TECHNICAL NOTE

AN APPROXIMATE METHOD TO PRESENT THE BUCKLING STRENGTH OF A GRILLAGE IN A PROBABILISTIC FORM

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SUMMARY

An approximate method for calculation in probabilistic terms of the buckling strength of a grillage under unidirectional in-plane compression is proposed. The geometric properties of longitudinals and transverses and the mechanical properties (yield stress and modulus of elasticity) of the material they are built from are treated as random parameters that may change over ship's service life. The cumulative distribution function of the grillage's critical buckling strength is calculated by using an analytical formula for multitude sets of input parameters while all of them having the same level of certainty. The assumption is that the critical buckling strength has the same (or very similar) level of certainty as that of the input parameters.

The accuracy of the proposed approximate method is relatively high (the maximal error is around 2%). It is recommended for use when specialized computer programs for application of Monte Carlo simulation method are not available. The method does not require a complicated specialized computer program and can be run on EXCEL computer program.

1. INTRODUCTION

The reliability of ship's hull structure depends on many parameters the majority of them being of a random nature. Each of these parameters has its own probabilistic distribution. So, the question may arise as to what is the probabilistic distribution of the end result of the calculations. The end result might be the reliability determined by several failure modes (e.g., buckling, ultimate strength, fatigue strength, etc.). If the end result can be expressed in an analytical closed-form format, the Monte Carlo simulation method can be used to calculate the probabilistic distribution of the end result corresponding to the applied failure criterion. The calculations require availability of a specialized computer program which is not always the case. In such a situation, an approximate method for probabilistic presentation of the end result is proposed here. It is applied to buckling strength calculations of grillages loaded by in-plane compression.

Many works have been published on buckling (or ultimate) strength estimation of grillages (gross panels). A very comprehensive review of these publications was done in [3]. To avoid repetition, one should only note that all publications treat the strength of grillages in a deterministic format. Its probabilistic treatment can be done by applying the Monte Carlo simulation method provided an analytical procedure for calculation of the buckling (or ultimate) strength of grillages exists. The method itself is well established and used in many industries. However, there are still cases when the engineer does not have a computer program for its application. Therefore, an approximate simple method was developed for such situations.

The method is based on the idea defined in [6] the very essence of which is: If all input parameters of the calculations have exactly the same probability of exceedance (POE), the result of the calculations will have POE very close to that of the input parameters. This approximate method was applied in [2] for calculation of the buckling strength of a grillage in probabilistic terms, considering the effect of corrosion over ship's service life. Here, comparison between the results obtained by Monte Carlo simulation and the proposed approximate method is given in order to clarify the error occurred in the calculations by the approximate method.

2. CALCULATION OF GRILLAGES' BUCKLING STRENGTH UNDER COMPRESSION IN DETERMINISTIC FORMAT

Detail explanation of the analytical method developed for assessment of the buckling strength of different types of grillages (in a deterministic format) can be found in [2]. Here, the calculations in a probabilistic format are applied to grillage configuration shown in Figure 1. The unknown critical buckling stress of each longitudinal can be calculated by the quadratic equation derived in [2]:

$$r_0 \sigma_{cr}^2 + r_1 \sigma_{cr} + r_2 = 0 \tag{1}$$

where:

σ_{cr} = critical buckling stress of each longitudinal

$$r_0 = -A_j \left(\varepsilon \sigma_y + C_M A_j \right) \tag{2}$$

$$r_1 = \sigma_y \left[A_j \left(\varepsilon A_M \sigma_y + A_j B_M \right) + \Delta \right] \tag{3}$$

$$r_2 = -\Delta A_M \sigma_y^2 \tag{4}$$

$$\varepsilon = \frac{f a^2}{\pi^2 E i} \quad (5)$$

$$A_j = \left(\frac{j}{n+1}\right)^2 \quad (6)$$

$$\Delta = \left(\frac{\mu}{\pi}\right)^4 \left(\frac{a}{L}\right)^3 \left(\frac{b}{L}\right) \frac{J}{i} \quad (7)$$

A_M, B_M, C_M = coefficients depending on the yield stress. In the example, they refer to shipbuilding high tensile steel with yield stress of 39.2 KN/cm^2. The data are taken from [2].

f = longitudinals' cross sectional area including the attached plate. The width of the attached plate is determined following the Classification Societies Rules (e.g. [1]). Buckling of attached plate and flanges is not considered;
a = spacing of transverses;
E = modulus of elasticity;
i = moment of inertia of longitudinals;
n = number of transverses;
b = spacing of longitudinals;
L = length of transverses;
J = moment of inertia of transverses
j = number of half-waves when deck longitudinals buckle

$$\left(\frac{\mu}{\pi}\right)^4 = \frac{1}{0.99 - 0.27\zeta - 0.51\zeta^2} \quad (8)$$

$$\zeta = \frac{1}{1 + \frac{\alpha E J}{L}} = \frac{1}{1 + \delta \frac{l_1}{L} \frac{J}{J_1}} \quad (9)$$

$\alpha = \frac{l_1}{J_1} \frac{\delta}{E}$ coefficient of pliability at transverses' ends (10)

l_1 = length of side frame
J_1 = moment of inertia of side frame including attached plate
$\delta \approx 0.29$ (coefficient depending on the boundary conditions at transverses' ends). Its upper boundary is 1/3 and the lower one –1/4. Here, the average value of 0.29 is used in the calculations.

Applying an iterative procedure, one should solve Eq. (1) for each A_j (i.e. for each j which represents the number of half-waves when deck longitudinals buckle) and find out the minimal root with physical meaning, i.e.

$$\sigma_{cr} = \frac{-r_1 - \sqrt{r_1^2 - 4 r_0 r_2}}{2 r_0} \quad (11)$$

Based on authors' experience, one could recommend the upper limit of j as j = 6.

To obtain the critical buckling force, one should multiply critical buckling stress of one longitudinal, σ_{cr}, by the cross section area of all longitudinals.

Eq.(1) was derived using the linear stability theory of a grillage consisting of plating, longitudinal and transverse beams. According to the applied beam theory, the plating is replaced with equivalent plates attached to the longitudinals and transverse beams. This allowed for application of the beam theory to the entire grillage structure. Also, a correction factor was introduced to account for deviation from Hooke's law due to plasticity. Thus, it became possible to calculate the critical buckling stresses when the Euler theoretical buckling stresses are known. The subject was investigated in [2] where the relationship between the material's yield stress, critical buckling stress and Euler theoretical buckling stress was given based on experimental data for different shipbuilding steel.

Results obtained from the beam theory (without the plasticity correction) have been verified by applying FEM simulation of a 3-D grillage elastic structure. The focus of the study was to find Euler theoretical buckling stress and critical moment of inertia of the deck transverses for a particular configuration of a deck grillage (Critical moment of inertia is a value above which grillage's buckling stress does not increase.) The FEM model has been assumed as linear in terms of material properties, geometry and loads. Systematic FEM simulations showed that in situations when the moment of inertia of transverses was lower than the critical value (which is desirable in grillage design) grillages buckled in "overall" mode, consistent with Euler instability modes typical for plates supported by beams.

3. INPUT PARAMETERS

The input parameters used here are very close to those used in [2]. The location of each girder and its length are treated as deterministic values. The geometric properties of longitudinals and transverses and the mechanical properties of the steel are treated as random variables which follow Gaussian distribution. In the example, the calculations are performed for as-built and 20 year old sample ship.

Parameters treated as random variables with mean values and standard deviations of Gaussian distribution:

For as-built-ship

	Mean	St. deviation
J	48807 cm^4	640.6 cm^4
i	598 cm^4	8.7 cm^4
σ_y	41.50 KN/cm^2	2.41 KN/cm^2
E	20600 KN/cm^2	1235 KN/cm^2
f	35.4 cm^2	0.47 cm^2
J_1	2700 cm^4	39.3 cm^4

For year 20

	Mean	St. deviation
J	41401 cm^4	1905.6 cm^4

i	511 cm^4	18.3 cm^4
σ_y	39.425 KN/cm^2	2.29 KN/cm^2
E	19570 KN/cm^2	1235 KN/cm^2
f	30.0 cm^2	1.38 cm^2
J_1	2308 cm^4	82.7 cm^4

Parameters treated as deterministic values (see Figure 1):

L = 1050 cm	a = 215 cm
l = 1505 cm	b = 35 cm
l_1 = 250 cm	n = 6
A_M = - 0.059	B_M = 1.474
C_M = 0.853	

The probability density functions (PDF) of the input parameters treated as random variables are shown in Figure 2 – Figure 7.

Data for the reduction due to corrosion of the yield and tensile stress over ship structures' life are given in [4]. Depending on the type of steel, the yield stress could be reduced by 16 - 40%.

Data for the reduction due to corrosion of material's modulus of elasticity over ship structure's life are given in [5]. These data show reduction of the modulus of elasticity in the order of 40 – 50% (relative to nominal values).

In the example, 5% reduction of the modulus of elasticity and the yield stress is envisaged as the first attempt to evaluate the sensitivity of the critical buckling stress to changes of the input data.

4. CALCULATIONS OF THE PROBABILISTIC DISTRIBUTIONS OF THE CRITICAL BUCKLING STRESS OF ONE LONGITUDINAL BY THE APPROAXIMATE METHOD

The critical buckling stress of each longitudinal was calculated by Eq. (11) using input parameters with multitude of levels of certainty (LOC) such as, e.g., 0.01, 0.05, 0.10, 0.20, etc. Thus, the cumulative distribution function (CDF) of σ_{cr} was built. Numerical results of the calculations are given in Table 1, Table 2 for j = 3 which determines the root of Eq. (1) with physical meaning (The results refer to 20 year old sample ship).

Differentiating the so derived CDF, one can calculate the corresponding PDF of σ_{cr}.

To evaluate the error resulting from application of the proposed approximate method, parallel calculations were performed by Monte Carlo simulation method. The results are illustrated in Figure 8. One can observe the fact that the PDFs and CDFs derived by the two methods are very close. To be more precise in evaluating the error implemented in the proposed approximate method, Table 3 provides data for the error all over the CDF of σ_{cr}. The maximal error for as-built ship is 1.13% and 2.27% for 20 year old sample ship. Such accuracy is acceptable for practical application of the proposed approximate method. Nevertheless, caution is required because this relatively high accuracy is typical only for cases when any change of any random parameter causes change of the final result in the same direction. Naturally, one should have an analytical procedure for calculation of the targeted result (e.g., buckling stress as in this example) and use it to perform sensitivity analysis before applying the proposed approximate method. Fortunately, there are many engineering problems that are very similar to the problem under consideration here which expands the area of its application beyond the problem under consideration here.

Once the accuracy of the proposed approximate method was confirmed, the PDFs and CDFs of σ_{cr} were built for as-built ship and 20 year old sample ship (see Figure 9). These functions allows for calculating the probability of exceedance or non-exceedance (PONE) of any given value of σ_{cr}. As an example, the probability of σ_{cr} being smaller than 95% of the nominal value of σ_{cr} was calculated. It was found that the probability of σ_{cr} being smaller than 95% of its nominal value (i.e. for the design or as-built value) calculated by the approximate method is 9.0% and by the Monte Carlo simulation – 11.9%.

The so derived CDFs of σ_{cr} can be of help in case when the input parameters in the calculations have different LOC. Thus, the question arises as to what is the LOC of the final result? Such a situation is quite typical in engineering calculations not only in the case under consideration here. Figure 10 illustrates how the LOC in such situations could be determined. In the example, the crossing point of the two arrows (one for the calculated P_{cr} and the other one – for given year of the ship's service life) determines the corresponding LOC of the calculated P_{cr}.

5. CONCLUSION

An approximate method for calculation in probabilistic terms of the buckling strength of a grillage under unidirectional in-plane compression is proposed. The geometric properties of longitudinals and transverses and the mechanical properties (yield stress and modulus of elasticity) of the material they are built from are treated as random parameters. Any change of each of them causes change of grillage's buckling in the same direction. Under these conditions, the accuracy of the proposed approximate method is relatively high, i.e., the maximal error is around 2%. It can be recommended for use when specialized computer programs for application of the Monte Carlo simulation method are not available. The method does not require a complicated specialized computer program and can be run on EXCEL computer program.

The basic idea of the proposed approximate method is building the CDF of the final result assuming that the level of its certainty is the same (or very similar) to that of the input parameters while all of them have the same LOC.

It is also concluded that the method could be applied for solving other engineering problems if any change of any input parameter causes change in the same direction of the final result. Major requirement for its application is the availability of an analytical method for calculation of the corresponding physical characteristic.

6. REFERENCES

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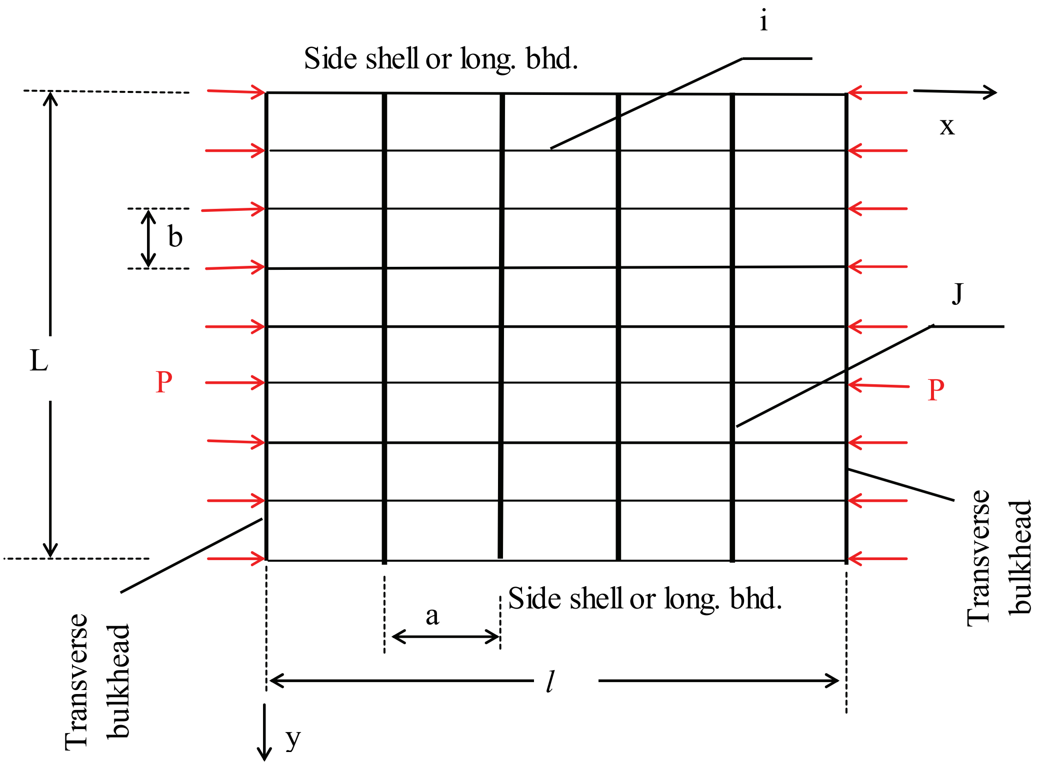

Figure 1: Grillage configuration used in the calculations

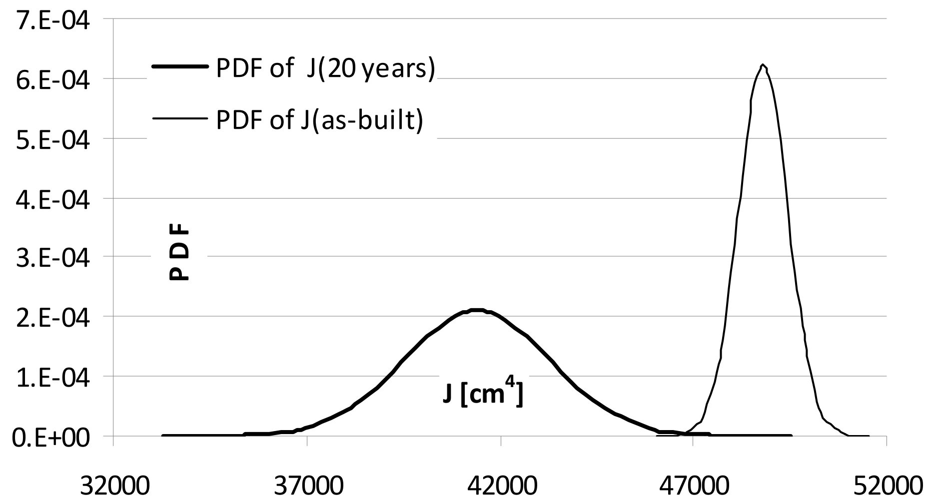

Figure 2: Probability Density Function of J (moment of inertia of transverses)

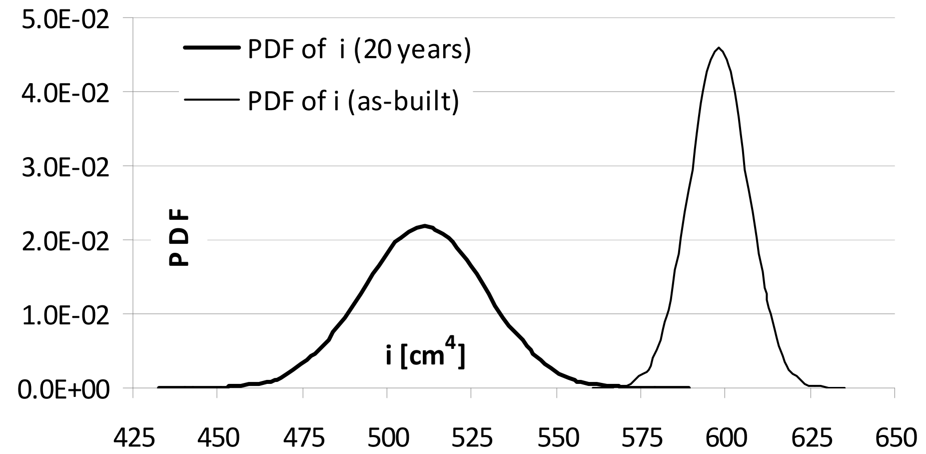

Figure 3: Probability Density Function of i (moment of inertia of longitudinals)

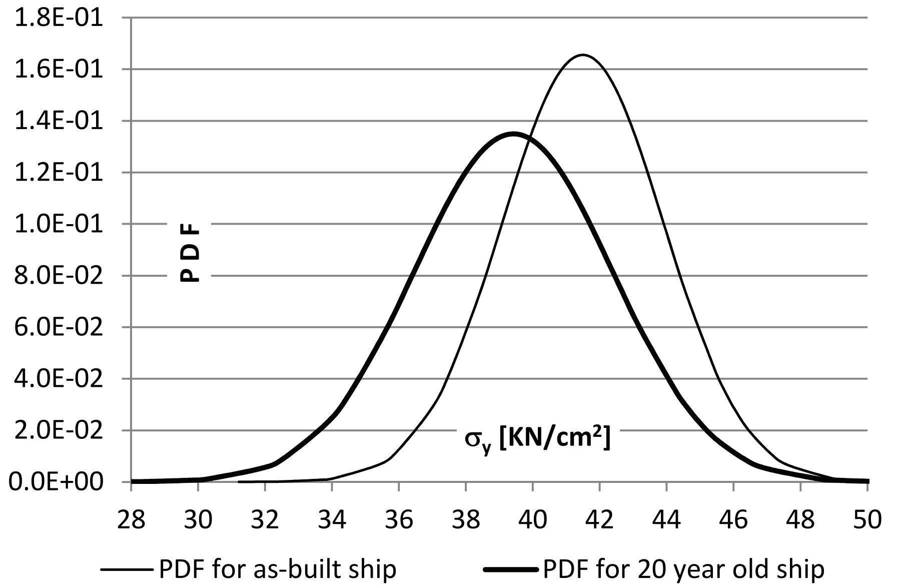

Figure 4: Probability Density Function of σ_y (yield stress)

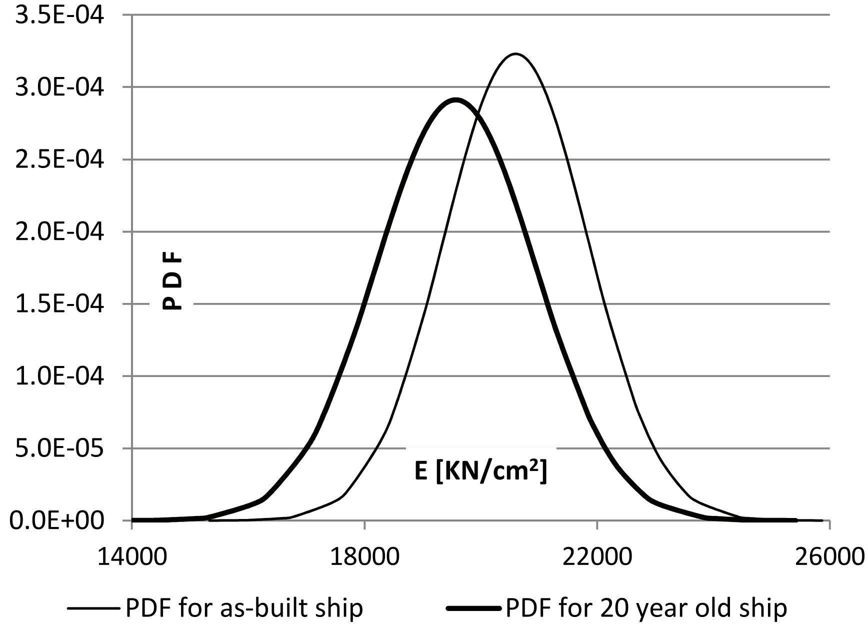

Figure 5: Probability Density Function of E (modulus of elasticity)

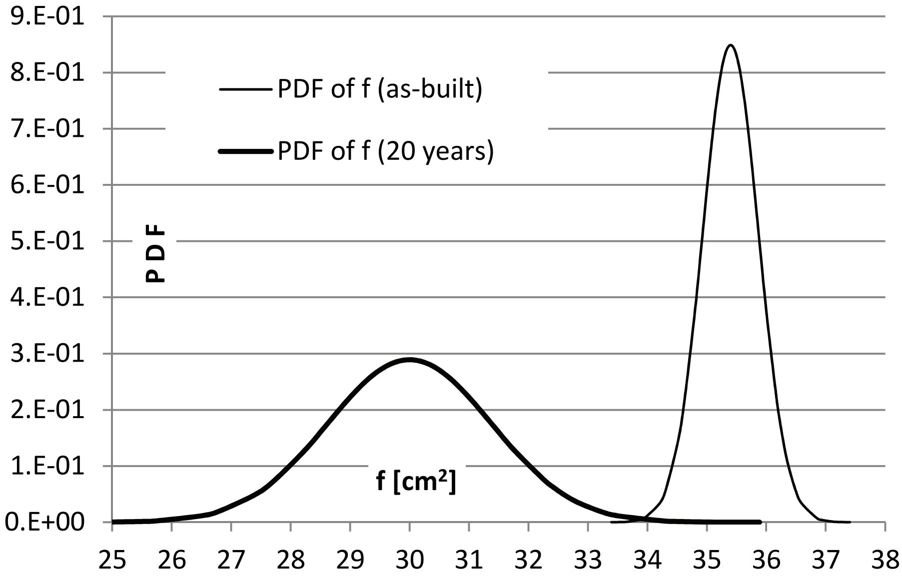

Figure 6: Probability Density Function of f (cross sectional area of a longitudinal)

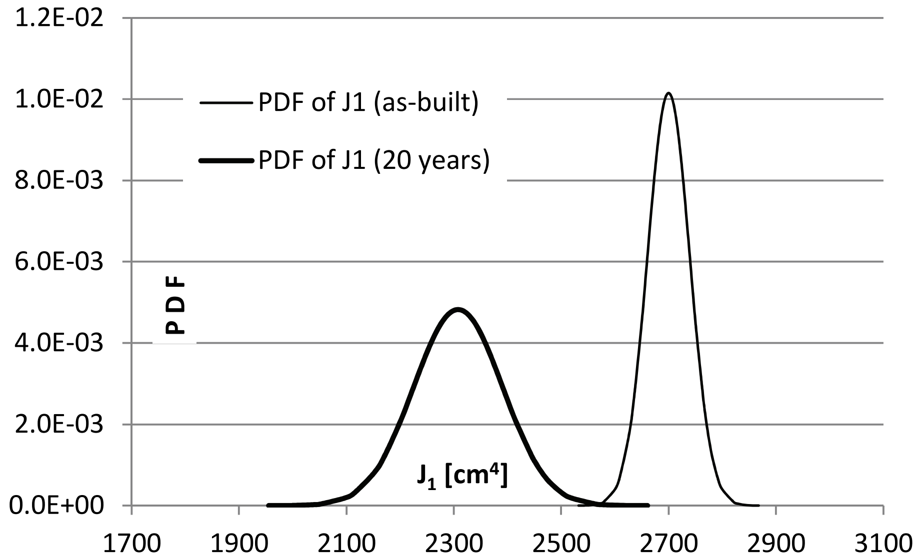

Figure 7: Probability Density Function of J_1 (moment of inertia of the side frame)

Table 1: Numerical results for calculated CDF and PDF of σ_{cr} (for CDF = 0.01 – 0.50)

Parameter	Dimensions	CDF of σ_{cr} [-]						
		0.01	0.05	0.10	0.20	0.30	0.40	0.50
J	[cm^4]	36967.9	38266.6	38958.9	39797.2	40401.7	40918.2	41401.0
i	[cm^4]	468.4	480.9	487.5	495.6	501.4	506.4	511.0
σ_y	[KN/cm^2]	32.55	34.56	35.64	36.94	37.87	38.68	39.43
E	[KN/cm^2]	16383	17317	17814	18417	18852	19223	19570
f	[cm^2]	26.8	27.7	28.2	28.8	29.3	29.7	30.0
J_1	[cm^4]	2115.6	2172.0	2202.0	2238.4	2264.6	2287.0	2308.0
α	1/(cm.KN)	2.09E-06	1.93E-06	1.85E-06	1.76E-06	1.70E-06	1.65E-06	1.61E-06
ζ	[-]	0.4532	0.4512	0.4501	0.4489	0.4481	0.4474	0.4467
$(\mu/\pi)^4$	[-]	1.3108	1.3083	1.3070	1.3055	1.3044	1.3035	1.3027
Δ	[-]	0.0296	0.0298	0.0299	0.0300	0.0301	0.0301	0.0302
A_j	[-]	0.1837	0.1837	0.1837	0.1837	0.1837	0.1837	0.1837
ε	$10^4 cm^2/KN$	0.0163	0.0156	0.0152	0.0148	0.0145	0.0143	0.0141
r_2	[KN/cm^2]^2	1.8501	2.0995	2.2392	2.4147	2.5456	2.6603	2.7699
r_1	[KN/cm^2]	2.3942	2.5464	2.6276	2.7260	2.7971	2.8578	2.9146
r_0	[-]	-0.1265	-0.1278	-0.1284	-0.1292	-0.1297	-0.1301	-0.1305
σ_{cr}	[KN/cm^2]	19.67	20.72	21.28	21.96	22.44	22.86	23.24
PDF of σ_{cr}	[cm^2 / KN]	0.016055	0.065472	0.113437	0.183566	0.229342	0.255517	0.264050

Table 2: Numerical results for calculated CDF and PDF of σ_{cr} (for CDF = 0.5 – 1.00)

Parameter	Dimensions	CDF [-]						
		0.50	0.60	0.70	0.80	0.90	0.95	0.99
J	[cm^4]	41401.0	41883.8	42400.3	43004.8	43843.1	44535.4	45834.1
i	[cm^4]	511.0	515.6	520.6	526.4	534.5	541.1	553.6
σ_y	[KN/cm^2]	39.43	40.17	40.98	41.91	43.21	44.29	46.30
E	[KN/cm^2]	19570	19917	20288	20723	21326	21823	22757
f	[cm^2]	30.0	30.3	30.7	31.2	31.8	32.3	33.2
J_1	[cm^4]	2308.0	2329.0	2351.4	2377.6	2414.0	2444.0	2500.4
α	1/(cm.KN)	1.61E-06	1.56E-06	1.52E-06	1.47E-06	1.41E-06	1.36E-06	1.27E-06
ζ	[-]	0.4467	0.4461	0.4454	0.4447	0.4436	0.4428	0.4414
$(\mu/\pi)^4$	[-]	1.3027	1.3020	1.3011	1.3002	1.2990	1.2980	1.2962
Δ	[-]	0.0302	0.0303	0.0303	0.0304	0.0305	0.0306	0.0307
A_j	[-]	0.1837	0.1837	0.1837	0.1837	0.1837	0.1837	0.1837
ε	$10^4 cm^2/KN$	0.0141	0.0138	0.0136	0.0134	0.0131	0.0128	0.0123
r_2	[KN/cm^2]^2	2.7699	2.8818	3.0042	3.1507	3.3599	3.5380	3.8851
r_1	[KN/cm^2]	2.9146	2.9715	3.0323	3.1036	3.2025	3.2843	3.4378
r_0	[-]	-0.1305	-0.1309	-0.1313	-0.1318	-0.1324	-0.1329	-0.1338
σ_{cr}	[KN/cm^2]	23.24	23.63	24.04	24.53	25.20	25.75	26.78
PDF of σ_{cr}	[cm^2 / KN]	0.264050	0.248365	0.218656	0.168701	0.090652	0.062294	0.013749

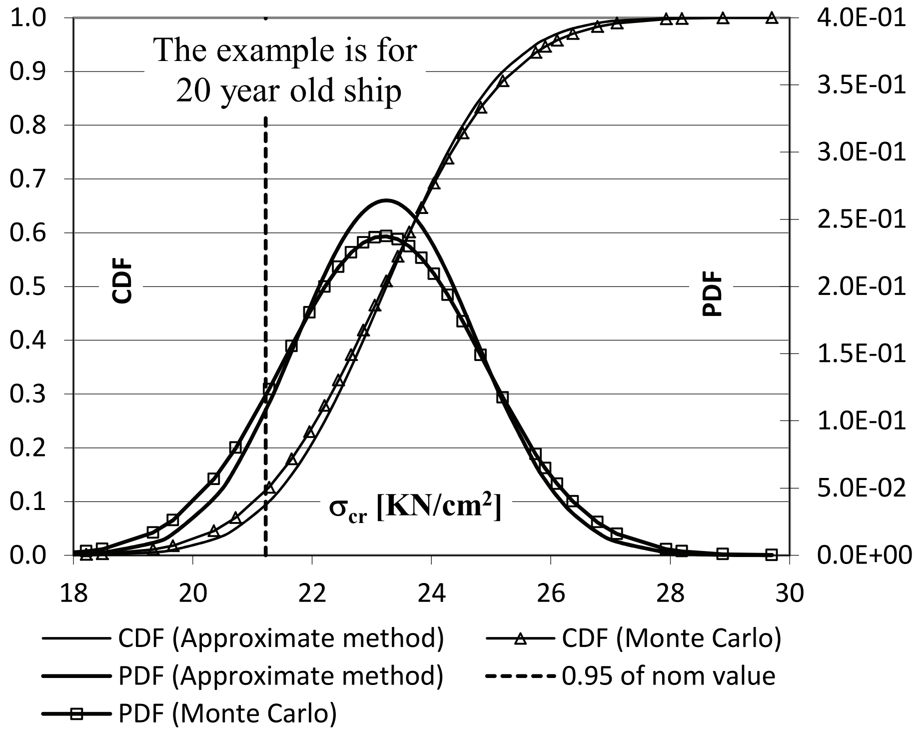

Figure 8: Probability Density and Cumulative Distribution Functions of critical buckling stress σ_{cr} of one longitudinal

Table 3: Error resulting from application of the approximate method for calculation of the CDF of σ_{cr} against results obtained by the Monte Carlo simulation method

	20 year old ship			as-built ship		
CDF of σ_{cr}	Monte Carlo	Approximation	error [%]	Monte Carlo	Approximation	error [%]
0.01	19.29	19.73	2.27	21.35	20.96	-1.82
0.05	20.44	20.76	1.58	22.25	21.96	-1.29
0.10	21.05	21.31	1.24	22.72	22.49	-1.02
0.20	21.79	21.97	0.86	23.30	23.13	-0.71
0.30	22.32	22.45	0.59	23.71	23.59	-0.50
0.40	22.77	22.86	0.38	24.07	23.99	-0.32
0.50	23.20	23.24	0.19	24.40	24.36	-0.16
0.60	23.63	23.63	0.00	24.73	24.73	-0.01
0.70	24.08	24.04	-0.19	25.09	25.13	0.15
0.80	24.61	24.52	-0.40	25.50	25.59	0.34
0.90	25.35	25.18	-0.68	26.08	26.23	0.58
0.95	25.96	25.73	-0.90	26.55	26.76	0.78
0.99	27.11	26.76	-1.29	27.45	27.76	1.13

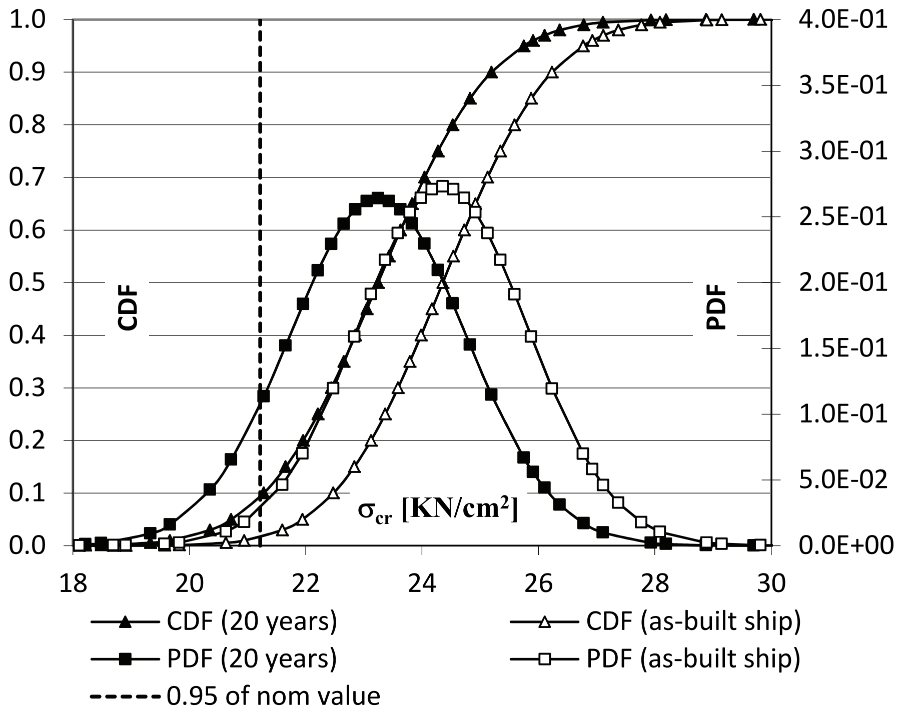

Figure 9: Probability Density and Cumulative Distribution Functions of σ_{cr} for as-built and 20 year old ship

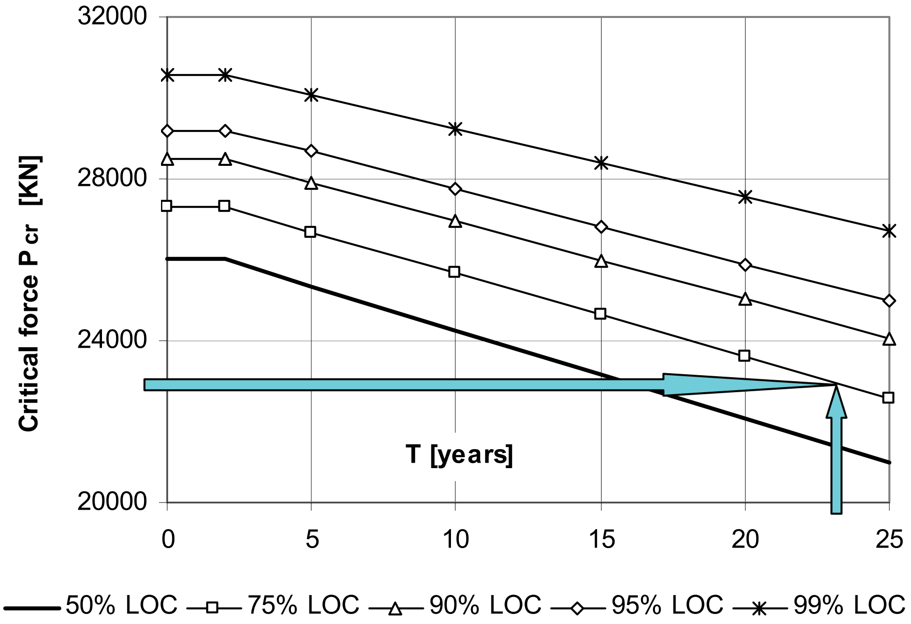

Figure 10: Application of the proposed approximate method for calculation of the level of certainty (LOC) of the final result when the input parameters have different LOC

