WHEN IS WATER SHALLOW?

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SUMMARY

A master of a vessel must at all times know where their vessel is operating. Traditionally this is only thought of in the geographical sense; however, there is a clear necessity, for safe vessel operations, that the master knows where their vessel is in the hydrodynamic sense. This knowledge is also of prime interest to designing naval architects and route planners alike. Water depth has profound effects on vessel performance and to know *When is Water Shallow?* is the key to successful vessel operation and wash mitigation. The authors propose a series of characterisations to aid the definition of shallow-water and hence provide greater operational understanding. These characterisations cover typical vessel performance indicators such as resistance, propulsion, manoeuvring, etc., but also wash-specific performance indicators such as wave angle, wave decay, soliton occurrence and spectral output.

NOMENCLATURE

A_x	Max. transverse sectional area of vessel (m^2)
b	Tank / channel breadth (m)
B	Maximum demi-hull beam (m)
C_b	Block coefficient = ∇ / L.B.T
C_p	Prismatic coefficient = $\nabla/L.A_x$
C_{wp}	Coefficient of wave pattern resistance = $R_{wp} / 0.5\rho.S.v^2$
C_r	Coefficient of residuary resistance
Fr_l	Froude length number = $v/\sqrt{(g.L)}$
Fr_h	Froude depth number = $v/\sqrt{(g.h)}$
g	Acceleration due to gravity (m/s^2)
H_w	Maximum wave height (m)
$H_{w'}$	Non-dimensional maximum wave height / vessel length ($H_w' = H_w/L$)
h	Water depth (m)
J	Propeller advance coefficient
K_T	Propeller thrust coefficient
K_Q	Propeller torque coefficient
L	Waterline length (m)
LCB	Longitudinal centre of buoyancy
$L/\nabla^{1/3}$	Length - volume ratio
n	Decay coefficient
η_0	Propeller open-water efficiency
R_{wp}	Wave-pattern resistance (N)
S	Surface area (m^2)
s	Demi-hull separation (m)
t	Time (s)
T	Draft (m)
T_W	Wave period (s)
v	Vessel speed (m/s)
W	Waterway or tank width (m)
x	Longitudinal axis (+ve fwd) (m)
y	Transverse offset from sailing line (m)
α	Bow wave angle (°)
Δ	Displacement (kg)
ρ	Density of water (kg/m^3)
γ	Gamma wave height constant
λ	Wavelength (m)
∇	Volume (m^3)

1. INTRODUCTION

Currently there is no accepted unambiguous definition (either numerical or descriptive) of when a vessel is affected by shallow-water. The multiple physical variables involved make this a complex and subjective problem. Furthermore it is understood that shallow-water wash (wave wake) is more complex than wash generated in deep-water [1], and in turn it can have significantly different (even fatal) outcomes for other water users [2].

With this in mind, the effects of shallow-water, as presently known, are now briefly described with a view to characterising the various operating modes in shallow-water more precisely.

Saunders [3], in his key work on ship hydrodynamics, offers a definition of shallow (restricted) water; *A body of water is considered to be shallow when the boundaries are close enough to the ship to affect its resistance, speed, attitude, manoeuvring, and other performance characteristics as compared to its corresponding behaviour in a body of water of unlimited depth.* Each of the performance criteria mentioned within Saunders' definition can be traced to other independent works.

Saunders goes further to quantify shallow-water, stating that a vessel can be considered to be in *restricted* waters if total vessel resistance (R_T) is increased by one percent (1%).

It is understood that there are four operating zones, each with a different set of boundaries: (a) open water, (b) depth restricted, (c) width restricted, (d) combined depth and width restricted. The two most common shallow-water restrictions, and those considered within this work, are (b) and (d). Typically, when considering these restrictions, the non-dimensionalised ratios of water depth to vessel draft, h/T, and vessel cross-sectional area to channel cross-sectional area (blockage) ratio, $A_x/(b.W)$, are utilised.

From a fluid dynamics perspective, the flow around a vessel operating in shallow-water can be described as a "restricted flow". Following the continuity equation [4], (i.e. constant mass flow rate) the fluid particles must accelerate past the restriction (i.e. the vessel and the bottom). From Bernoulli's principle [5], this acceleration in turn leads to a local pressure reduction around the vessel, which is nominally described as the "venturi effect"[6]. Accordingly the increased local flow velocity increases skin frictional (viscous) resistance. These viscous and pressure effects are increased when the flow is restricted by both depth and width.

The British Ship Research Association's (BSRA) Ship Design Manual [7] lists the shallow-water performance criteria and summarises them in a convenient plot, reproduced in Figure 1. The manual describes three recognisable flow regimes and their effect on vessel performance: (a) Infinite Depth: essentially deep-water with normal unrestricted flow occurring around the hull form; (b) Intermediate Depth: significant flow regime changes occurring which are noticeable, but do not have a significant effect on performance; and (c) Shallow-water: the restriction of flow under the keel which has a dominant effect on vessel performance.

The key words within the BSRA definition here are *noticeable*, *significant* and *dominant* which while somewhat subjective and non-specific fit with it being a generic definition of shallow-water. The BSRA plot Figure 1 links various vessel performance criteria with the depth – draft ratio, h/T. Additionally it indicates that vessel performance parameters are affected progressively as h/T decreases. A secondary finding from this figure is that not all performance parameters are affected at the same depth – draft ratio.

The BSRA figure is generic and descriptive in nature. It is probable that the performance parameters of long slender hull forms are affected differently to those of short squat hull forms, either by experiencing shallow-water effects earlier or later, or the vessel performance parameters being affected in a different order.

It should be noted that the h/T parameters in the physical tests conducted by the authors in previous works [8, 9, 10] have ranged from 8.0 to 1.8, which covers the deep, intermediate and shallow-water ranges as presented by the BSRA manual.

2. SHALLOW-WATER VESSEL PERFORMANCE

2.1 RESISTANCE

The most noticeable effect of vessel operations in shallow-water is the change in resistance, which is most commonly felt as a loss in vessel speed [11]. Schlichtling's [12] experimental work provides an excellent semi-empirical guide to the typical magnitude and form of the speed loss in open shallow-water. This work was later extended by Lackenby [13] to incorporate more "realistic" areas of operation, and accounted for correct skin frictional allowance, Figure 2.

2.2 WAVE PATTERN

Kelvin [1] first described the wave pattern generated by a moving point source at the water surface. The Kelvin ship wave pattern is well understood and is covered here only briefly. The deep-water wave pattern consists of divergent and transverse wave systems. The cusp line of the divergent system intersects the sailing line at a fixed angle of 19°28'.

As the water depth is reduced, for a given speed, for example, the divergent wave system angle progressively increases to approximately 90°, whilst the transverse wave system is ultimately lost with only the divergent system remaining. As the water depth is reduced even further, the wave angle progressively reduces [14]. The simplified Kelvin wave patterns for varying conditions are shown in Figure 3.

2.3 PROPULSION

It follows that, as for vessel resistance, due to the restricted flow around the hull, the performance of propellers will also be affected. Harvald [15] conducted open-water and self-propulsion model experiments in shallow-water for a bulk carrier hull form. These experiments showed that the propeller performance coefficients, (i.e. K_T, K_Q, η_0, and J), were affected and were non-linear functions of h/T. For a given power, the typical reductions in propeller rpm are of the magnitude of 15% or more, see Figure 4.

2.4 SINKAGE AND TRIM

Being a free-floating body, a vessel will sink and trim according to the static and dynamic forces upon it. In shallow-water the restricted flow causes a change in pressure around a vessel, causing a suction effect towards the boundary. This in turn causes the vessel to sink and trim by the head, the combination of which is known as squat [16]. The magnitude of ship squat is a direct function of vessel speed, depth-draft ratio and hull form. At high speed, a ship lifts and trims by the stern. The vessels initial trim can have a significant effect on the extent of which grounding may occur [17, 18], Figure 5.

It should be noted that similar dynamic effects occur when vessels come close to a bank or even to other vessels (albeit in a lateral sense). This is known as interaction [19] and, from a safety point of view, can be just as significant as squat.

2.5 MANOEUVRING

In shallow-water, it is appropriate to replace the term "manoeuvrability" with "controllability", as a vessel is considered to be controllable when the navigator is able to manoeuvre the vessel within acceptable limits [20]. Vessel controllability can be separated into three distinct areas or functions, being: (i) course keeping, (ii) manoeuvring and (iii) speed changing, all of which are affected by shallow-water.

A vessel which is difficult to manoeuvring in shallow-waters may be described as “sluggish” [20]. The added mass of the vessel increases, and the relationship between drift angle and side force is significantly altered, to the point where the vessel becomes uncontrollable. This “sluggishness” may also be explained partially by flow changes due to propeller wake onto the vessel’s rudder, leading to reduced hydrodynamic performance, in a three areas mentioned above.

An additional consideration is that, as a vessel enters shallow-water, its resistance increases and the vessel slows (for a constant power or shaft speed). As a vessel’s manoeuvring characteristics are directly a function of speed, this speed reduction has an added effect on vessel controllability. Dand [21] showed that ship-handling characteristics (manoeuvring), as well as vessel resistance, are affected by restrictions in water depth and width. Typically, for a given speed, a vessel’s turning circle diameter increases as water depth decreases.

2.6 MOTIONS

As vessel resistance, trim, propulsion and manoeuvring are affected by shallow-water, so too are vessel motions. As mentioned above, the reduced underkeel clearance creates a suction effect on the hull form which increases “hydrodynamic damping” [22]. This in turn leads to a reduction in vessel motions (i.e. pitch, heave and roll) [23]. The magnitudes of these reductions are a direct function of vessel heading, speed and water depth, see Figure 6. A summary of shallow-water on vessel performance is shown in Table 1.

3. WASH PERFORMANCE

The previous section has shown that the effect of shallow-water on vessel performance is well known. However, the effect of shallow-water on wash-specific performance parameters is less well understood. In this section, we consider these performance parameters and extend previous work in deep-water to water of limited depth with the intention of providing a more precise definition of various types of shallow-water.

Within the authors’ previous work [9] (an extension of previous shallow water works [24, 25 and 26]) deep-water wash performance parameters, or characterisations, were assessed, i.e. wave height, wave period, wave energy, etc. The assessment evaluated their suitability for use in deep and shallow-water. From the review additional shallow-water wash characterisations were recommended which fully describe wash across the trans-critical range. These new characterisations increase the understanding of trans-critical wash and, in turn should enable effective wash mitigation strategies, to be developed.

3.1 OPERATIONAL CONDITIONS

Only two operational conditions, deep and shallow-water, were considered in Section 0. However, with respect to vessel wash, the shallow-water condition can be described more precisely. This is significant as knowing in which shallow-water condition a vessel is operating will assist in developing effective mitigation strategies.

Following on from Havelock’s work [14], the onset of shallow-water effects is typically defined in terms of Froude depth number, Fr_h. It has been generally considered that the deep-water condition covers the Fr_h range up to 0.5, after which the vessel can be considered to be operating in shallow-water. From Havelock’s work, three discreet shallow-water operational conditions can be defined: (i) sub-critical, (ii) critical, and (iii) super-critical.

The deep- and shallow-water operational condition ranges are typically specified in terms of Fr_h, providing “ball park” estimations. However, such a definition is a simplification of a highly complex problem, since each operational condition cannot be so clearly defined using a single parameter.

It should be understood that there are no set numerical boundaries between conditions but, rather, transitional zones in between identifiable states. Furthermore, it is recommended that outcome or evidence based classification, which varies between vessel type and environment, rather than that based on simple (empirical) numerical formulae, be used.

4. WASH CHARACTERISATIONS

The following key wash characterisations have been investigated by the authors and are proposed as being suitable for defining trans-critical wash and boundaries of shallow-water zones. It is understood that some characterisations may require a towing tank or full scale observation to be realised. However, enough characterisations are presented to satisfy operators and naval architects alike.

4.1 LEADING WAVE ANGLE

The change of Kelvin wave pattern with water depth and / or speed has been covered in Section 0. The deep- water wave pattern comprises both transverse and divergent wave systems. For a given speed, as the water depth is progressively decreased, the transverse system will gradually reduce until, at a Froude depth number of unity, it is lost completely.

Another key feature is the accompanying change in leading wave angle. The leading wave angle is a key characterisation for trans-critical wash, being simple to measure and understand, [27].

Within the authors' previous work [9] the leading wave angle, α, was measured for two hull forms, across the trans-critical Fr_h range, as shown in Figure 7. The results showed that the measured values closely matched Havelock's theoretical predictions [14], except that the maximum leading wave angle occurred at around Fr_h = 0.9, and not at Fr_h = 1.0.

4.2 WAVE DECAY

It is known from Havelock [28] that, in deep-water, the divergent wave system decays at a rate proportional to the inverse cube root of the distance from the sailing line, equation (1).

$$H_w \propto . y^{-1/3} \qquad (1)$$

Renilson and Lenz [29] determined that a vessel's maximum wave height, H_w, can be calculated for a given offset from sailing line, y, once gamma, γ, in equation (1) is known.

$$H_w = \gamma . y^{n} \qquad (2)$$

For deep-water the decay coefficient, n, is constant (n=-1/3). However for shallow-water it is known that n varies with depth Froude number [30, for example], which made it a potentially valuable wash characterisation. This was further investigated in the authors' following work [8].

Two hull forms were tested in three water depths covering the trans-critical Fr_h range. For the hull forms and conditions tested, the main conclusion was that the decay coefficient, n, was a function of Fr_h, and that it may vary between -1.0 to -0.2 across the trans-critical Fr_h range, see Figure 8.

4.3 SOLITON

A soliton is a single non-dispersive wave with no preceding or following trough. Solitons are cyclical and time dependant in nature. John Scott Russell first observed the "wave of translation" (soliton) in 1844 [31]. It should be clarified that, for steady conditions, multiple individual solitons will be continuously formed ahead of the vessel, see Figure 9.

Within Ertekin et. al.'s [32] experiments, solitons were observed occurring over a range of Froude depth numbers from 0.9 to 1.2, (not just at the critical value). It was further reported that, for both measured and numerical experiments, the resistance oscillated about a mean value, with a period equal to that of soliton generation. This is confirmation that solitons are time-dependant in nature.

In the authors' previous work [10] it was determined that, for a vessel travelling near the critical depth Froude number, a time-dependent "unsteadiness" was present within the results, as a precursor to a soliton forming.

It is noted that solitons are not confined to narrow channels, case in point being a tsunami. Generally (near the critical number) an increase in blockage results in an increase in wave height.

4.4 WAVELET ANALYSIS

The technique of spectral wavelet analysis of vessel wash has been covered in the authors' earlier work [33]. Wavelet analysis is similar to Fourier analysis, with both methods breaking down time-domain signals into their individual components and plotting them in the frequency domain. However, whereas in the fast Fourier transform process all the localised time information is lost, wavelet analysis has the key benefit of being able to describe <u>when</u> an event took place within the signal.

Wavelet analysis provides both numerical and visual outputs. The visual outputs utilised within the earlier work [33], are two dimensional (2D) and three dimensional (3D) combination plots. Both plot types represent signals as a combined time-frequency.

Each 2D plot (Figure 10) is a combination of two separate yet related sub-plots. The upper sub-plot is a standard longitudinal wave cut, of wave amplitude as a function of time. The lower sub-plot is the associated wavelet analysis output of the longitudinal wave cut. The 3D plot (Figure 11) contains the same information as the 2D plot, however, it is presented in a three-dimensional form.

The authors' suggested metrics for reviewing wavelet analysis results are:

(i) the <u>value</u> of the peak spectral energy;
(ii) the <u>location</u> of the peak spectral energy;
(iii) the <u>frequency</u> of the peak spectral energy;
(iv) the frequency <u>range</u> of the global spectral energy;
(v) the <u>form</u> of the global spectral energy.

5. CHARACTERISATION SUMMARIES

Considering the aforementioned wash characterisations, which are good engineering approximations, the following operational zone summaries can be made for all water depths.

5.1 DEEP-WATER

The deep-water operational zone can be characterised by (in no order of significance):

a) transverse and divergent wave systems both present;
b) leading wave angle of 19° 28’ (cusp locus line intersection);
c) divergent wave system decays at $n = -⅓$;
d) wave system is dispersive;
e) no solitons are present;
f) wavelet analysis metrics (i) to (v) constant with time; and
g) vessel performance[1] (resistance, trim, heave, and squat, etc.) is constant with time.

By definition, if any one of these conditions are not met, then the vessel can be considered to be operating in shallow-water.

5.2 SHALLOW-WATER SUB-CRITICAL

The shallow-water sub-critical operational zone can be characterised by:

a) transverse and divergent wave systems present, with the transverse system diminishing as Fr_h increases;
b) leading wave angle increasing from 19° 28’ towards a maximum value of approximately 90°;
c) leading wave decay rate variable, (i.e. not constant at $n = -⅓$);
d) the wave system transforming from a dispersive system to a combined dispersive/non-dispersive system;
e) no solitons are present;
f) wavelet analysis metrics initially constant at low Fr_h but becoming time dependant (variable) as Fr_h increases; and
g) vessel performance[1] differs from that in deep-water.

5.3 SHALLOW-WATER CRITICAL

The shallow-water critical operational zone can be characterised by:

a) critical wave system present;
b) leading wave angle at a maximum value of approximately 90°;
c) leading wave decay rate is variable with Fr_h, (i.e. not constant at $n = -⅓$);
d) wave system is non-dispersive;
e) solitons are generated;
f) wavelet analysis metrics vary with time, indicating non-dispersive conditions; and
g) vessel Performance[1] differs from that in deep-water;

5.4 SHALLOW-WATER SUPER-CRITICAL

The shallow-water Super-Critical operational zone can be characterised by:

a) divergent wave system only present;
b) leading wave angle decreasing from the maximum value;
c) leading wave decay rate variable with Fr_h, (i.e. not constant at $n = -⅓$);
d) the wave system transforming from a non-dispersive to a dispersive system;
e) no solitons present;
f) wavelet analysis metrics becoming less time dependant (variable) as Fr_h increases (i.e. transforming from non-dispersive to a dispersive system); and
g) vessel performance[1] differs from that in deep-water.

The above deep-water and shallow-water characterisations have been summarised in Table 2.

6. CONCLUDING COMMENTS

For safe vessel operations a master of a vessel must at all times know where their vessel is operating, in both the geographical and hydrodynamic senses.

By deduction, if one of the aforementioned deep-water characteristics is not present, then the vessel can be assumed to be operating in shallow-water. The question *When is water shallow?* has been answered.

Furthermore, from the characterisations it is possible to further classify shallow-water into Sub-Critical, Critical, and Super-Critical operational zones

These characterisations, while providing greater understanding of shallow-water, also have fundamental implications for safe vessel operations.

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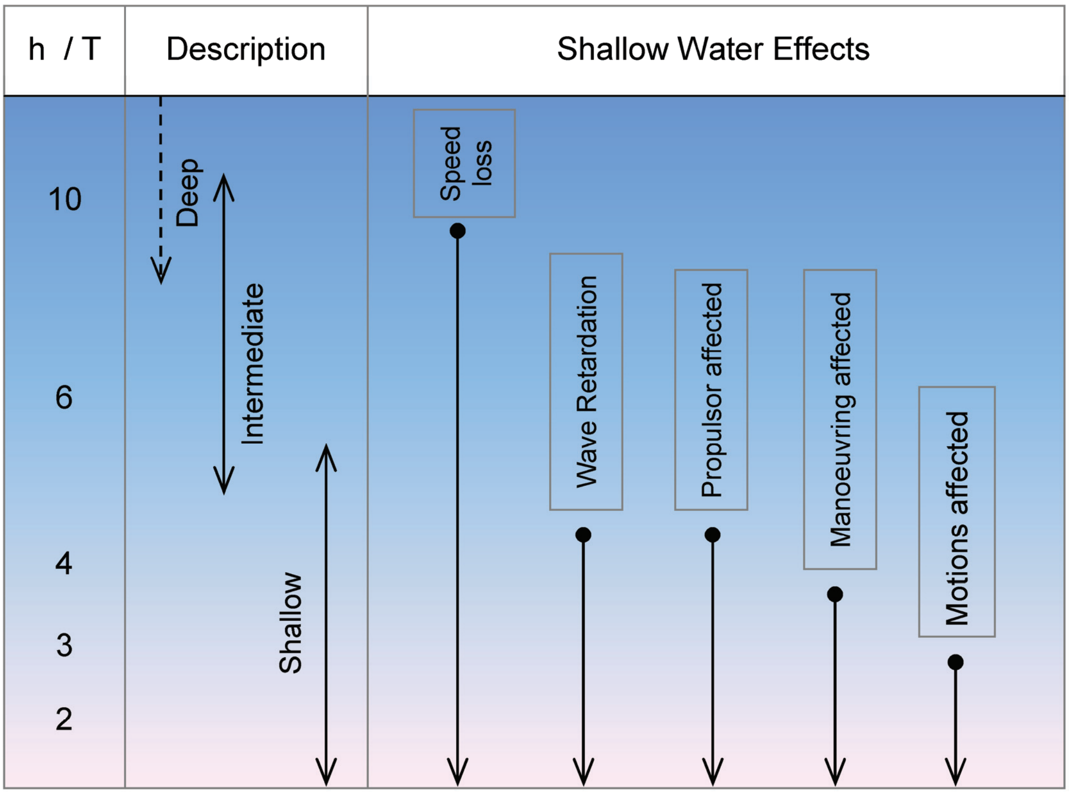

Figure 1 – BSRA Diagram [7]

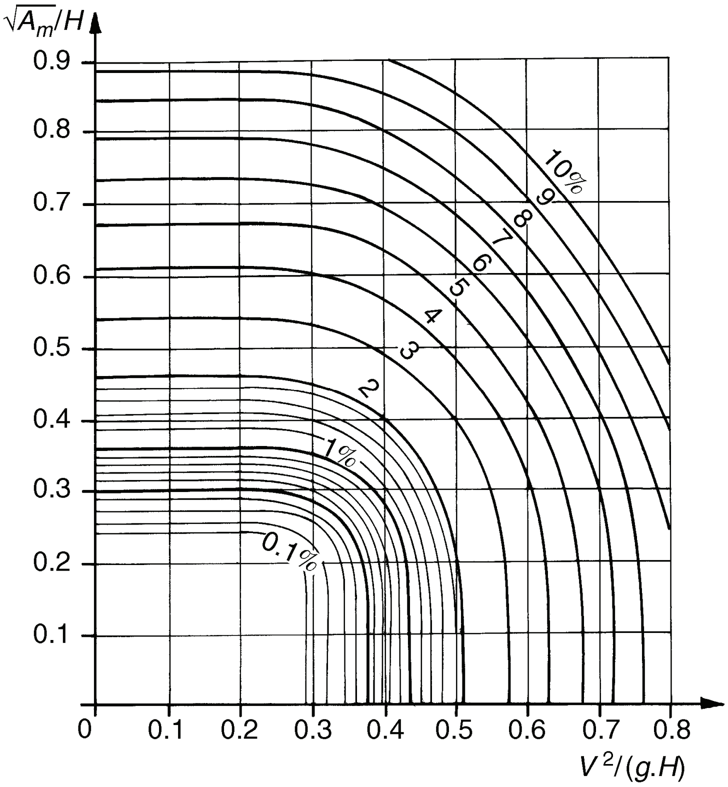

Figure 2 – Shallow-water Speed Loss Diagram [13]

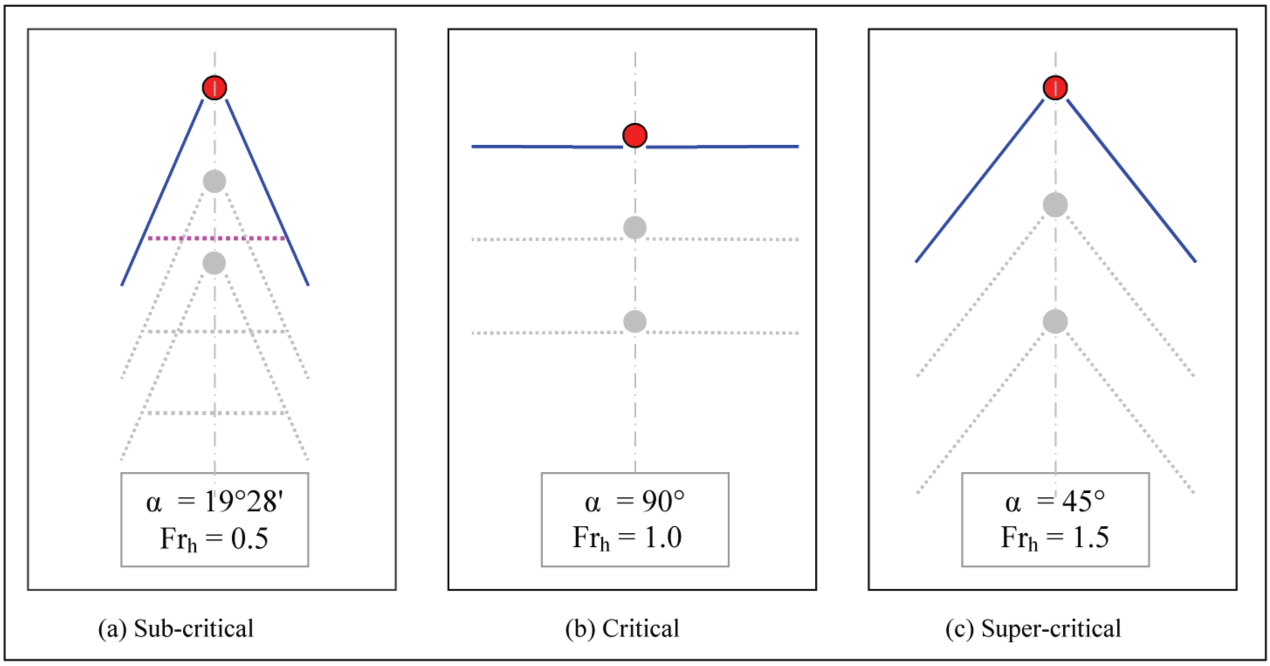

Figure 3 – Kelvin Wave Pattern (Point Source)

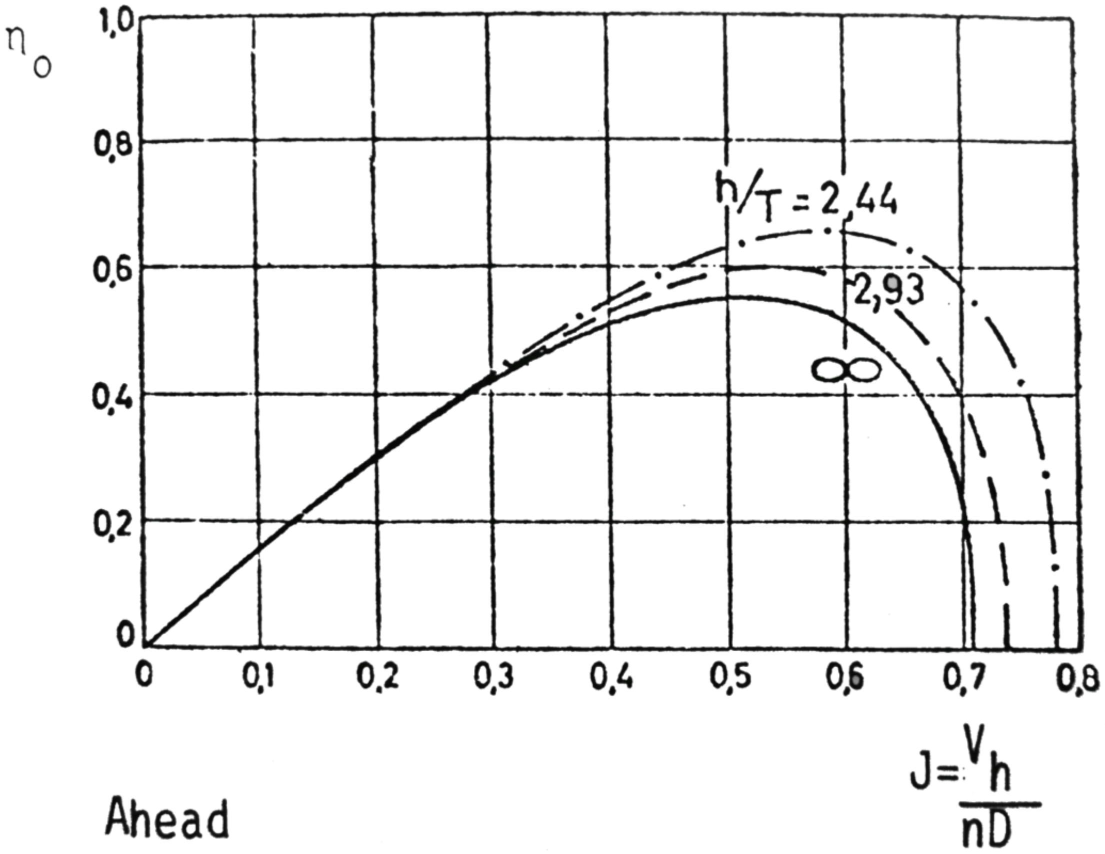

Figure 4 – Propeller Open Water Efficiency [7]

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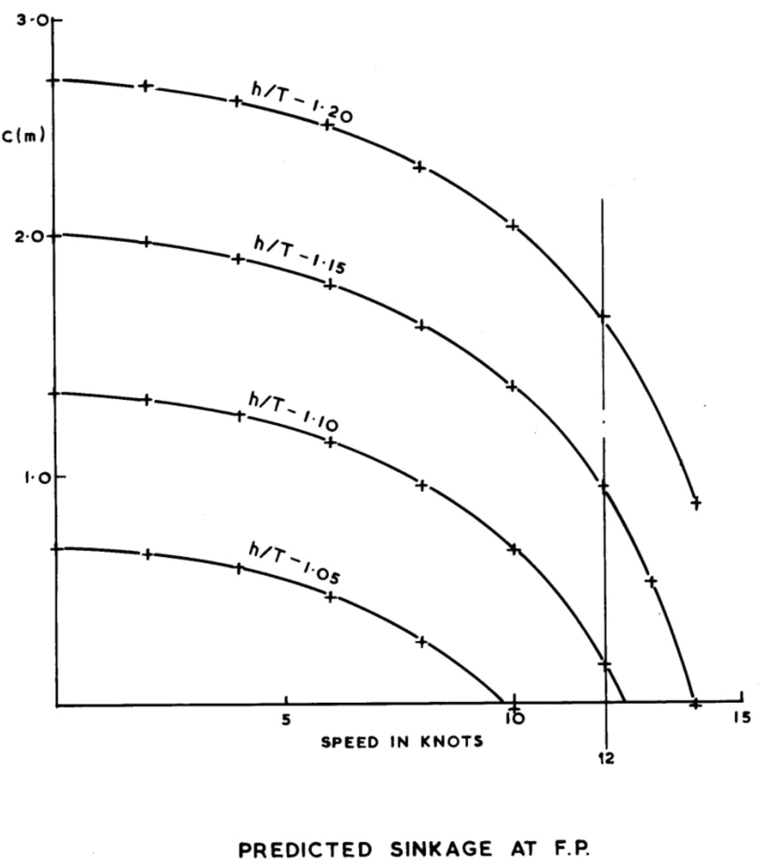

Figure 5 – Sinkage and Trim [18]

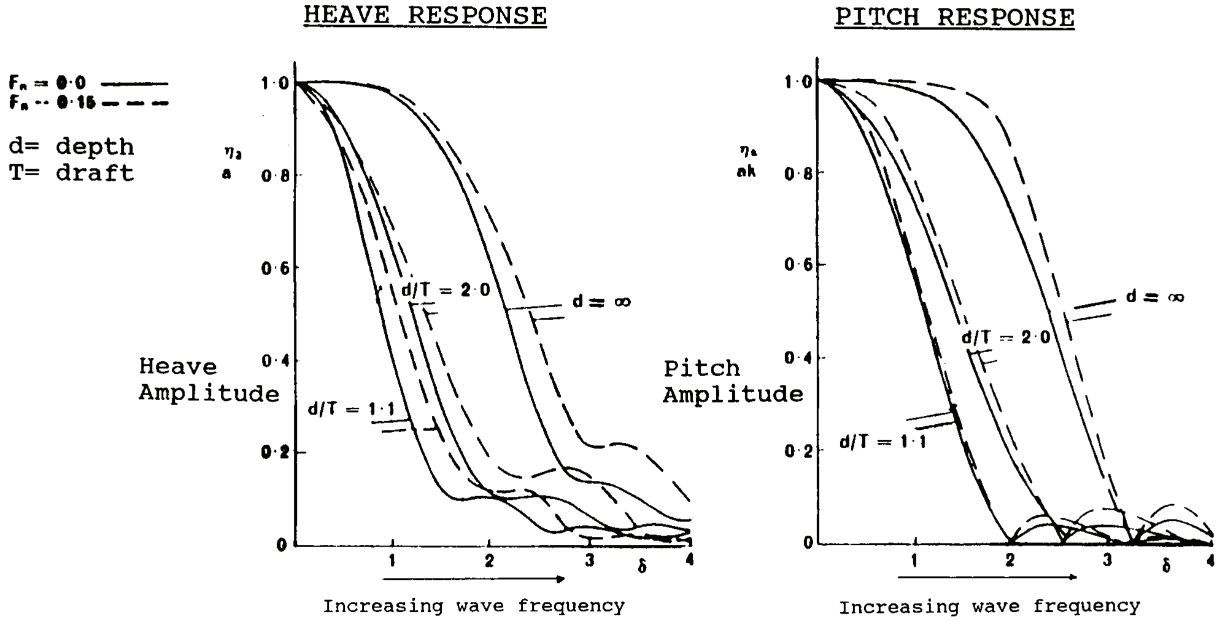

Figure 6 – Vessel Motions [3]

Table 1 – Shallow-water Vessel Performance

Characterisation	**Hydrodynamic Effect**	**Outcome on Vessel**
Resistance (friction)	Skin friction increase	Speed decrease
Wave pattern	Local pressure change	Wave angle change
Propulsion	Modified wake	Loading change
Squat	Hydrodynamic suction	Draught and trim increase
Manoeuvring	Boundary back flow	Reduced responsiveness
Motions	Hydrodynamic added mass	Dampened motions

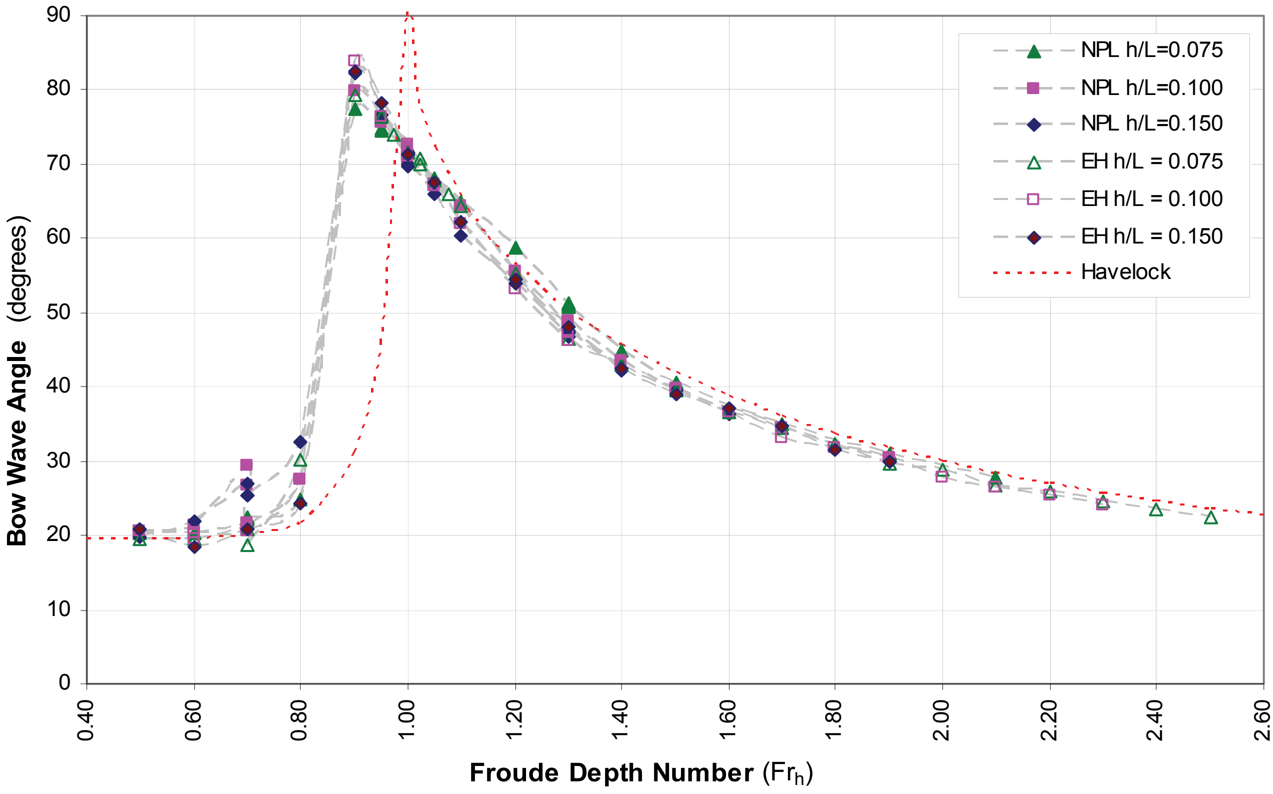

Figure 7 – Bow Wave Angle as a function of Froude Depth Number angle [9]

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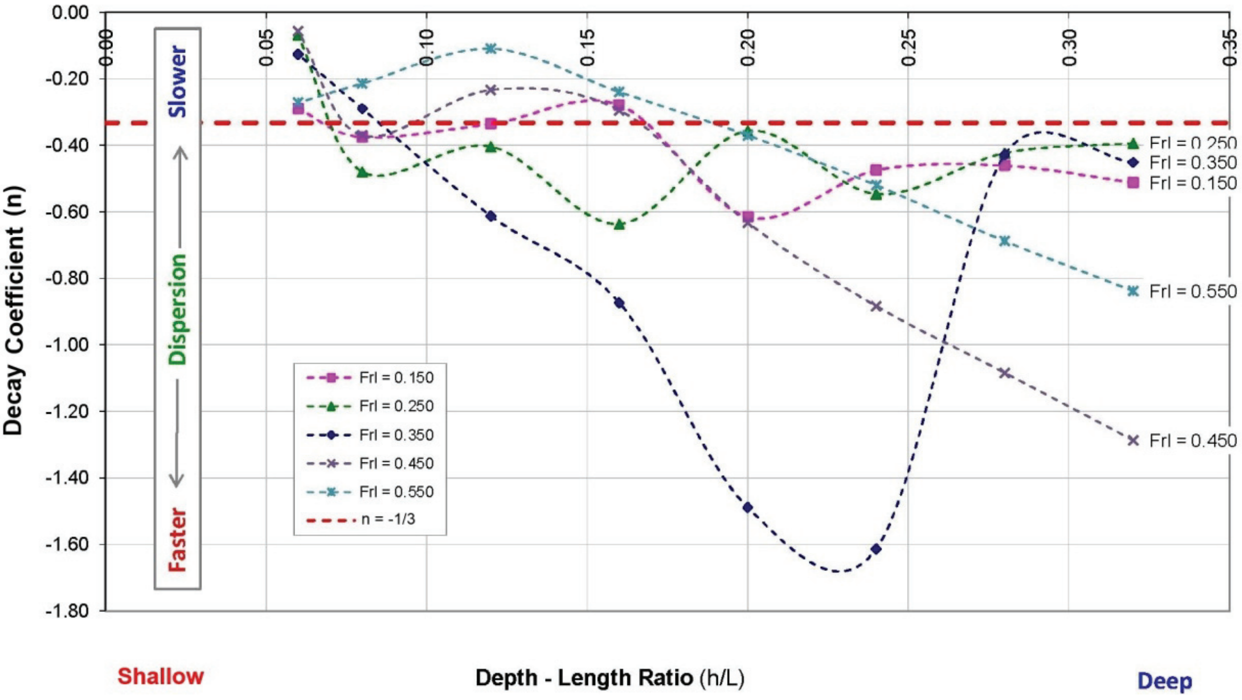

Figure 8 – Leading Wave Decay (n) as a Function of Depth / Length Ratio [9]

Figure 9 – Solitons generated from a mono hull in the AMC Towing Tank

This photo shows several solitons generated by a model of a full form vessel operating in shallow water. The solitons propagate ahead of the model, but are 'disturbed' close to the sidewalls of the tank as there is a 45 degree fillet in the corners of the tank (approximately equal to the water depth of 100mm). As can be seen, this has caused the solitons to form their own diverging wave system at the tank boundary, (i.e. non-dispersive waves creating a dispersive wave system).

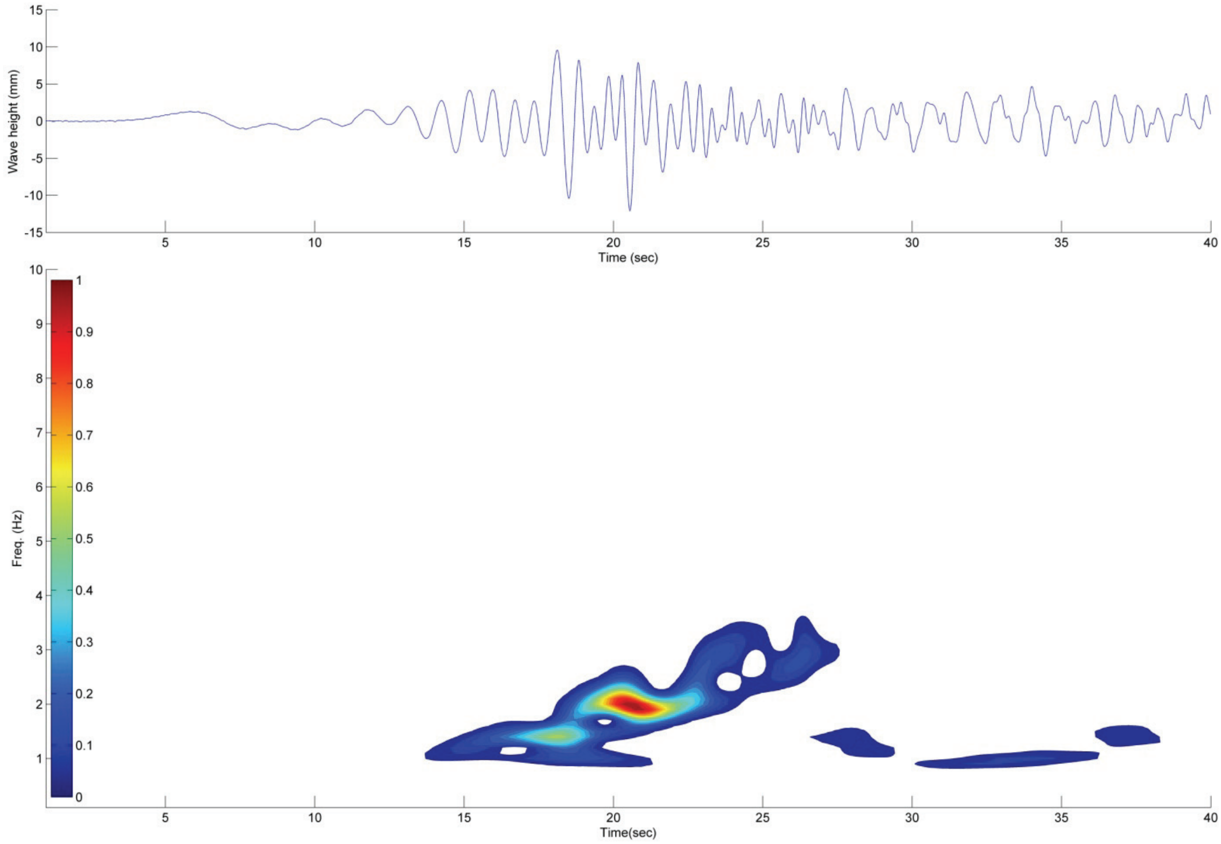

Figure 10 – Wavelet Analysis – Typical 2D plot [33]

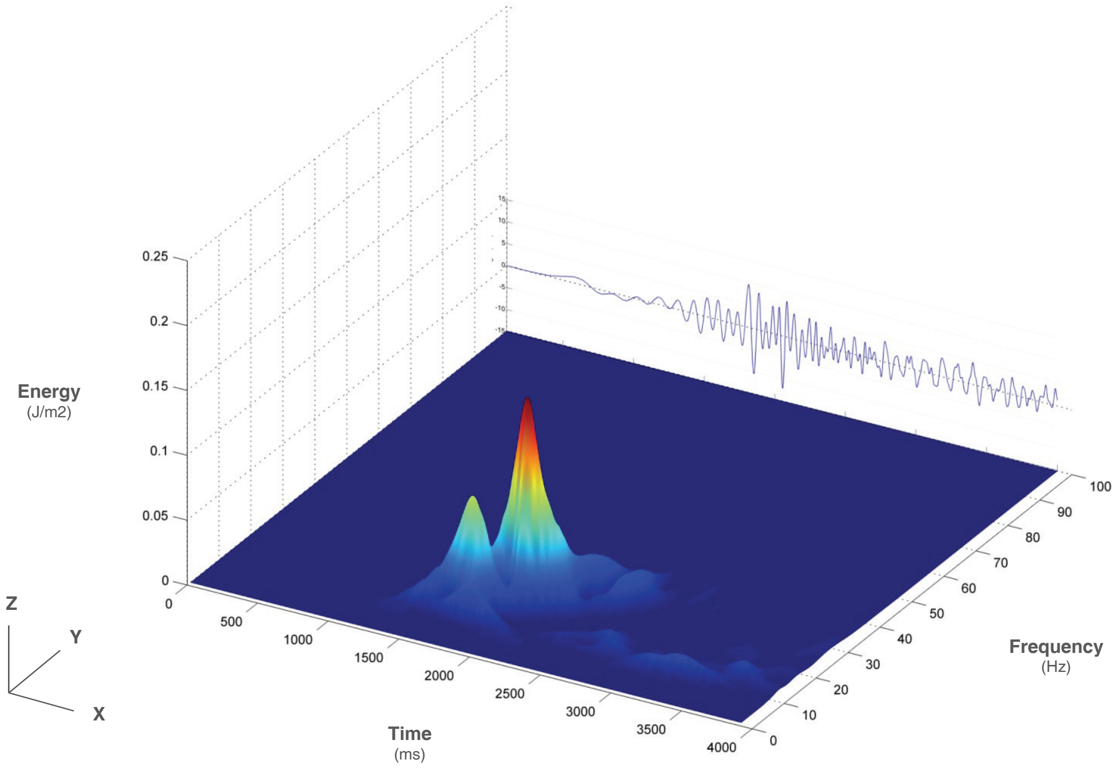

Figure 11 – Wavelet Analysis – Typical 3D Plot [33]

Table 2 – Wash Characterisation Summary Table

	Operational Zone			
Characterisation	**Deep-water**	Shallow-water **Sub Critical**	Shallow-water **Critical**	Shallow-water **Super Critical**
Froude Depth Number	$Fr_h < 0.5$	$0.5 < Fr_h < 1.0$	$Fr_h \cong 1.0$	$Fr_h > 1.0$
Divergent Wave System	Yes	Yes	Yes	Yes
Transverse Wave System	Yes	Diminishing	None	None
Leading Wave Angle	Constant @19° 28’	$19° 28’ \leq \theta \leq 90°$	$\theta \cong 90°$	$90° \geq \theta$
Leading Wave Decay	Constant @ -1/3	Variable -$f(Fr_h)$	Variable - $f(Fr_h)$	Variable - $f(Fr_h)$
Wave System Dispersive	Yes	Diminishing	No	Increasing
Solitons	No	No	Yes	No
Spectral (WA)	Constant	Variable with time	Variable for fixed Fr_l + Variable with time	Variable with time
Performance [1]	Constant with time	Increasing	Variable with time (Oscillating)	Reducing

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[1] (Vessel) Performance (i.e. resistance, trim, heave, squat, etc.) is not a wash characterisation as such but is a valuable co-dependent indicator. See also Table 1.