

Ten different submarine concepts will be evaluated. In this example, the following characteristics and performances will be varied:

- Type of torpedoes.
  - Type A torpedoes;
  - Type B torpedoes;
- Number of torpedoes;
- Operational endurance, possible time at sea measured in days;
- Passive signature, radiated noise (acoustic);
- Target echo strength (acoustic);
- Type of submarine sensors.
  - Type A sensors;
  - Type B sensors;
  - Type C sensors;
- Speed given in knots;
- Propulsion.
  - Diesel electric with batteries, i.e. Air Dependent Propulsion (ADP);
  - Stirling/Diesel electric with batteries, i.e. Air Independent propulsion (AIP).

These parameters have been chosen to reflect some of the technology developments that have taken place during the past fifty years and also show the impact of some possible development for the coming ten years. Torpedoes have been preferred as more cost-effective compared to missiles in this scenario, due to torpedoes' greater damage effect and missiles' obvious signatures

which reveal the submarine's position during firing in a littoral sea as the Baltic.

The Type A: Heavy and light weight torpedoes are wire-guided with maximum speed of 40 and 25 knots, respectively, with a passive acoustic homing sensor.

The Type B: Heavy and light weight torpedoes are wire-guided with maximum speed of 50 and 45 knots, respectively, with an active/passive acoustic homing sensor.

The general characteristics for the submarine concepts are presented in Table 4 and the MoP design variations are presented in Table 5. In this scenario only acoustic signatures will be used as the patrol area is located in a magnetically disturbed area and that no bottom sensors are used in the scenario. The magnetic and electric signatures have therefore been omitted. Base-line for the acoustic signatures is older serving submarines from the 1970-80s, from which the broadband radiated noise in spectrum level presented by Miasnikov (1998) [20] and the target echo strength as presented in example from Bossér & Nordin (2014) [6] as a reference level, has been reduced in steps of 3 dB. See Table 5.

To reflect the development on sonars, three different sonar setups are used in the concepts.

The Type A: One cylindrical hull array, diameter 3 m and 1 m high, and intercept sonar.

The Type B: One cylindrical array sonar (CAS), diameter 3 m and 1 m high, and flank array sonar (FAS) length 30 m and intercept sonar.

The Type C: One conformal hull array sonar (CHA), of 1 m height, and flank array sonar (FAS) length 30 m and intercept sonar.

Table 4: General characteristics and design variations for the ten alternative submarine concepts

| Name/<br>Property | Length<br>(m) | Diameter<br>(m) | Submerged<br>displacement<br>(m <sup>3</sup> ) | Operative<br>endurance | Diving depth<br>(m) | Crew |
|-------------------|---------------|-----------------|--|------------------------|---------------------|------|
| AX1               | 50            | 6.1             | 1100   | 14                     | 200                 | 25   |
| AX2               | 50            | 6.1             | 1250   | 21                     | 200                 | 22   |
| AX3               | 50            | 6.1             | 1250   | 21                     | 200                 | 22   |
| AX4               | 50            | 6.1             | 1250   | 21                     | 200                 | 22   |
| AX5               | 52            | 6.2             | 1350   | 30                     | 200                 | 25   |
| AX6               | 52            | 6.2             | 1350   | 30                     | 200                 | 25   |
| AX7               | 60            | 6.5             | 1800   | 30                     | 300                 | 25   |
| AX8               | 60            | 6.5             | 1800   | 30                     | 300                 | 25   |
| AX9               | 63            | 6.8             | 2150   | 45                     | 300                 | 28   |
| AX10              | 63            | 6.8             | 2150   | 45                     | 300                 | 28   |

Table 5: Design variations for the ten alternative submarine concepts

| Name/<br>Property | Weapons load<br>type | Number of<br>weapons | Passive signature | Target echo<br>strength | Sensor type<br>(acoustic) | Speed | ADP or AIP |
|-------------------|----------------------|----------------------|-------------------|-------------------------|---------------------------|-------|------------|
| AX1               | A                    | 8+4                  | 0                 | 0                       | A                         | 18    | ADP        |
| AX2               | A                    | 12+6                 | -3                | -3                      | B                         | 16    | ADP        |
| AX3               | B                    | 12+6                 | -3                | -3                      | B                         | 18    | ADP        |
| AX4               | B                    | 12+6                 | -6                | -3                      | B                         | 16    | ADP        |
| AX5               | B                    | 12+6                 | -3                | -3                      | B                         | 16    | ADP        |
| AX6               | B                    | 20+8                 | -6                | -6                      | B                         | 18    | ADP        |
| AX7               | B                    | 12+6                 | -3                | -3                      | C                         | 16    | AIP        |
| AX8               | B                    | 20+8                 | -6                | -6                      | C                         | 18    | AIP        |
| AX9               | B                    | 12+6                 | -9                | -9                      | C                         | 16    | AIP        |
| AX10              | B                    | 20+8                 | -9                | -9                      | C                         | 18    | AIP        |

### 6.3 SIMULATION SETUP

The simulation is set up by combining the following components in the simulation administration dialog:

- Batch of submarine concepts;
- Submarine mission profile for F9: ASuW;
- Scenario, including.
  - Map, The Baltic;
  - Environment, summer mid Baltic;
  - Conflict level, RoE, high crisis;
  - Tactics and combat procedures;
  - ACO profiles and rules;
- Number of runs: 400;
- The connection table options for building up the modular MoE and MoC.

Following the example of the SR and ASW-missions, an equivalent approach can be used for an ASuW-mission. The objective is to detect, classify and destroy an opponent's surface ships in an operational area during the time frame of  $T_M$ . The measures of interest are:

- Total number of opponent ships,  $DE_{SH}$ , the raiding ships in this scenario;
- Number of detected opponent ships,  $ME_{Dt}$ , that is passing through the area during  $T_M$ ;
- Number of correctly classified opponent ships,  $ME_{Cl}$ , during  $T_M$ ;
- Number of times that the submarine has to hunt and approach a detected or classified opponent ship,  $ME_{Hunt}$ ;
- Number of times that the submarine has enough speed,  $ME_{ESpeed}$ , to approach within a distance  $d$  from the opponent ship of interest;
- Number of times that the submarine has enough energy,  $ME_{EEnergy}$ , to approach within a distance  $d$  from the opponent ship of interest;



- Number of correctly fired weapons that hit the opponent ships,  $ME_{Hi}$ ;
- Number of destroyed opponent ships,  $ME_{Ki}$ .

From these MEs we can put together necessary CEs for the  $MoE_{ASuW}$ , including definitions of CEs and MEs from the SR and ASW-missions above.

The  $MoE_{ASuW}$  can be expressed as:

$$MoE_{ASuW} = CE_{Dt} * CE_{Cl} * CE_{Tm} * CE_{Mb} * CE_{Hi} * CE_{Ki}$$

#### 6.4 RESULTS FROM OA SIMULATIONS

Results from the OA simulations of an Anti-Surface mission (F9) are presented in Table 6 for the following data; MoE,  $MC_{Sig}$ ,  $MC_{Sur}$ ,  $MC_{End}$ , and MoC.

Table 6: OA results for the ten alternative submarine concepts

| Name/Property | MoE  | Signature | Survivability | Endurance | MoC   |
|---------------|------|-----------|---------------|-----------|-------|
| AX1           | 0,49 | 4,88      | 0,76          | 0,93      | 5,91  |
| AX2           | 2,63 | 5,09      | 0,78          | 0,93      | 15,01 |
| AX3           | 2,89 | 4,92      | 0,80          | 0,93      | 15,79 |
| AX4           | 3,39 | 4,90      | 0,80          | 0,93      | 17,50 |
| AX5           | 3,31 | 4,82      | 0,78          | 0,91      | 16,58 |
| AX6           | 6,02 | 5,08      | 0,81          | 0,91      | 27,71 |
| AX7           | 3,35 | 4,76      | 0,80          | 1,00      | 15,19 |
| AX8           | 5,24 | 5,33      | 0,88          | 1,00      | 24,20 |
| AX9           | 2,88 | 5,86      | 0,94          | 1,00      | 18,31 |
| AX10          | 6,24 | 5,89      | 0,95          | 1,00      | 33,73 |

These results can then be normalised against AX1 results to show the differences in result.

Table 7: Normalised OA results for the ten alternative submarine concepts

| Name/Property | MoE   | Signature | Survivability | Endurance | MoC  |
|---------------|-------|-----------|---------------|-----------|------|
| AX1           | 1     | 1         | 1             | 1         | 1    |
| AX2           | 5,35  | 1,04      | 1,03          | 1,00      | 2,54 |
| AX3           | 5,89  | 1,01      | 1,06          | 1,00      | 2,67 |
| AX4           | 6,90  | 1,00      | 1,06          | 1,00      | 2,96 |
| AX5           | 6,73  | 0,99      | 1,03          | 0,99      | 2,81 |
| AX6           | 12,25 | 1,04      | 1,07          | 0,98      | 4,69 |
| AX7           | 6,82  | 0,98      | 1,06          | 1,07      | 2,57 |
| AX8           | 10,66 | 1,09      | 1,16          | 1,07      | 4,09 |
| AX9           | 5,87  | 1,20      | 1,25          | 1,07      | 3,10 |
| AX10          | 12,71 | 1,21      | 1,25          | 1,07      | 5,71 |

These normalised OA results for MoE and MoC are presented in Figure 13.

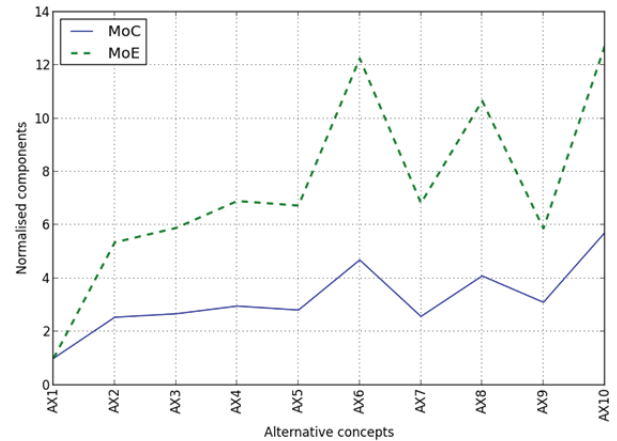


Figure 13: Normalised MoE and MoC results for the ten alternative submarine concepts.

The results for the ASuW-mission MoE clearly show the results dependence on weapon performance and loads for concepts AX6, AX8 and AX10, and the importance of low signatures in combination with sensitive sensors, see Table 5.

The choice of ADP or AIP influences the result to a lesser extent. This can be explained by the fact that no prior ASW patrol, air-borne or ship-borne, is operating before the raiding group meet the submarine and that the encounter is short compared to the battery endurance. Thereby the submarine cannot enjoy the bonus of prolonged submerged endurance that the AIP options give the submarine.

There is no indication that the operational endurance has any effect except for AX1, where the simulated operational length and AX1 operational endurance are of equal length.

In the same way, there is no indication that the diving depth had any effect, which is to be expected as the bottom depth is the same or less compared to the diving depth for the studied concepts.

These tactical results can then be traced to related functions, systems or installations by going backwards from MoC and MoE via MCs and CEs to the individual MEs and DEs/DRs. By tracing the tactical results back to the technical design, with all the requirements and parameters well documented, it is possible to evaluate all requirements from the start of the design process. In this example, with technologies ranging from the 1970 to 2020 it was shown that it is possible to evaluate the impact of alternative technologies, functions and systems through the development process.

## **7. VALIDATION**

The validation was performed according to the Validation Square approach, Pedersen (2000) [32]. The OA-model was exposed to OA-teams from FOA, who participated in the evaluation of the OA-model as well as performed independent audits. These audits were also done by the OA-model design team as well as a group of independent submarine designers and submarine officers. Experiences were documented and implemented as the development process converged to a mature OA-model. The teams concluded that it was useful, behaved as required, and that relevant information could be collected during simulation.

Data Elements (DE), Measured Elements (ME), and Calculated Elements (CE) were logged during simulation in a flexible way and were used to build Measure of Capability (MoC) and Measure of Effectiveness (MoE) in a modular way. The process was incrementally refined as a result of continuous audits during the development.

Real submarines in operation encounter a unique mix of circumstances, which will form new appreciations of missions. It was agreed by the OA-teams that the OA-model could handle upcoming events from adversaries in a tactically meaningful way. The participating operational teams were Submarine Commanding Officers and Executive Officers and the design teams. After initial adjustments it was concluded by these teams, that the OA-model's performance and the tactical rules were realistic.

The same conclusion was also later reached by an independent group of experienced officers, who audited tactics and combat procedures as a complement to the officers and designers who took part in development of tactics and combat procedures for the OA-model. The validation groups have successfully completed their tasks, to test and validate that the new OA-model is useful for its purpose in relation to older OA-models.

## **8. DISCUSSION**

The main purpose of this paper was to introduce new methods of OA for NICS exemplified with submarines. In the paper a new flexible and adaptable OA-model is developed and described. An event based Monte Carlo model was developed. It can simulate submarine missions under different conditions. Technical design and features are utilised to quantify the relation between the technical system and the resulting tactical result quantified by measures of effectiveness.

Based on a systematic hierarchical structure, MoEs and MoCs are built from individual design parameters via different measures and calculations to an aggregated overall submarine effectiveness, for different mission types and under diverse conditions. A further purpose was to relate the different measures not only to the

technical structure in the functional domain, but also to the system and installation domains.

The tactical results could then be traced to related functions, systems or installations. By tracing tactical system results back to the technical design, with all the requirements and parameters well documented, it is considered possible to evaluate all requirements from the start of the design process. In the same way it is possible to evaluate the impact of alternative technologies, functions and systems through the development process.

This method is not a rigid utilization of OA for submarine design. It includes a flexible, modular, and adaptable set of measures so that the analysts and designer can influence the design process to achieve relevant measures of effectiveness related to given objectives. This freedom of design of new measures will also make it possible to more freely explore the different paths of analysis. Thereby new knowledge may be generated.

With the introduction of an Artificial Commanding Officer (ACO) in combination with a developed tactical and combat procedural model, it is possible to produce a flexible operational analysis, including sensitivity analysis of different tactics. During audits and validation it has been shown that the model behaved according to the current appreciation of tactics and combat procedures in general. However, such audits can only address specific scenarios. Commanding Officers change and tactics and combat procedures develop. Thus it was concluded that audits must be used regularly to check the approach's ongoing validity.

The historic cumbersome and time consuming OA runs have now been reduced to reasonable response times within the coherent design method. This is a result of the implementation of a multi-core parallel processing approach, reducing the computational time from years to months and now down to days and hours.

For the final choice of a design, SubOA is used as an evaluation tool by the design team and SubOA is used as a handrail so that all alternative Play-Cards and concepts or submarines of interest, are evaluated in a systematic and consistent way. SubOA is used as part of a toolbox, including not only OA for systems effectiveness but also tools for technical design, cost calculations, and systems analysis. This toolbox, Submarine Analysis (SubAn), is an integrated submarine design and analysis model, which forms the basis of a coherent method utilising a simulation based design approach in search of best designs, see Nordin 2014d [31].

## **9. CONCLUSIONS**

In order to generate more precise decisions and to facilitate a more rapid knowledge growth from the very start of the design process of submarines in comparison

to classic design approaches; a new OA-model has been developed – SubOA. Three main objectives were achieved as a result of the OA-model:

- a) The model produced provided traceability between its tactical results and the technical design of the analysed submarine. The traceability was validated by the SubOA design team.
- b) The analysis was made flexible and adaptable by allowing the analyst to design appropriate measures of effectiveness and capability in a modular way. This approach was established after studies and audits by independent OA-teams.
- c) The adaptable tactical model implemented in SubOA allowed different dynamic tactical behaviours. It was shown through audits with naval officers and validation with the design team that these OA-models behaved in a correct way and the tactical rules and outcomes reflected real situations.
- d) As a result the OA-model, SubOA, has been deemed useful for its purpose of evaluating submarine concepts and real submarines.

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