DISCUSSION

ON THE APPLICATION OF THE EXTREME VALUE THEORY IN SHIP'S STRENGTH CALCULATIONS

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COMMENT

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The problem undertaken by the Author is fundamental for ship safety and was discussed many times in the past (e.g. Boistov, 2000). However, the discussions did not dispel the Author's or my doubts for the same reason: "If the calculations with the Extreme Value Theory of the probability of exceeding of the design hull girder bending moment are correct, ships would suffer more severe casualties than previously observed."

The same arguments as mentioned by the Author crossed my mind with doubts regarding the "two types of statistics: individual amplitude and extreme value statistics" considered with no reference to the theory assumption, or to their applicability in developing safety standards.

Random waves are assumed to be a stationary stochastic process defined as an ensemble of its realizations: $\{\zeta_i = \zeta_i(t),\ i = 1, 2,\ldots,\ t \in [0, T]\}$. It is normally assumed (Ochi, 1998) that random waves (sea state) are:

- a steady-state and ergodic process;
- wave elevations are distributed following the normal probability distribution with zero mean value $\mathrm{E}[\zeta_k(t)] = 0$, and a variance $\mathrm{E}[\zeta_k^2(t)] = \sigma^2$, representing the sea severity;
- wave spectral density $S(\omega)$ is narrow-banded;
- wave peaks and troughs are statistically independent.

Area under the wave spectrum is equal to the variance of the process: $\int_0^\infty S(\omega)d\omega = \sigma^2$, and in practice standardized spectrums are used, let us say the Pierson-Moskowitz spectrum (IACS, 2001), determined by significant wave height, H_s, and the average zero up-crossing wave period, T_z.

For random waves statistical properties of the stochastic process are equal to those for a single realization $\zeta_k(t)$, so we can focus on one realization of the process $\zeta = \zeta(t) \equiv \zeta_k(t)$ – the wave. If we consider the exceeding of a certain level x by wave elevation $\zeta(t)$, its distribution will follow the Rayleigh distribution (Ochi, 1998).

$$\mathrm{F}_X(x) = 1 - e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}, \qquad \mathrm{f}_X(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, \qquad (19)$$

where σ^2 is the variance of the normal distribution of the wave elevation. It describes the wave amplitude distribution.

The wave is composed of n oscillations, represented by wave amplitude or wave height occurring in each oscillation. They compose a sequence of amplitudes $(x_1, x_2,\ldots, x_n)$ – a random sample as the realization represents random process (sea waves) and the statistics of the wave elevation in each cross section of the process for fixed $t = t_f$ is followed by a normal distribution. This sample has a maximum: $y_n = \max(x_1, x_2,\ldots, x_n)$, which is also a random variable. As each element x_i, $i = 1, 2,\ldots, n$ is assumed to be an independent random variable and has the same probability distribution, so the extreme random variable Y_n has the following distribution (eg. Sobczyk, 1992):

$$\mathrm{F}_{Y_n}(y) = \mathrm{P}(Y_n \le y) = \mathrm{P}(\forall_k x_k \le y) = \mathrm{F}^n(y)$$
$$\mathrm{f}_{Y_n}(y) = n\,\mathrm{F}_X^{n-1}(y)\mathrm{f}_X(y). \qquad (20)$$

In the linear theory the above is also valid for the ship response in waves. Consider, e.g. the vertical wave bending moment M_W (in hogging) as a random variable. The impact of the number of wave bending moments m_{wi}, $i = 1, 2,\ldots, n$ in the sample sequence on the extreme wave amplitude probability density function is shown in Figure 1.

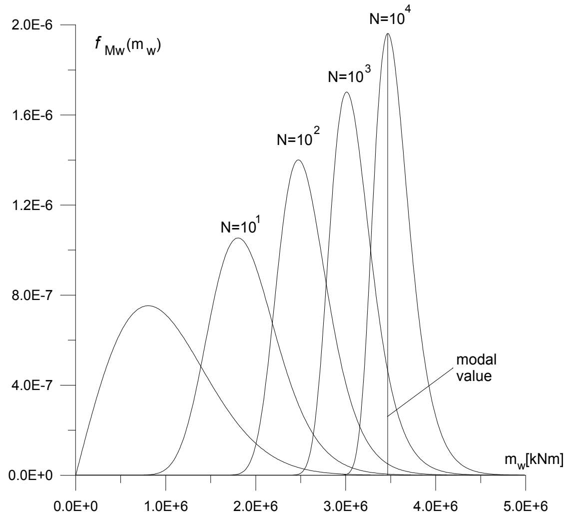

Figure 13. Initial and extreme value probability density function of bending moment M_W in a sea state with H_s = 16.5 m and T_z = 12.5 s

The probability of exceeding a level m_w* in a given sea state (H_s, T_z), called the short-term prediction, is equal to:

$$\mathrm{P}(M_W > m_w^*) = 1 - \mathrm{F}_{M_W}\ , \qquad (21)$$

where F_{M_W} is given by equation (1) if we consider individual amplitude statistic, or

$$\mathrm{P}(M_W > m_w^*) = 1 - \mathrm{F}_{M_W}^n \qquad (22)$$

if we consider extreme value statistics.

The probability of exceeding level m_w* in the long term for the ship response $Y = M_W$ is expressed in the form:

$$\mathrm{P}(\mathrm{Y} > \mathrm{m_w}) = \sum_m \sum_l \sum_k \int_0^\infty \int_0^\infty \int_{m_w}^\infty \mathrm{f}_{klm}\left(y \mid (h_s, t_z)\right) \mathrm{g}(h_s, t_z) \mathrm{d}y \mathrm{d}h \mathrm{d}t p_{kl} p_l p_m\ ,$$

where $\mathrm{f}_{klm} = \mathrm{f}_{klm}(y \mid (h_s, t_o))$ is the probability density function of Y in the sea state condition (H_s, T_z) having the probability density function $g(h_s, t_z)$. Replacing the third integral in the above equation by the probability distribution (3) or (4) and approximating the second and first integral by their partial sums, the following formula is obtained:

$$\mathrm{P}(y > m_w) = \sum_m \sum_l \sum_k \sum_j \sum_i (1 - \mathrm{F}_Y^{ijl}(m_w)) p_{ijl} p_{kl} p_l p_m$$

where:

- p_m is the probability of the ship's loading condition occurrence (different drafts for different loading conditions);
- p_l is the probability of ship presence in sea area A_l, $l = 1, \ldots, n$;
- p_{kl} is the probability of ship course in relation to waves in sea area A_l (uniform distribution in the interval $[0,2\pi]$ is used);
- p_{ijl} is the probability of the short term sea state, determined by (H_S, T_0), occurrence in the sea area A_l. $l = 1,\ldots, n$.

The probability distributions of the sea states occurrence is given in the form of a matrix called the scatter diagram (e.g. IACS, 2001, for North Atlantic), which presents the probabilities p_{ijl} of sea state occurrence in the interval product $[H_{si}, H_{si+1}]_1$ x $[T_{zj}, T_{zj+1}]_l$, $i = 1,\ldots,$r, $j = 1,\ldots,s$, l=1,...,t.

Let us assume one ship loading condition, $m = 1$, one heading in relation to waves, $k = 1$, and one sea area – the North Atlantic, $l = 1$, to simplify computations. Then the probability of exceeding the level m_w* is expressed by the following equations:

$$\mathrm{P}(y > m_w) = \sum_j \sum_i (1 - \mathrm{F}_Y^{ij}(m_w)) p_{ij} \qquad (23)$$

$$\mathrm{P}(y > m_w) = \sum_j \sum_i (1 - [\mathrm{F}_Y^{ij}(m_w)]^N) p_{ij} \qquad (24)$$

Computations were carried out according to equations (5) (or (6)) in the following steps:

1. Simulation of the bending moment m_w (Figure 13) as a response to one realization of the waving process $\zeta = \zeta(t)$, being a superposition of 128 harmonic components of the random waves, determined by (H_s, T_z);
2. Computation of the variance $\mathrm{E}[\zeta_k^2(t)] = \sigma^2$, determining the Rayleigh distribution (19);
3. Computation of the probability of exceeding the level m_w* according to equations (23) and (24).

In applying equation (24) it was assumed that the number of moment cycles $N = 10^4$. The magnitude of the extreme bending moment (and any other response) increases substantially only in the first hour. Computations were carried out for different levels m_w* for a panamax bulk carrier, presented in Table 1.

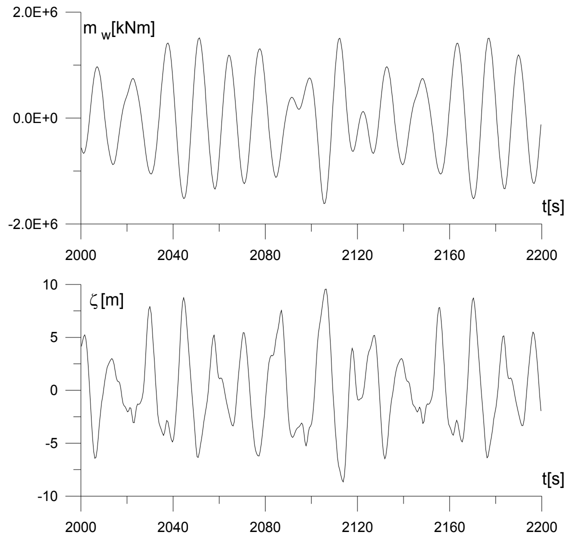

Figure 14. Simulation of the bending moment m_w as a ship response to one realization $\zeta = \zeta(t)$ of sea waves: H_s = 16.5 m and T_z = 12.5 s

Table 1. Probability of exceeding level m_w^*

m_w^* [kNm]	Acc. to (5)	Acc. to (6)
2850000	0.000000044	0.0002618
2900000	0.000000032	0.0002077
2900000	0.000000023	0.0001638
3000000	0.000000017	0.0001285
3050000	**0.000000012**	**0.0001001**
3100000	0.000000009	0.0000775
3150000	0.000000006	0.0000595

Ochi showed that there is a 63.2% chance that the largest wave will exceed the modal value of the extreme value distribution. This modal value is not referred to as a design value. In his book he continued the following reasoning: "This probability is extremely high; hence, it is highly desirable in the design of marine system to consider sufficiently large wave amplitude for which the probability of being exceeded is very small. In other words, a very small α should be chosen which may be called the *risk parameter,* and extreme $\bar{y}_n$ evaluated for which the following relationship holds:

$$\lim_{n\to\infty}\{1-\mathrm{F}^n(\bar{y}_n)\} = \alpha \ .$$

The design extreme value with α = 0.01 is approximately 33% greater than the modal value, but the design extreme value with α = 0.005 is only 4% greater than that with α = 0.01." So there is no need to apply smaller *risk parameter* than α = 0.01, as the safety will not be substantially improved."

Further Ochi wrote: "The design extreme waves with the parameter discussed so far are for a short–term sea state. Suppose a marine system is designed to withstand a severe wave amplitude $\bar{y}_n$ with α = 0.01 in a specified sea, this implies that the design extreme wave provides 99% assurance of safety when the marine system encounters this state once in its lifetime. Therefore, if marine system is expected to encounter seas of this severity five times, for example, in his lifetime, it is necessary to divide the risk parameter by 5 in order to maintain 99% safety assurance throughout its lifetime." (Ochi, 1998)

If we accept the design life $T = T_d$ = 25 years, then the mean number of bending moment cycles for this period is $N \approx 10^8$ and the solution to the equation $\mathrm{P}(M_W > m_w{*}) = 1/N$ (using (23)) gives the value $m_w{*}$ which is exceeded on average once in 25 years. Computations of the probability of exceeding the level $m_w{*}$ (Table 1) show that the probability of exceeding the value $m_w{*}$ = 3.05 10^6, computed according to (23), is equal to 1.2 10^{-8}, while the probability of exceeding this value, computed according to (24), has the *risk parameter* equal to $0.0001113 \approx \alpha/100$, where α = 0.01 is set for short–term sea state and provides 99% assurance of safety. This result may be interpreted that the ship, with the design bending moment of $m_w{*}$ = 3.051^6 is expected to encounter 100 severe seas in her lifetime, on average 4 per year, maintaining 99% safety assurance of the ship hull girder strength throughout its lifetime. Scatter diagram analysis confirms this interpretation for North Atlantic (IACS, 2001).

It should be noted that mainly the high seas contribute to the long–term value.

Referring to the paper, the probability distribution developed with the use of the EVT and presented in Figure 9 should have, in my opinion, the shape as in Figure. 13 of the discussion. Maybe, the constant duration of window applied to 3.5 years of records of measurements caused that the probability distribution is different. Another possible factor causing the difference could be the records of measurements referred to relatively moderate seaway.

It would be interesting if the Author divide the records of measurements into groups corresponding to the steady sea states and assign to them frequencies of occurrence of the sea states in the first step (if it is possible) and then apply the EVT with windows of constant duration to the records corresponding to each sea state. Finally, the probability of exceeding a given level could be computed according to equation (24).

In Table 1, I presented the computations of probabilities of exceeding different levels of the wave bending moments using two different distributions: distribution of amplitude (peak) and distribution of extreme amplitude, and I did not refer to any bending moment level as the design value. Then, I focused on the wave bending moment (level) = 3.05 106, where the probability of exceeding its amplitude is equal to 1.1 10-8 and the probability of exceeding the extreme amplitude is equal to, which wave bending moment is the long–term risk parameter.

The process of determining these probabilities (formula (23) and (24) of the discussion) is an "averaging process", and thus, I interpreted that a vessel is expected to encounter average severe seas condition (bending moment = 3.05 106) 100 times in her lifetime. This means that safety assurance of the ship hull girder is 99% throughout her lifetime, and in this sense, it is the long-term design wave bending moment. In one such sea state the short-term design wave bending moment is much less than 3.05 106 and the probability of exceeding it is equal to 0.01 = α.

I did not extend this interpretation to the probability of exceeding = 3.05 106 by wave bending moment amplitude (equal to 1.1 10-8). This level is exceeded in average once in 25 years. It can be showed that the probability of exceeding value (level) = 3.05 106 at least once is 63.2%, at least twice is 26.4%, and so on.

Still we have two different random variables with different distributions and interpretations. The only relation between them is that the probability distribution of the wave bending moment amplitude is the initial distribution of the extreme wave bending moment amplitude.

The discussion stirs up the problem of defining design value (e.g. wave bending moment). Prof. Ochi says that the predicting of extreme values is "invaluable" as it "provides information for the design and operation of marine systems", which means that the design value of marine systems is the predicted extreme value. Thus, design value is strictly defined. However, this is not the only one possible approach.

But the key issue of your paper is the statement: "the major reason for these huge differences is in the fact that only the maximal values within each time window are extracted without taking into account the differences in their probabilities of occurrence". That conclusion is not fully clear for me and therefore I did not comment it in the discussion. For example, if we have the probability density function, the probability of occurrence of the random variable in a certain interval or domain is determined as an integral of the probability density function over this interval/domain. Referring to this definition, I have problems in understanding the way of arranging the "Groups of probabilities" and why the POO of the random variable within each group "i" is formulated as:

$$10^{-(i+0.5)} \leq \text{POE of max } M_{W,h,i} \leq 10^{-(i-0.5)}$$

where i = 1, 2, 3, 4, 5, etc. (I cannot see the backup theory behind this formulation)

Concluding, on the level of my understanding (my profession is hydrodynamics), "the huge difference" is the result of different random variables with their different distributions and interpretations. It does not mean that the wave bending moment determined by class rules is incorrect but that it may require relevant defining of the design value.

The main dimensions of a Panamax bulk carrier, which I used to carry out simulations and computations of the probability distributions are as follows: L = 227m; B = 32.5m; T = 12.5m and Cb = 0.8. The wave bending moment determined by the class Rules for the ship is equal to 2.76 106 kNm. It is normal that computations, necessarily based on simplified models, yield slightly higher results and they should be calibrated against the reality.

I agree that random variables of a sequence of the sample: (X1, X2, …. Xn), used to determine the maximum value distribution, are not fully independent (I computed the autocorrelation function which supports this observation) and they are quite identically distributed (this is the main assumption of the random wave theory).

In my opinion the data from records of wave bending moment over a period of around 3.5 years should be divided into steady sea states with assigned to them probabilities of occurrence. And then the time window of constant duration could be used to determine the maximum value (one maximum value for one time window). They should create numerical distribution of the maximum value for each sea state. Then, using these distributions and the probabilities of sea states occurrence, the maximum value distribution for the 3.5 year period can be computed. However, I do not know whether such computations are possible using the records; if yes, they could be used to verify theoretical models.

AUTHOR'S REPONSE

The description in the discussion of the ideas in the extreme value statistics may serve as a good illustration of the fundamental difference between the individual amplitude statistics and extreme value statistics. The former considers all available values of the random variable (in the example – wave bending moments) while the latter considers only their maximal values within given time windows. The considerable disparity in the different treatment of the random variables explains the differences of several orders of magnitude in the probability of exceeding the design wave bending moment (See Table 1 of the discussion). In this author's view, the major reason for these huge differences is in the fact that only the maximal values within each time window are extracted without taking into account the differences in their probabilities of occurrence.

Faulkner and Sadden (1979) wrote: "…we may note that for the special case where the wave bending moment is chosen such that Q = 1/n (which leads to the most probable extreme moment) the probability of exceedence is $1 - e^{-1} = 0.632$". The design wave bending moment given in the class Rules relates to wave bending moment that can be exceeded only once within 100 million cycles, i.e. $n = 10^8$ and $Q = 10^{-8}$.

Prof. Ochi introduces a *risk parameter* to be used for selection of a new design wave amplitude $\overline{y_n}$ for which Q = *risk parameter a.* He does not directly refer to Q = 0.632 as a value corresponding to the class Rules design bending moment. However, there are some attempts to treat the mode as a class Rule for the design wave bending moment using the reasoning of Faulkner and Sadden given above and Prof. Ochi's example for changing the design wave amplitude $\overline{y_n}$ until the probability of its exceedence becomes equal to the *risk parameter a.*

In the discussion, *a* = 0.01 for short term distribution and Q = 0.01. For the Panamax bulk carrier in the discussion, the probability of exceeding the design wave bending moment calculated by Eq. (5) is 1.2×10^{-8} and 1.001×10^{-4} when Eq. (6) is used. It is shown that the ship will meet 100 severe sea states in her lifetime (This conclusion can be referred to any sea-going ship designed with the present class Rules for the design wave bending moment.) In the class Rules, the design wave bending moment (obtained by individual amplitude statistics) is assumed to be exceeded only once in a ship's lifetime. What could one expect to happen if one designs a ship with wave bending moment that is assumed to be exceeded only once in her lifetime but in reality happens 100 times in her lifetime? The expectation is that ships would suffer numerous structural failures during their service life. However, the experience from real ships' operation does not support such an expectation.

In moderate sea-states, the wave bending moments are not so large but they are met more frequently. The reverse is true for high sea-states. Most of the analyzed available records refer to relatively moderate sea state. This explains why Figure 9 is different from Figure 1 in the discussion. It also explains why there are only a few bins in Figure 7 and Figure 8. (There are five bins in Figure 6 but the numbers within them are so small that the graph does not show it.) However, the insufficient number of records from high sea-states does not change the major point in the Technical Note – in the statistical analysis, one should consider not only the maximal values of the random parameters in each time window but also one should take into consideration the probability of their occurrence.

The proposal for rearranging the time windows' duration to correspond to the sea states occurrence and applying after that the extreme value theory is very interesting and useful. If time permits, it can be realized in my future work.

During my work and studies, I have become familiar with the work of Prof. Boitsov and his appeal to the class societies to accept design wave bending moment with a relatively higher safety margin compared to the factor chosen for the still water bending moment. As to his publication referred to, there was no agreement in the discussion section of the published paper (Boitsov, 2000). It does not seem there is an agreement at this time as well.

I think the class Rules for the design wave bending moment of the Panamax bulk carrier will be very close to $m_w^* = 3.050.000\,[\text{KN.m}]$ you have used in the example in Table 1. If individual amplitude statistics is used, the probability of its exceedence once will be around 10^{-8}. If extreme value statistics is used, the probability of its exceedence at least once will be 63.2%, at least twice - 26.4%, etc. (Your calculations with the formulas of the extreme value theory would provide similar numerical results for any value of the wave bending moment in Table 1 around $m_w^* = 3.050.000\,[\text{KN.m}]$). Again, we have returned to the basic question about the reason for the huge difference between the numerical results obtained by individual amplitude statistics and extreme value statistics applied in the calculation of the probability of exceedence of the Rules design wave bending moment. The calculations with the extreme value statistics are correct if the basic assumptions in it are fulfilled - i.e. when all X_1, X_2,…X_n (see Eq. (19)) are independent and identically distributed random variables. However, in the general case, they are not.

In the Technical Note, data from records of wave bending moment over a period of around 3.5 years have been used. The effect of several durations of the time windows has been analyzed. For each of them, the corresponding probabilistic distribution of the wave bending moment has been found with a specialized computer program "EasyFit". The calculations have shown that the probabilistic distributions within each time window were not identical. Hence, the probabilities of exceedence of the maximal value in each time window are different but this is not taken into consideration in the traditional interpretation of the extreme value statistics. So, the key question remains unanswered, i.e. shall we continue taking into consideration only the maximal value from each time window or shall we also consider its probability of occurrence?

Your proposal for another way of analyzing the records of wave bending moment is interesting and reasonable. If time and circumstances permit, one could try to implement and test it.

ADDITIONAL REFERENCES

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