VESSEL WAVE WAKE CHARACTERISATION USING WAVELET ANALYSIS

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SUMMARY

This work focuses on characterising vessel wave wake (wash) using wavelet analysis when a vessel is operating in the sub-critical and critical zone. Such characterisation complements other wash characteristics: Froude depth number, bow wave angle, solitons and decay coefficient. The examination of experimental results indicates that differences in characteristics with respect to water depth, Froude depth number, vessel displacement, hull form and soliton generation can be identified through wavelet analysis. The results demonstrate "proof of concept" that wavelet analysis is a powerful tool for characterising vessel wash and captures the effects of key operational and vessel changes.

NOMENCLATURE

A_x	Midship section area
b	Tank breadth (m)
B	Maximum demi-hull beam (m)
C_p	Prismatic coefficient = $\nabla/L.A_x$
E	Wave energy (J/m^2)
Fr_l	Froude length number = $v/\sqrt{(g.L)}$
Fr_h	Froude depth number = $v/\sqrt{(g.h)}$
g	Acceleration due to gravity (m/s^2)
H_w	Maximum wave height (m)
H_c	Maximum wave height (Benchmark craft) (m)
H_{hsc}	Maximum wave height (High Speed Craft)
H_{lead}	Leading wave height (m)
h	Water depth (m)
L	Waterline length (m)
$L/\nabla^{1/3}$	Length - volume ratio
n	Decay coefficient
s	Demi-hull separation (m)
t	Time (s)
T	Draft (m)
T_W	Wave period (s)
T_c	Wave period associated with H_c (s)
T_{hsc}	Wave period associated with H_{hsc} (s)
v	Vessel speed (m/s)
x	Longitudinal axis (+ve fwd) (m)
y	Transverse dist. from sailing line (m)
Δ	Displacement (kg)
α	Bow wave angle (°)
α_{lead}	Leading wave angle (°)
β	The decision parameter
Δ	Displacement (kg)
γ	wave height constant
λ	Wavelength (m)
∇	Volume (m^3)
ρ	Density of water (kg/m^3)
$\psi(t)$	wavelet function

1. PROBLEM INTRODUCTION

1.1 WASH IMPACT

As a wave enters shallow water the dynamics of the wave change; wave speed and wave length decrease and, (following the law of conservation of energy), the wave height, (i.e. steepness), increases. Within wave mechanics this process is known as "shoaling" [1].

The typical wave spectra encountered by a shoreline are those from wind waves, sea swell, and the tide. These spectra are naturally occurring and can essentially be considered as continuous in form; (the tide generally comprising a number of regular sinusoidal waves, albeit of extremely long period). As the shoreline (or river bank) has been formed by long-term exposure to the wave swell and/or tide spectra, the introduction of vessel-generated "foreign" transient wave spectra, can have significant impact on the shoreline and shore goers [2].

These foreign vessel waves may have a wave height and period differing significantly from those of naturally occurring waves. In turn this may lead to greater shoaling wave surge or breaking, both of which can endanger shore goers, facilitate shore erosion (redistribution of sand, sediment or rocks) and long term potentially change the local coastal morphological processes [3]. The level of erosion and danger to shore goers is dependent on the form of the shoreline.

Marine traffic in coastal or inland waterways has two immediate effects on the environment: bank erosion and fine sediment re-suspension. This environmental impact has led to "green" criteria being placed on vessel design and operation [4, 5]. Environmental impact is no longer seen as of secondary importance, but in some cases has become the prime concern and its assessment a contractual requirement [6, 7].

Not only can vessel wash affect the environment, but it can also have an effect on other waterway users, (e.g. commercial and recreational vessels, marine infrastructure, swimmers and shore-goers). It can be a minor irritant affecting the enjoyment of others within the littoral zone; however it can also be a significant public safety hazard.

There has been one recorded death which can be directly attributed to vessel wash; the Purdy Incident

[8]. A 10m long monohull fishing vessel, *Purdy*, was swamped by high speed craft (HSC) wash. A crew member was washed overboard and subsequently lost at sea. This was despite the weather being fine and clear, the vessel being headed into the oncoming wash and speed restrictions being placed on the HSC. The official accident report suggested that a soliton may have been to blame, which passed across shoaling water, thereby increasing in size as it approached the Purdy. This incident highlighted the possible extreme danger and unpredictability of wash.

Wash can be identified as a form of pollution and considered in the same vein as noise pollution, (both typically being transient wave forms). Its effects are very much dependant on amplitude, frequency, (i.e. energy) and exposure time [9]. While there is common understanding and awareness of noise pollution, and its effects, there is little for vessel wash. This may be because its impact is often only indirectly experienced by humans.

The aforementioned environmental, safety, and pollution risks have led authorities to implement wash mitigation strategies. Typically these have taken the form of vessel route planning, speed restrictions, access restrictions and shore warnings [5, 10]. Clearly there is a need to understand vessel wash parameters (i.e. characteristics). Ideally such understanding will lead to more focussed mitigating strategies.

1.2 WASH CHARACTERISATION

For any given speed (v), the wave pattern is dependent on water depth (h). Each flow regime has its own characteristic wave pattern. There are no set numerical boundaries between regimes, (sub-critical, critical, super-critical), but rather transitional zones in between identifiable states. A suitable characterisation must fully capture this dynamic.

Macfarlane and Renilson, [11] stated that a suitable (deep water) wash measure should be; (i) independent of location, (ii) independent of sample size, (iii) easy to understand, (iv) easy to measure, and (v) able to rank vessel's wake performance.

Current wash characterisations are a combination of observation and calculation. Said characterisations are focussed primarily on the deep water condition. Therefore these measures are unlikely to be effective in assessing shallow water wave wake. What is required is a wave wake metric that is applicable to both the deep and shallow water conditions, (i.e. trans-critical). Following on from Macfarlane and Renilson's guidelines, such a metric should provide additional information as to the energy (form and magnitude) contained within the wave system. Wash can be characterised utilising a combination of observation and calculation methods.

Wave height, (H_w) is the simplest and most obvious wash characteristic, (being the largest elevation from peak to trough within an individual wave). Wave period, (T_w), is another simple wave characteristic, (being peak to peak zero crossing time of H_w). The T_w of individual waves or groups of waves is important, especially when coupled with H_w. While good initial indicators of vessel wash, H_w and T_w do not fulfil the requirements for a suitable measure of wash, specifically the requirement for a more global understanding of the wash event. It follows that more descriptive measures are needed.

Wave angle is another characterisation primarily based on observation. The angle of the leading wave (α_{lead}) has been proposed as a key wash characterisation [12], as opposed to the angle of the maximum wave (α_{max}). This is because height of the leading wave H_{lead} is easily identifiable across the trans-critical range and still provides similar decay characteristics to the H_{max} utilised in sub-critical (deep water) work.

A more complex, and commonly used, wash characterisation is the Froude depth number, (Fr_h), equation 1. Typically utilised to classify the flow regime a vessel is travelling in and hence its wash.

$$Fr_h = v / \sqrt{(g.h)} \quad (1)$$

Four recognised transitional zones between identifiable states are known for the Froude depth number; (i) Deep water ($Fr_h < 0.5$); (ii) Sub-critical ($0.5 < Fr_h < 1.0$), (iii) Critical ($Fr_h = 1.0$); and (iv) Super-critical ($Fr_h > 1.0$). It is understood that the trans-critical Froude depth number range is nominally $0.5 < Fr_h < 1.5$.

These zones are based on Havelock's 2D point theory [13], and each corresponds to a Froude depth number range. The physical actuality is of course far more complicated than a single pressure source, and accordingly the actual Fr_h differ from those theoretically proposed.

Wave energy (E), equation 2, is a measure of the energy contained in a single wave [7]. It is a useful attempt to combine the simplistic maximum wave height and period measures in a more meaningful and complex characterisation. The output units are expressed as Joules per metre of wave front.

$$E = 1961.H_w^2.T_w^2 \quad (2)$$

It is important to note that the wave height (H_w) and wave period (T_w), variables are both squared. This means that a wave of low H_w, yet high T_w, could have the same energy value as a wave of low T_w and high Hw. This fact can provide misleading results when comparing wave energies.

Kofoed-Hansen and Kirkegaard [14] describe a complex wash criterion which was originally developed for the operation of high-speed craft (HSC) in Danish waters, equation 3. This criterion is a measure of wave effects upon a shore, and compares the wash from a HSC with the benchmark wash of a conventional vessel.

$$H_{hsc} = \beta^{3/2} . \sqrt{(T_c / T_{hsc})} . H_c \qquad (3)$$

H_{hsc} is the maximum wave height (of the high speed craft), measured in a water depth of 3m, and T_{hsc} its corresponding period. H_c is maximum wave height (of the benchmark craft), measured in 3m deep water, and T_c its corresponding period. β being the decision parameter, usually taken as unity.

This criterion, while representing the concerns of shore goers and recreational vessels can prove difficult to measure. This is mainly due to the 3m water depth requirement, but also due to the criticality of the wave as it enters shallow water. Fundamentally the local topography of the coast where the measurement occurs may significantly affect results.

A characterisation which fulfils most of the requirements for a suitable wave wake measure is the wave decay function, [15, 16], (equation 4). The method works on the principle that the transverse and divergent wave trains generated from a vessel decay at different rates. Also as the (deep water) divergent waves are dominant, it follows that if the decay of this wave train can be determined, the wave measurement goals will be achieved.

$$H_w = \gamma . y^{n} \qquad (4)$$

It is known that, for deep water, the divergent wave system decay rate (n) is approximately equal to the inverse cube root of the distance from sailing line, [17]. A vessel's maximum wave height can be calculated for a given offset from sailing line, (y), once the wave height constant (γ) is known. To obtain the gamma value for a vessel at a particular speed, a series of longitudinal wave cuts parallel to the sailing line must be taken at varying offsets from the vessel. The cuts are analysed for maximum wave elevation, and these maxima plotted as a function of $H / y^{-1/3}$. From this plot a curve of best fit is obtained, and γ determined.

It must be noted that the wave decay function, as stated in equation 4, has application only in the deep water condition. From the Authors' previous work, [18], it is known that a vessel's shallow water decay rate is significantly different to its deep water value of -1/3. Furthermore the decay coefficient (n) was determined as a direct function of Fr_h and varied between -1.0 to -0.2, across the trans-critical range.

Another observed, yet less common wash characterisation, is solitary waves or solitons. A soliton can be typified as a single wave with no preceding, or following trough. Of significance is that solitons are only generated by vessels travelling in shallow water, at or near the critical speed. Therefore it is not observed in the sub-critical (deep water), or super-critical speeds. It is known that if a vessel maintains "critical" speed, (i.e. $Fr_h \cong 1.0$), for any significant length of time, it is unable to maintain a steady state and the periodic generation of solitons occur. The unsteady flow state is also reflected in the vessel's sinkage, trim and resistance measurements [19, 20].

In considering the existing aforementioned measures, H_w, T_w, Fr_h, α_{lead}, while assisting in characterising wash, are too simplistic in nature and too restricted in scope. The more complex measures, (E, H_{hsc},γ), while having greater scope, are either open to misinterpretation, too specific, or are not suitable for trans-critical analysis. An alternate view or perspective of the problem may provide a suitable measure to help further characterise vessel wash. This is now discussed.

2. SIGNAL ANALYSIS

By considering a wave in a mathematical context, (that is not as a physical entity but as a complex signal), a typical wave cut can be described as a transient signal. Such a signal can be analysed using modern signal analysis techniques.

Fourier analysis relies on the core assumption that the waveform being analysed is periodic. Wash signals are non-periodic or transient, and therefore have an important time domain component. The Fast Fourier Transform of a transient wave will produce a continuous spectrum in the frequency domain only and therefore will have limited application. Additionally choosing the length of wave elevation time signal to be analysed by Fourier analysis can be problematic, with the results being highly dependent on the sample length.

A mathematical tool for analysing transient signals uses the concept of wavelets. Wavelets allow a transient signal to be broken down into its constituent frequencies. Such frequencies, (and their relevant amplitudes), provide a unique identification of such a discrete signal, and its energy potential.

The term wavelet first appeared in Alfred Haar's 1909 thesis [21], as "Haar Wavelets". Following Haar there were some minor advances in wavelet analysis from the 1930s to the 1970s. However the next significant leap in understanding, (and application), was by Jean Morlet, and his later combined work with Alex Grossman [22]. Their concept was that a signal could be transformed into wavelet form and then transformed back into the original signal without loss of information. This is fundamental to the application of (wavelet based) data compression.

Following on, Mallat, [23], advanced wavelet analysis techniques through his work in digital signal processing. The current "state of the art" wavelet technique was originally developed by Daubechies [24]. Daubechies proposed a new class of wavelet, which has most significance within the field of digital filtering. By combining Daubechies and Mallat's work, it is possible to transform signals, with minimal information loss using low computational requirements.

Fundamentally wavelet analysis is similar to Fourier analysis, with both methods breaking down time domain signals into their individual components and plotting them in the frequency domain. However where all the localised time information is lost in the FFT process, (i.e. the ability to describe when an event took place within the signal), wavelet analysis has the key benefit being of able to represent signals as a combined time-frequency representation.

Wavelet analysis output provides a time-scale frequency map of a signal, which enables the identification of time specific features. Clearly such a capability is ideal for analysing transient signals, and within this paper wavelet analysis is utilised to attempt to identify key wash characteristics.

A detailed mathematical background of wavelets is beyond the scope of this paper, but can be found within the following key references; [25, 26, 27, 28, 29, 30, 31, 32 and 33].

3. PHYSICAL TESTING PROGRAMME

3.1 TEST PROGRAMME

The test program undertaken to generate the data utilised within the following wavelet analysis has been well covered within the Authors previous works, [12, 18, 34]. For the sake of brevity only outline details have been provided here.

Parameter	Range
v (m/s)	0.56 – 3.07
h (m)	0.188 – 0.800
Fr_h	0.30 – 2.25
Fr_L	0.14 – 0.57
h/L	0.075 – 0.320

Table 1 - Test conditions

All model tests were conducted at the Australian Maritime College's (AMC) Model Test Basin (MTB). In total over 275 runs were completed for two hull forms, with the summary outline test programme shown in Table 1. Initially the tests covered an extended trans-critical Froude depth range, ($0.5 < Frh < 2.5$); however later tests concentrated on the sub-critical to critical range ($0.5 < Frh < 1.0$).

3.2 MEASUREMENT AND DATA ACQUISITION

Measurement of the resultant model wave pattern was undertaken utilising an array of 17 resistance-type wave probes, (Figure 1). The transverse array 1 measured wave height, decay and leading wave angle, and the longitudinal array measured wave variation (growth). The primary data used in the analysis presented in the present work was acquired from wave probe 9.

3.3 HULL FORMS

Two catamaran hull forms were tested, one a low wash type (EH), the other a conventional fuller form (NPL+). The demi-hull form parameters for both models can be seen in Table 2, and the body plans are shown in Figure 2 and Figure 3.

Parameter	EH	NPL+	
		Heavy Δ	Light Δ
L (m)	2.50	2.50	2.48
B (m)	0.125	0.23	0.22
T (m)	0.071	0.11	0.10
Δ (kg)	23.16	51.28	40.00
$L/\Delta^{1/3}$	11.14	8.48	9.14
L/B	20.00	10.87	11.27
B/T	1.75	2.09	2.20
C_P	0.625	0.693	0.652
s/L	0.4	0.4	0.4
Static Trim	0°	0°	0°

Table 2 – Hull form parameters

4. RESULTS

4.1 RESULTS PRESENTATION

Wavelet analysis provides both numerical and visual outputs. For this work the key numerical outputs utilised are peak spectral energy and its associated frequency. The visual outputs utilised are two dimensional (2D) and three dimensional (3D) combination plots.

Each 2D plot is a combination of two separate yet related sub plots. The upper sub-plot is a standard longitudinal wave cut, of wave amplitude against time. The lower sub-plot is the associated wavelet analysis output of the longitudinal wave cut. Said output consists of: time (x-axis), plotted against frequency (y-axis), and is also plotted against spectral energy (z-axis).

For this work, and to facilitate comparison, the 2D wavelet analysis sub plots have their spectral energy (z-axis) normalised to a unity value. This enables the form and peak energy values of the output to be reviewed easily.

The 3D plots represent the same information as the 2D plots but in three dimensional form. Additionally the 3D plots do not have the spectral energy (z-axis) normalised, but it is left in engineering units. This enables the relative magnitude as well as form of the spectral energy to be shown.

It should be noted that for purposes of clarity, within the 2D plots, that the peak / maximum spectral energy is coloured red whilst the minimal spectral energy is coloured blue, (as per the legend bar). This colouring is carried over to the 3D plots; however the vertical scale of each 3D plot is set to the maximum value for that series and not to each maximum on that plot.

Additionally it should be noted that all results presented are derived from longitudinal wave cuts, measured at wave probe 9, (Figure 1). With the exception of the results presented in Section 4.7 (Effect of time), which utilise probes 15, 16, 12, 17, 9, 18 and 9.

4.2 REVIEW METHOD

The devised metrics for reviewing wavelet analysis results (plots + numerical) are;

(i) the <u>value</u> of the peak spectral energy.
(ii) the <u>location</u> of the peak spectral energy,
(iii) the <u>frequency</u> of the peak spectral energy,
(iv) the frequency <u>range</u> of the global spectral energy,
(v) the <u>form</u> of the global spectral energy,

4.3 EFFECT OF WATER DEPTH (h)

The effect of water depth was investigated utilising wavelet analysis for the NPL+ hull form. A matrix of four water depths, (h = 200 / 400 / 600 / 800 mm), and three Froude depth numbers, (Fr_h = 0.5 /0.7 / 1.0), was tested. It should of course be noted that, for a fixed Fr_h value, over a range of water depths, the Fr_l value must vary. For the sake of brevity results from only one Froude depth number, (Fr_h = 0.7) have been reproduced.

From a review of 2D (normalised E) plots (Figure 4 to Figure 7) it is observed that, as the water depth progressively increases; (i) the average energy frequency reduces; (ii) the energy frequency range / band becomes narrower and (iii) the form of the energy becomes less intermittent or scattered. Additionally the location of the peak spectral energy (shown in red) stays relatively constant within the first wave packet.

From a review of the 3D plots (Figure 8 to Figure 11), a significant change in the peak spectral energy (E) is clear. This is in addition to the aforementioned changes in spectral energy average frequency and form.

Values of E were measured for each case and the following was observed. For Fr_h = 0.5 and Fr_h =0.7 (both sub-critical), as h increases, so too does the E value. However for Fr_h = 1.0 (critical) the E value does not increase in the same manner.

As discussed in [12], when operating at or near the critical Fr_h, the measured wave height results are time dependant. Accordingly without a full time history of the event, any single result is essentially a snap-shot, which may or may not be at the maximum value. This was reported as "unsteadiness" due to soliton generation. This phenomenon would seem to also hold true when measuring E at the critical Fr_h number.

4.4 EFFECT OF FROUDE DEPTH NUMBER (Fr_h)

The effect of Fr_h (for fixed water depth) was investigated utilising wavelet analysis for the NPL+ hull form. A matrix of three Froude depth numbers, (Fr_h = 0.5 /0.7 / 1.0), and four water depths, (h = 200 / 400 / 600 / 800 mm), was tested. Again it should be noted that for a fixed Fr_h value, over a range of water depths, the Fr_l value must vary. For conciseness only one water depth (h = 400mm) has been reproduced.

From a review of the 2D (normalised E) plots (Figure 12 to Figure 14) it is observed that as Fr_h progressively increases: (i) the average energy frequency reduces and (ii) the energy frequency range / band is narrower. Similar trends were found for constant Fr_h but with changing water depth, as per Section 4.3.

From a review of the 3D plots (Figure 15 to Figure 17), a significant change in E value is clear. This is in addition to the aforementioned changes in spectral energy average frequency and form.

Values of E were measured for each case and it was observed that, at all water depths, as the Fr_h increased so too did the E value. However it is noted that the E values for Fr_h = 1.0 may be time dependant (unsteady).

4.5 EFFECT OF DISPLACEMENT (fixed h)

The effect of displacement was investigated utilising wavelet analysis for the NPL+ hull form. A matrix of two displacements (Δ = light / heavy), three Froude depth numbers, (Fr_h = 0.5 /0.7 / 1.0), for one water depth (h = 400 mm), was tested. For the sake of brevity only Fr_h = 0.5 is shown.

From a review of the 2D (normalised E) plots (Figure 18 and
Figure 19), interestingly it is observed that the spectral energy characteristics, (i.e. average frequency, range and form), are very similar between displacements. This finding is common across all Fr_h tested. As shown in the 3D plots (Figure 20 and Figure 21) the form remains consistent with the change in displacement but the energy value increases with an increase in displacement

A review of the measured peak spectral energy shows that, (for all Fr_h values), an increase in displacement leads to an increase in E.

4.6 EFFECT OF HULL FORM

The effect of hull form was investigated utilising wavelet analysis for a fixed water depth (h = 250mm). A two hull forms (NPL+/ EH) were tested for five depth Froude Numbers (Fr_h = 0.5 / 0.7 / 1.0 / 1.3 / 1.5). From a review of the 2D & 3D plots it was observed that there was a small difference in spectral energy form between the two hulls. For brevity no results have been presented.

4.7 EFFECT OF TIME (SOLITON)

The effect of time was investigated for the NPL+ hull form. The hull was run at a fixed water depth (h = 200 mm), and for a fixed Froude depth number, (Fr_h = 1.0).

In the Authors' previous paper on unsteadiness, [12], a comparison of the measured wave heights along the main longitudinal probe array (Figure 1) was utilised to determine (soliton) wave growth. It was believed that Wavelet analysis would capture the dynamics of the soliton cycle.

The 2D (normalised E) plots (Figure 22 to Figure 27) show that the average spectral energy frequency and its associated range / band are relatively similar. From close inspection it can be observed that there is a slight yet noticeable difference in the form of spectral energy across the probes. The measured peak spectral energy (E) though shows little change in value, and there is no change in the associated peak frequency value.

5. DISCUSSION

The devised methods for reviewing wavelet analysis results were successful in characterising vessel wash due to changes in; (i) water depth, (ii) Froude depth number, (iii) displacement & (iv) hull form.

With regard to the effect of displacement & the effect of hull form, it is noted that, as per the Authors' earlier works [34], the EH and NPL+ hull forms have high $L/\Delta^{1/3}$ and L/B ratios, which is a characteristic of their relatively low wave heights.

Additionally both hull forms are catamarans, and it is possible that a mono hull would have significantly different wavelet characteristics. Accordingly (for these hull forms) wavelet analysis does not strongly identify the hull form variation. A systematic series of fuller-form mono hulls (e.g. the AMECRC Systematic Series [35]), may be more suitable for illustrating the influence of hull form on wave wake through wavelet analysis.

With regard to the effect of time (soliton), the wavelet analysis results are inconclusive. It is probable that Wavelet analysis of a complete soliton (time) event, (initiation – growth – shed), would show more clearly the change in spectral characteristics.

6. CONCLUDING REMARKS

The key benefit of wavelet analysis lies in its graphical output, which highlights features not clearly discernible from (traditional) numerical values alone.

The 2D plots enable side by side comparison of the spectral form. The 3D plots clearly show the change in spectral magnitude as well as form.

These graphic outputs combined with the key wavelet analysis characteristic of being able to describe when events take place within a signal, make wavelet analysis a powerful and useful tool for wash characterisation.

Wavelet analysis has been shown to be a useful tool (metric) in characterising vessel wash. It has been able to capture the effects of water depth (Fixed $_{Frh}$), Fr_h (Fixed h), displacement (fixed h), hull form, and of time (soliton)

7. FURTHER WORK

An expected outcome of this wavelet analysis "proof of concept" work was the generation of more questions, and in turn the extension of the wash programme. Directly from this work there are four areas requiring further investigation.

Firstly wavelet analysis of a fuller-form mono hull would further highlight the effects of displacement. It may also enable comparison between mono hulls and catamarans. Secondly wavelet analysis on a complete systematic series (i.e. AMECRC), would enable a detailed review for changes in various hull form coefficients. Thirdly wavelet analysis of an entire soliton cycle may enable full characterisation and an alternate understanding of the event. Finally further exploration, (utilising wavelet analysis), of the effect of Fr_l in Deep Water.

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"nani gigantum humeris insidentes"

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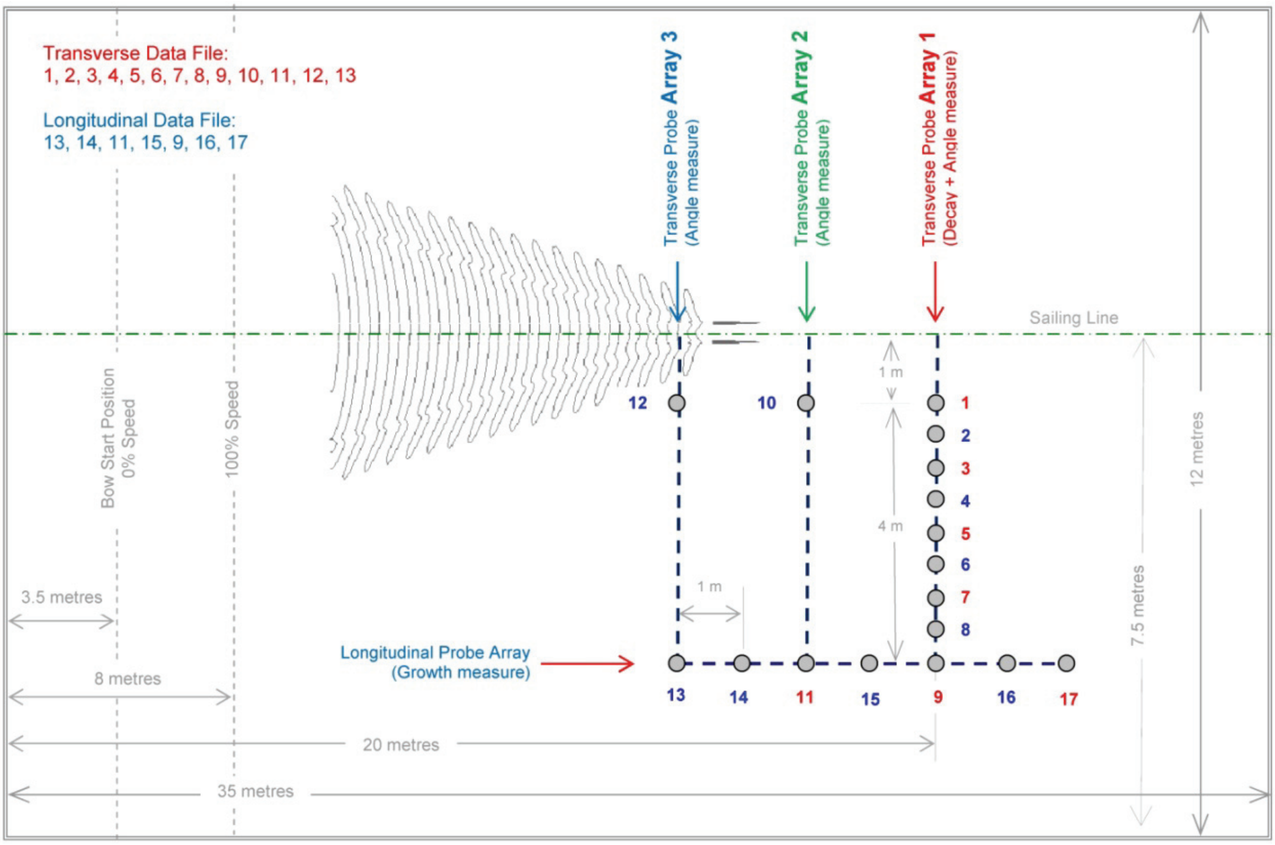

Figure 1 – AMC MTB Probe Arrangement

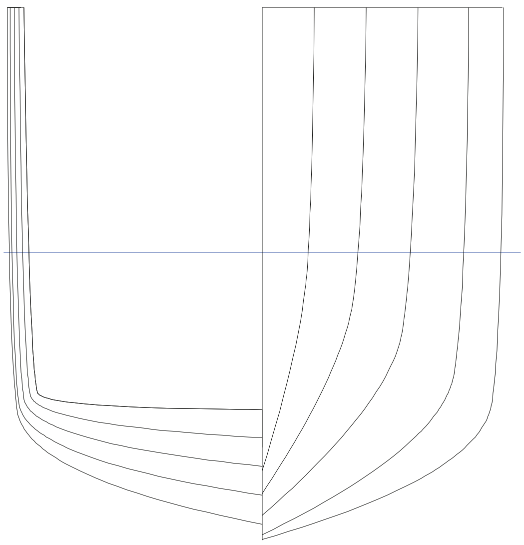

Figure 2 – EH Series Demi - hull Form (nts)

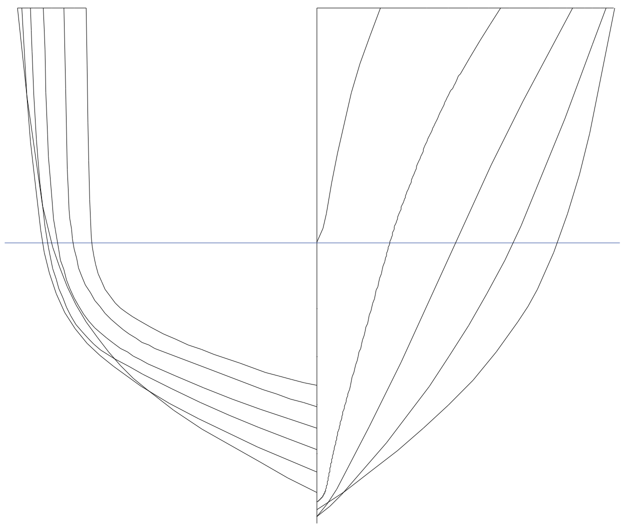

Figure 3- NPL+ Demi - Hull Form (nts)

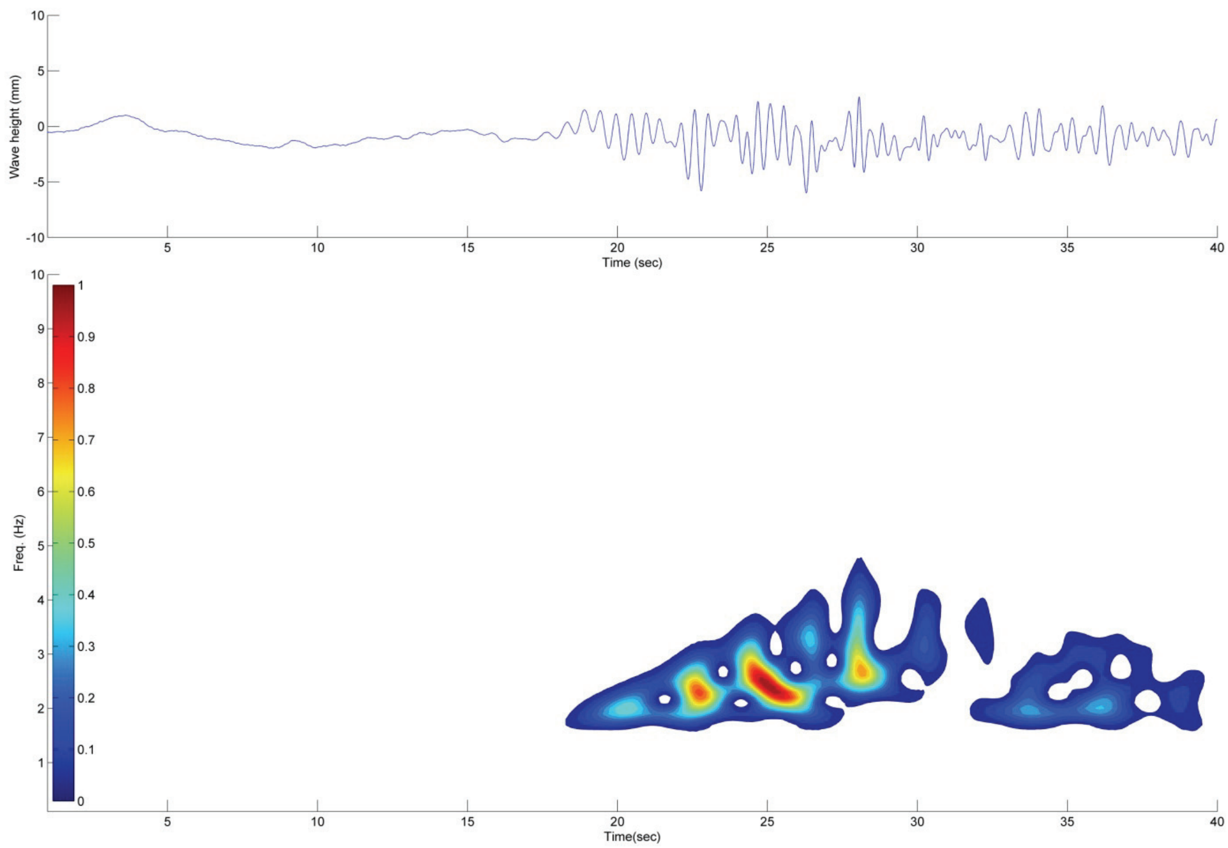

Figure 4 – Effect of Water Depth - 2D Plot - $Fr_h = 0.7$ - h = 200mm

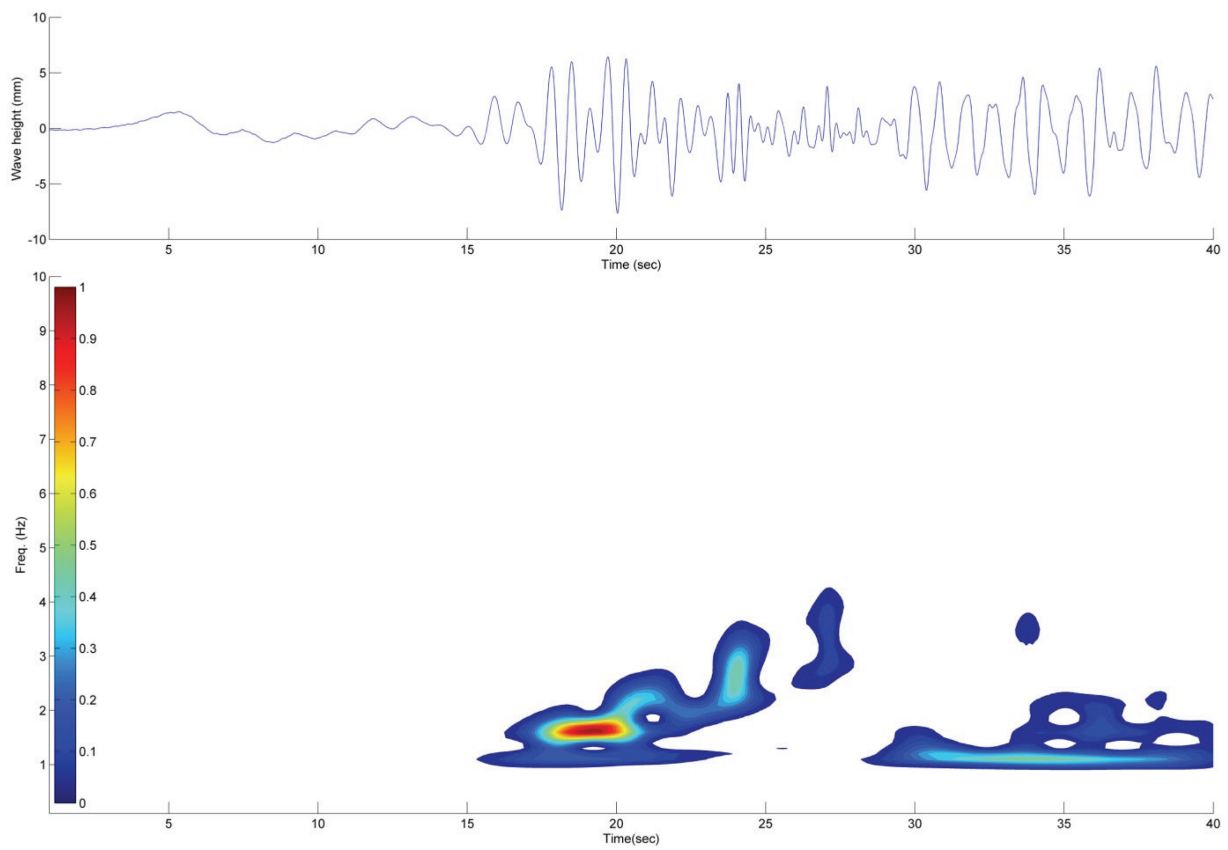

Figure 5 - Effect of Water Depth - 2D Plot - $Fr_h = 0.7$ - h = 400mm

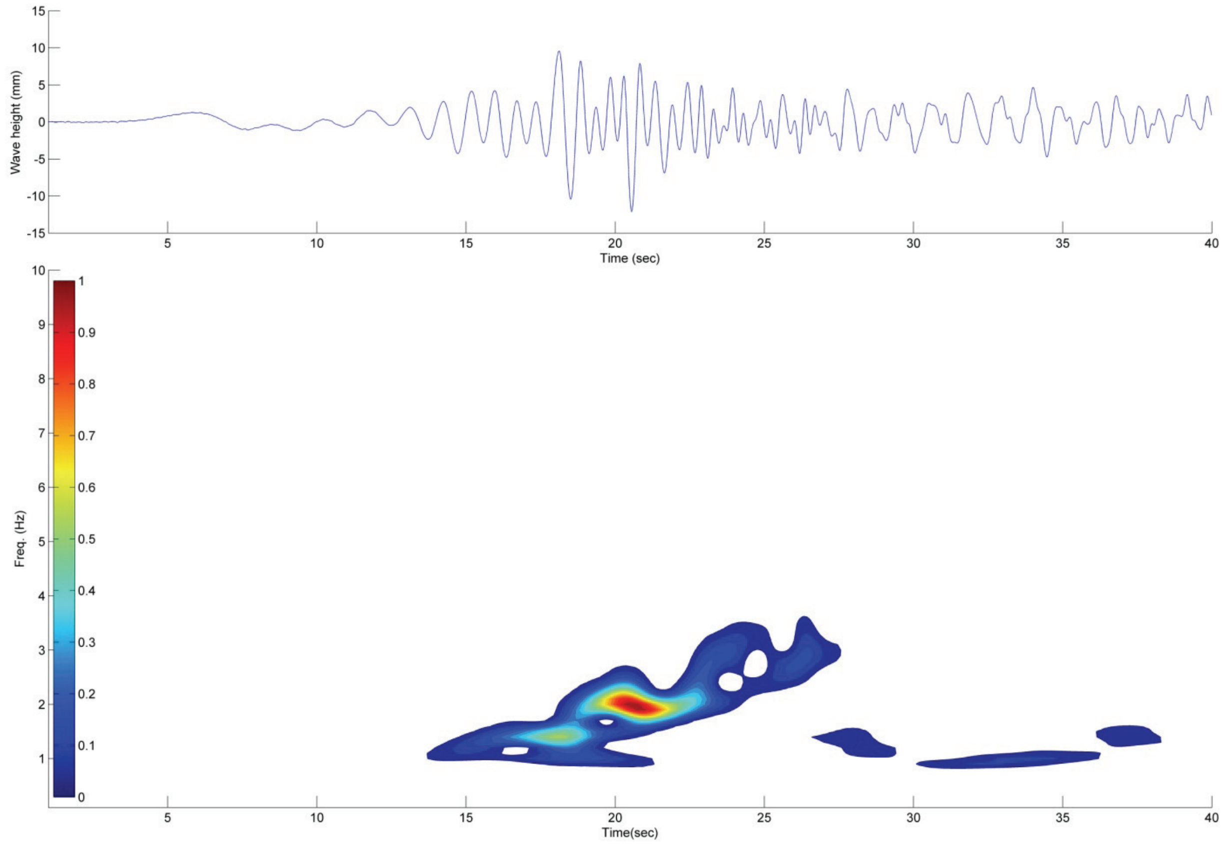

Figure 6 - Effect of Water Depth - 2D Plot - $Fr_h = 0.7$ - h = 600mm

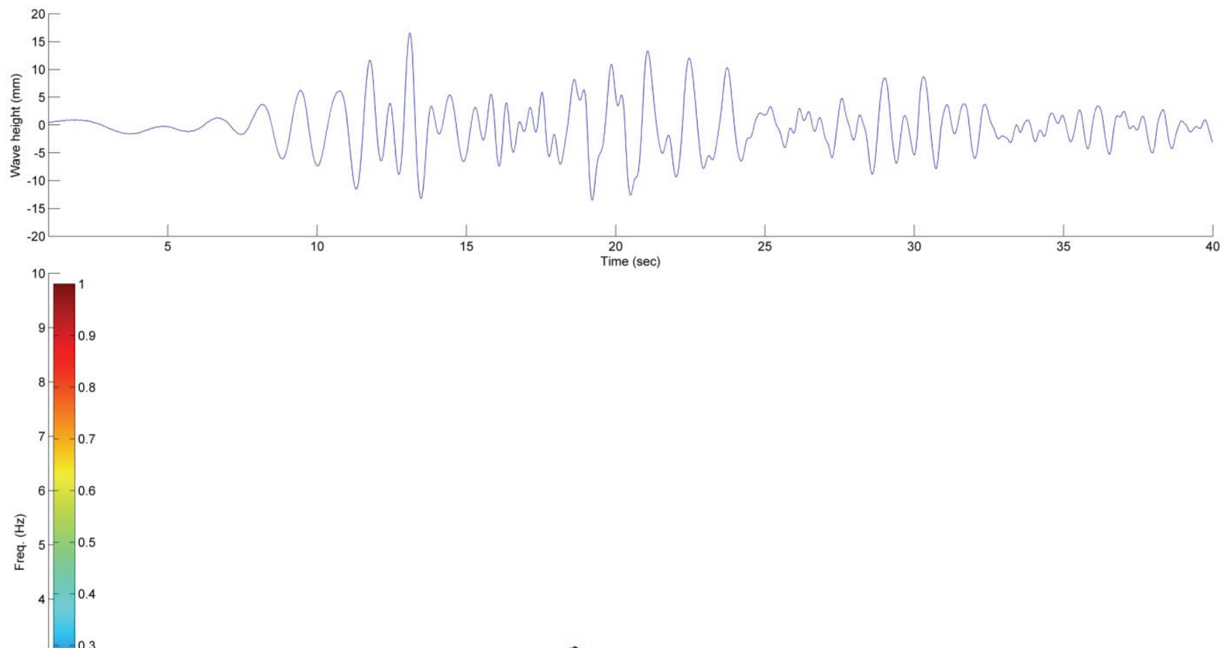

Figure 7 - Effect of Water Depth - 2D Plot - $Fr_h = 0.7$ - h = 800mm

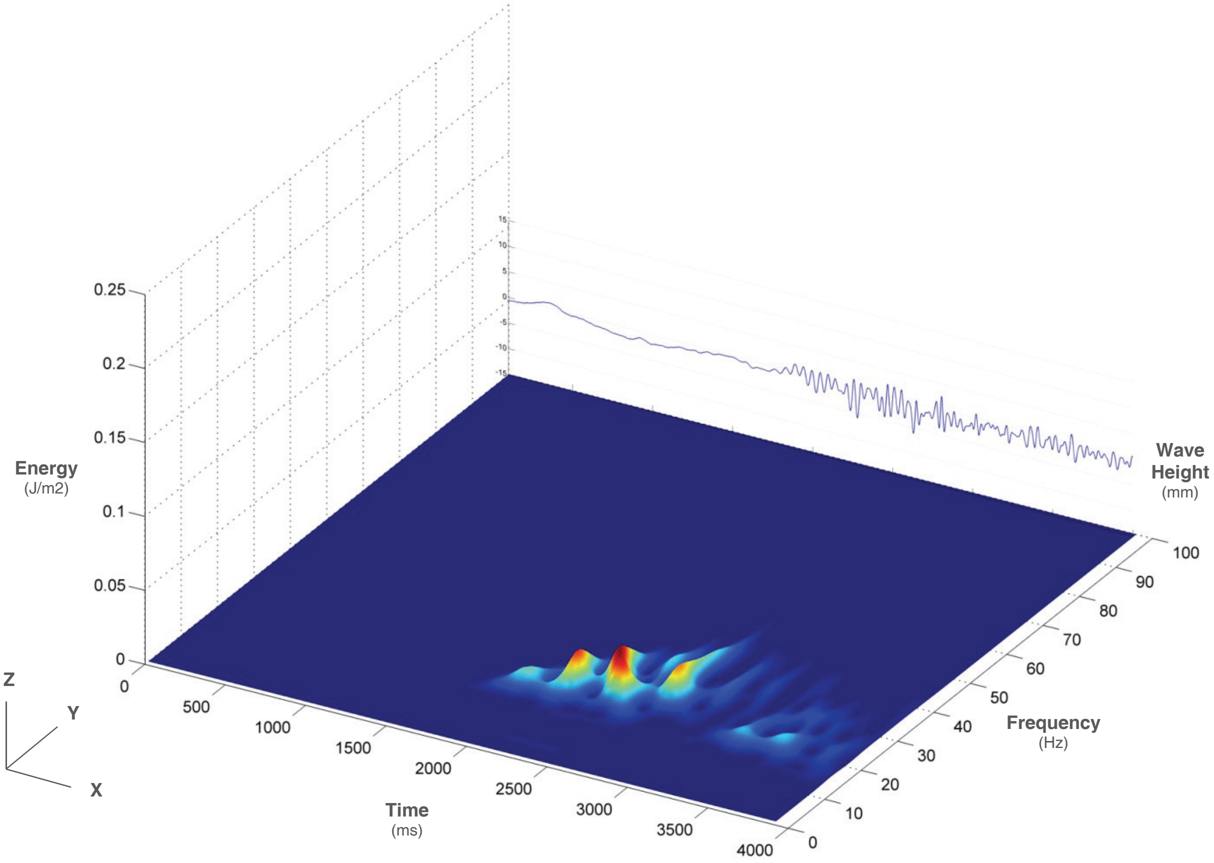

Figure 8 - Effect of Water Depth - 3D Plot - $Fr_h = 0.7$ - h = 200mm

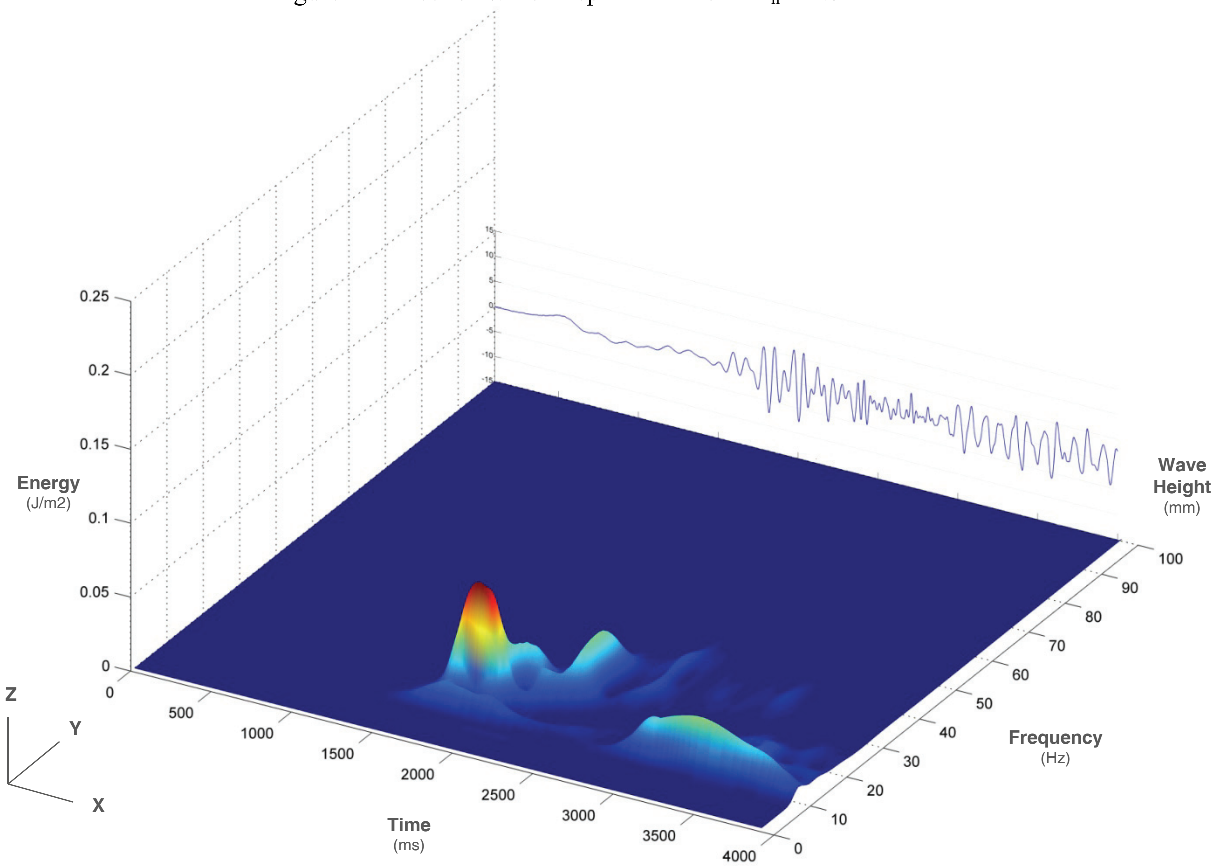

Figure 9 – Effect of Water Depth - 3D Plot - $Fr_h = 0.7$ - h = 400mm

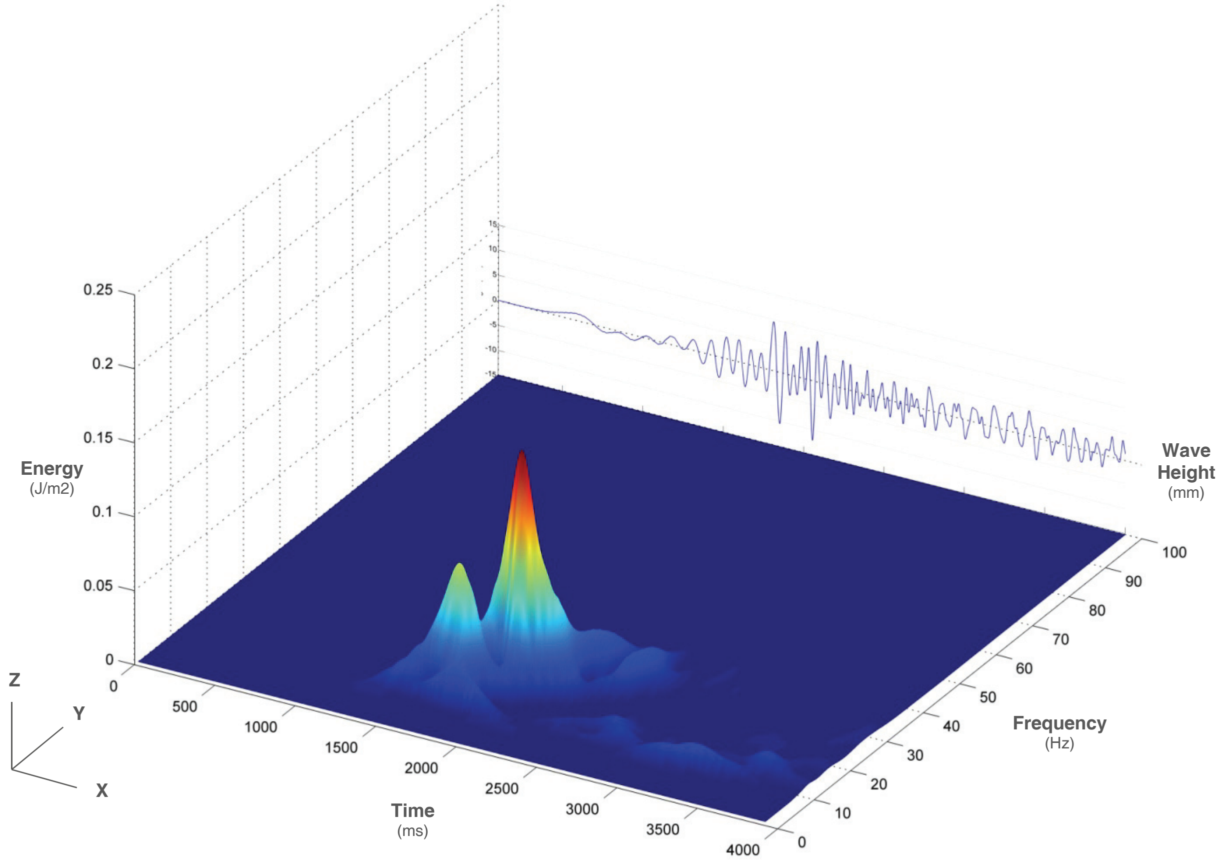

Figure 10 – Effect of Water Depth - 3D Plot - Fr_h = 0.7 - h = 600mm

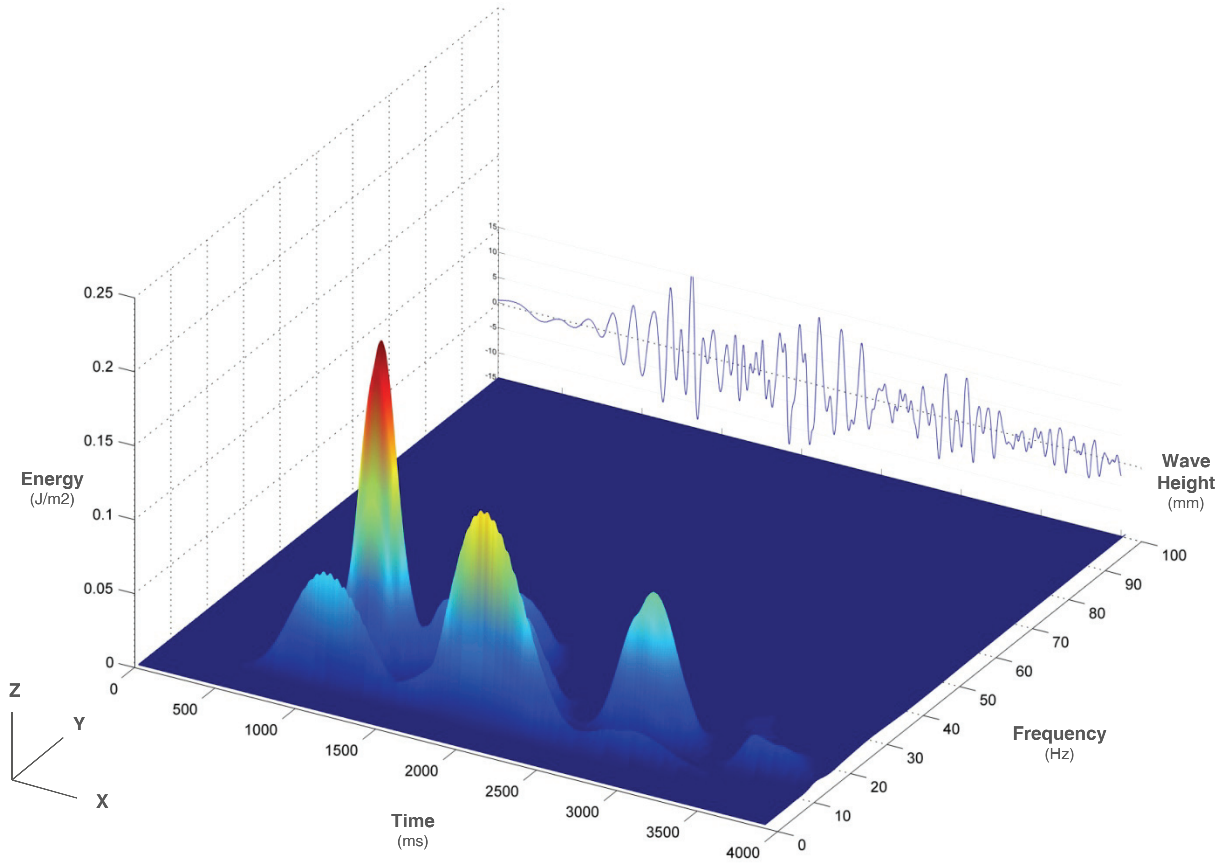

Figure 11 – Effect of Water Depth - 3D Plot - Fr_h = 0.7 - h = 800mm

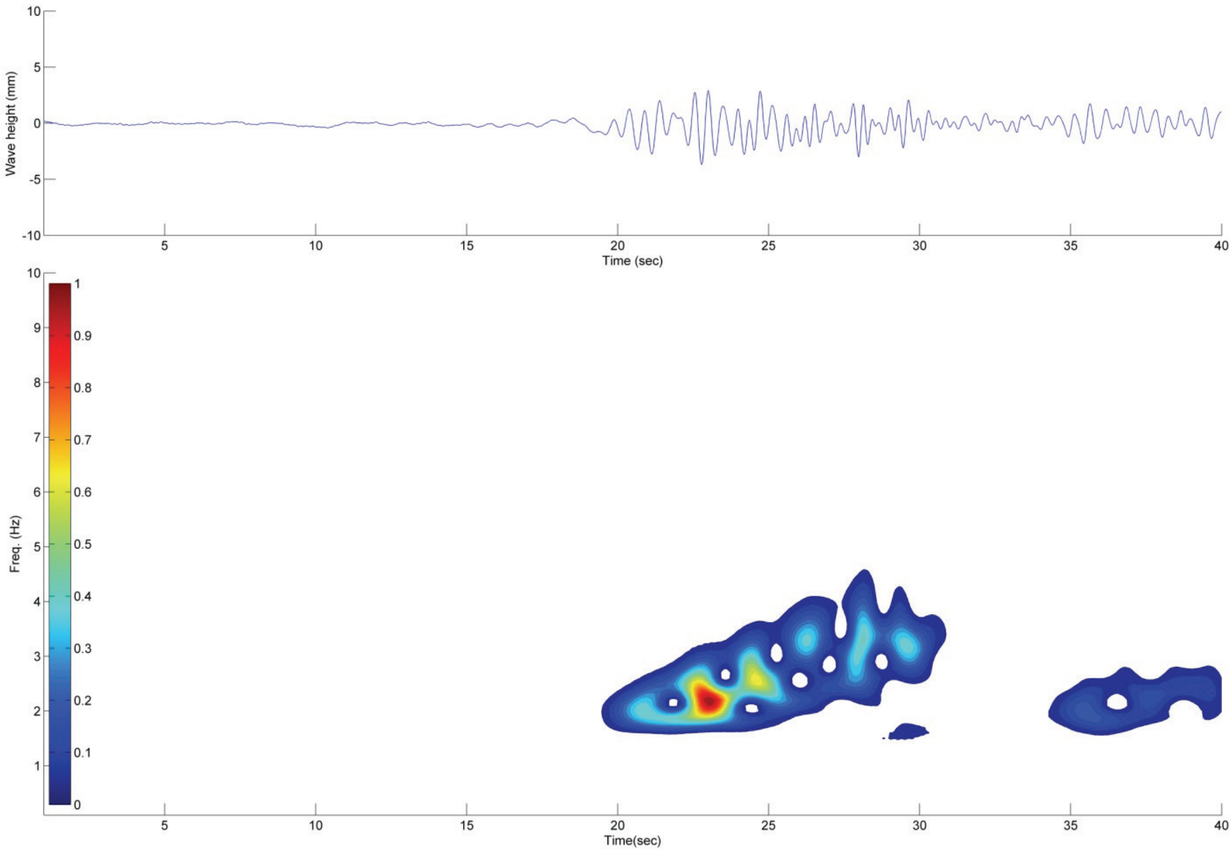

Figure 12 – Effect of Fr_h - 2D Plot - h = 400mm - Fr_h = 0.5

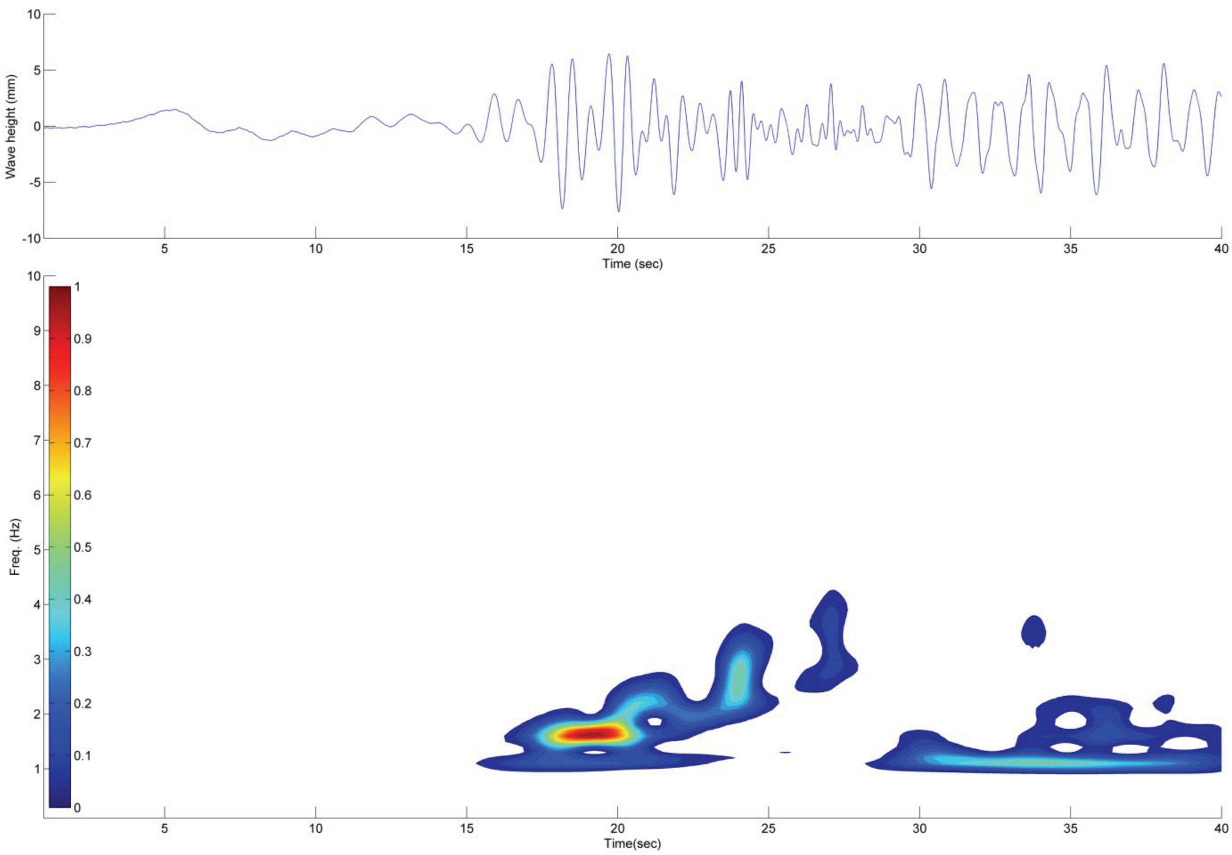

Figure 13 - Effect of Fr_h - 2D Plot - h = 400mm - Fr_h = 0.7

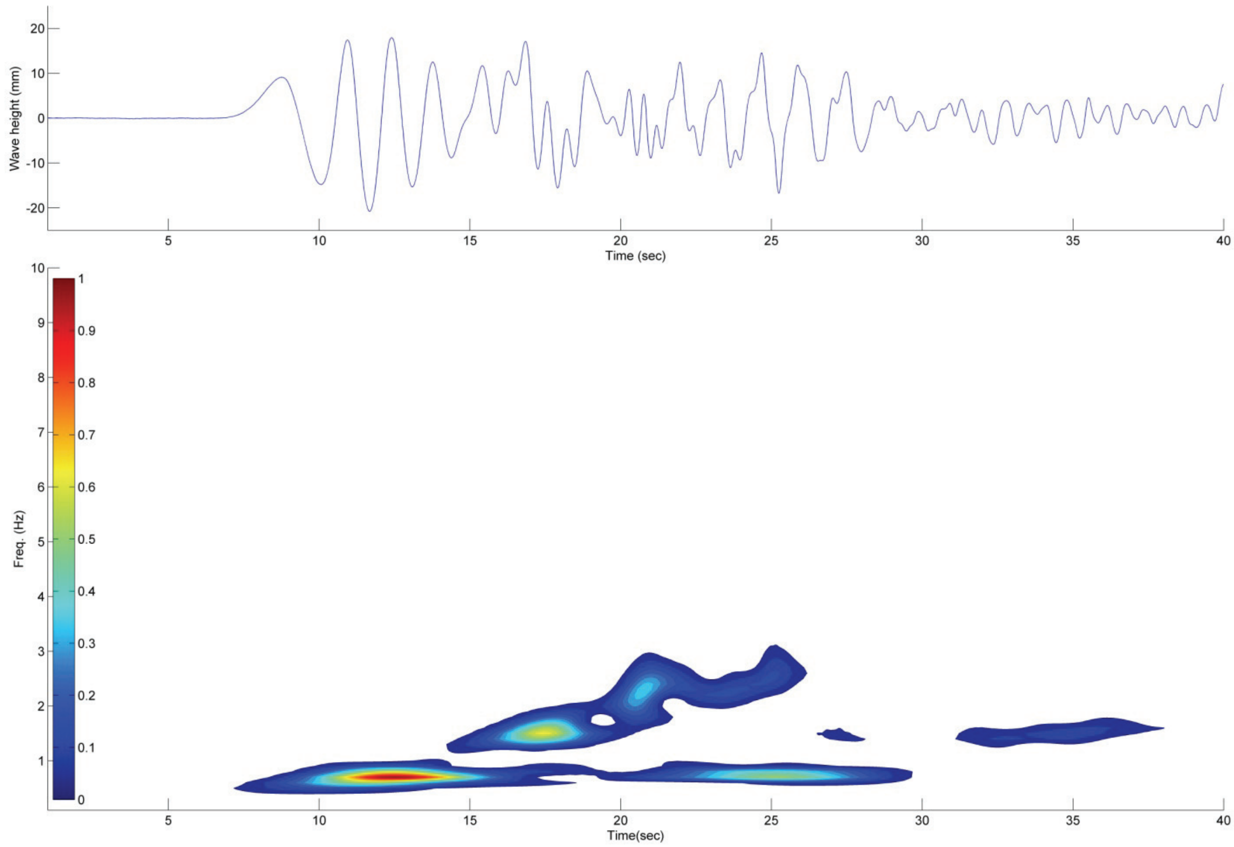

Figure 14 - Effect of Fr_h - 2D Plot - h = 400mm - Fr_h = 1.0

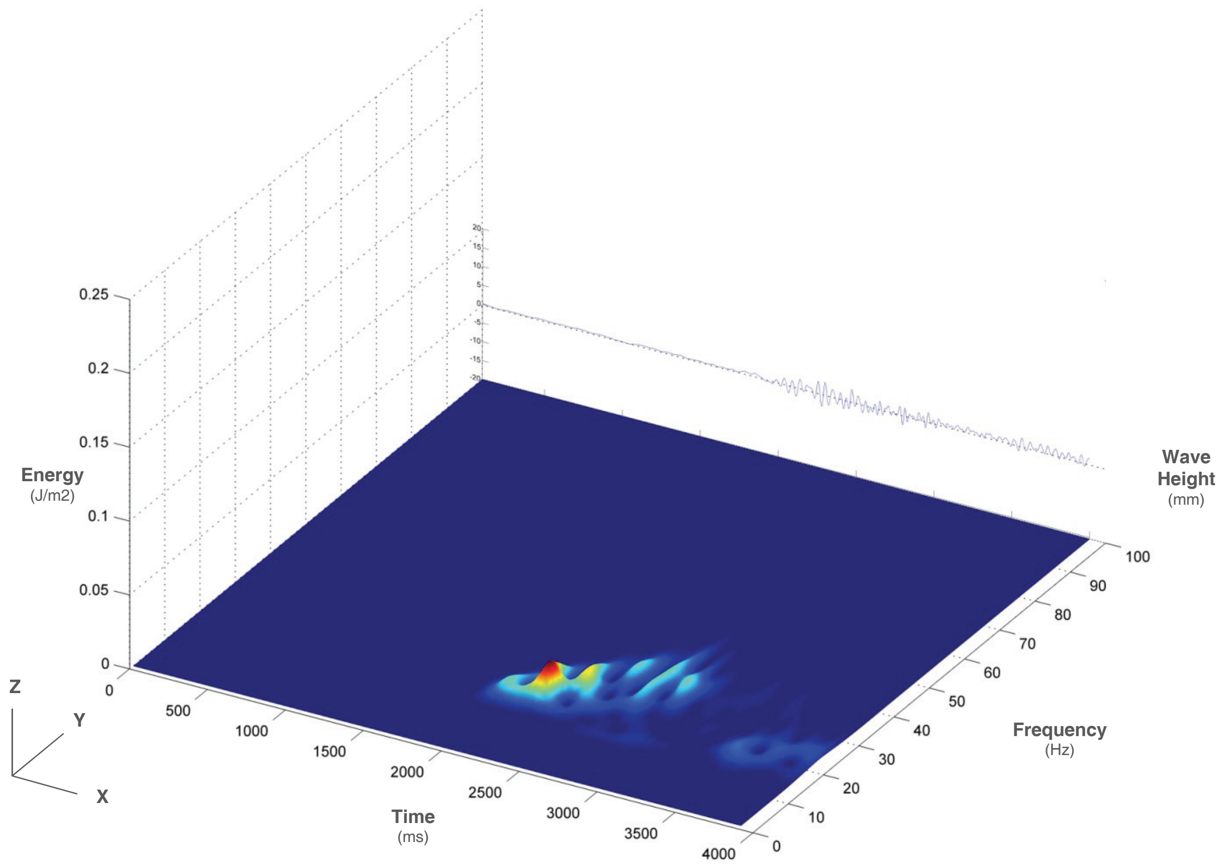

Figure 15 – Effect of Fr_h - 3D Plot - h = 400mm - Fr_h = 0.5

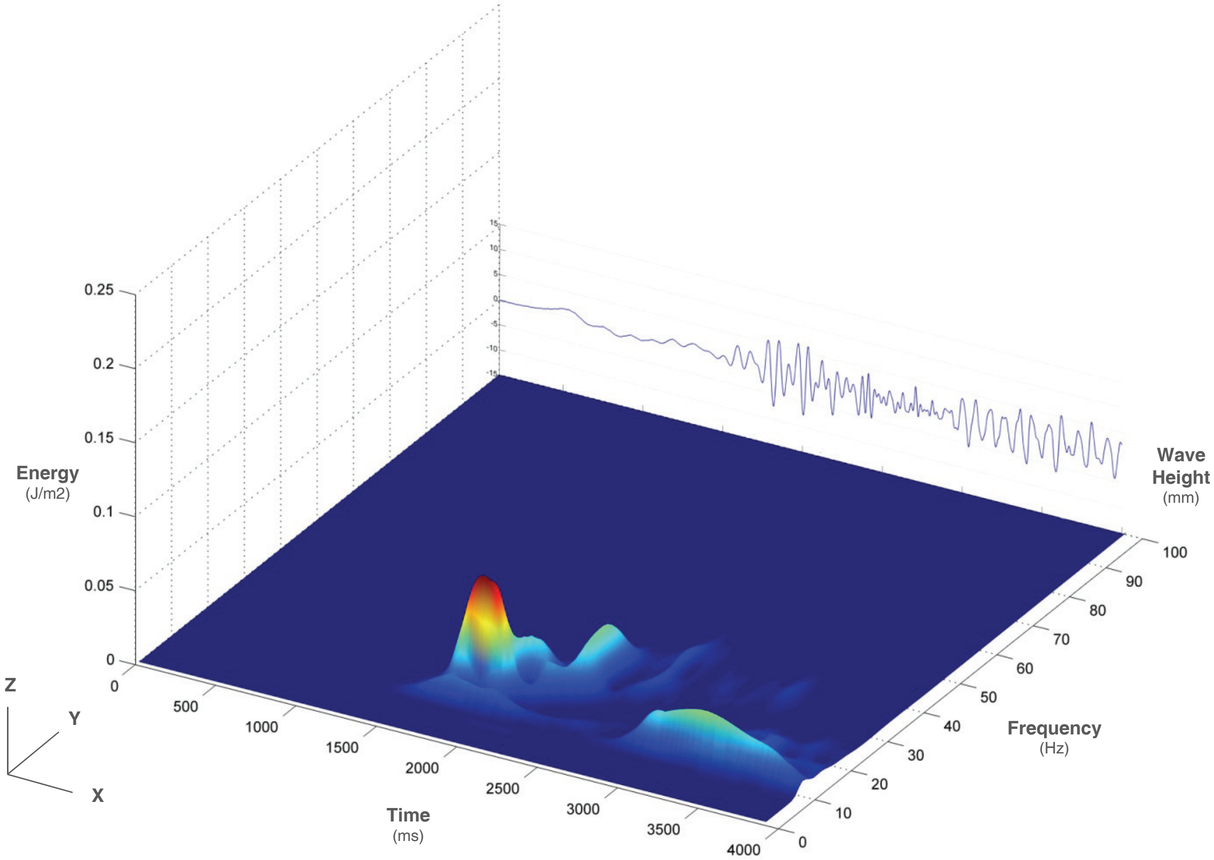

Figure 16 – Effect of Fr_h - 3D Plot - h = 400mm - Fr_h = 0.7

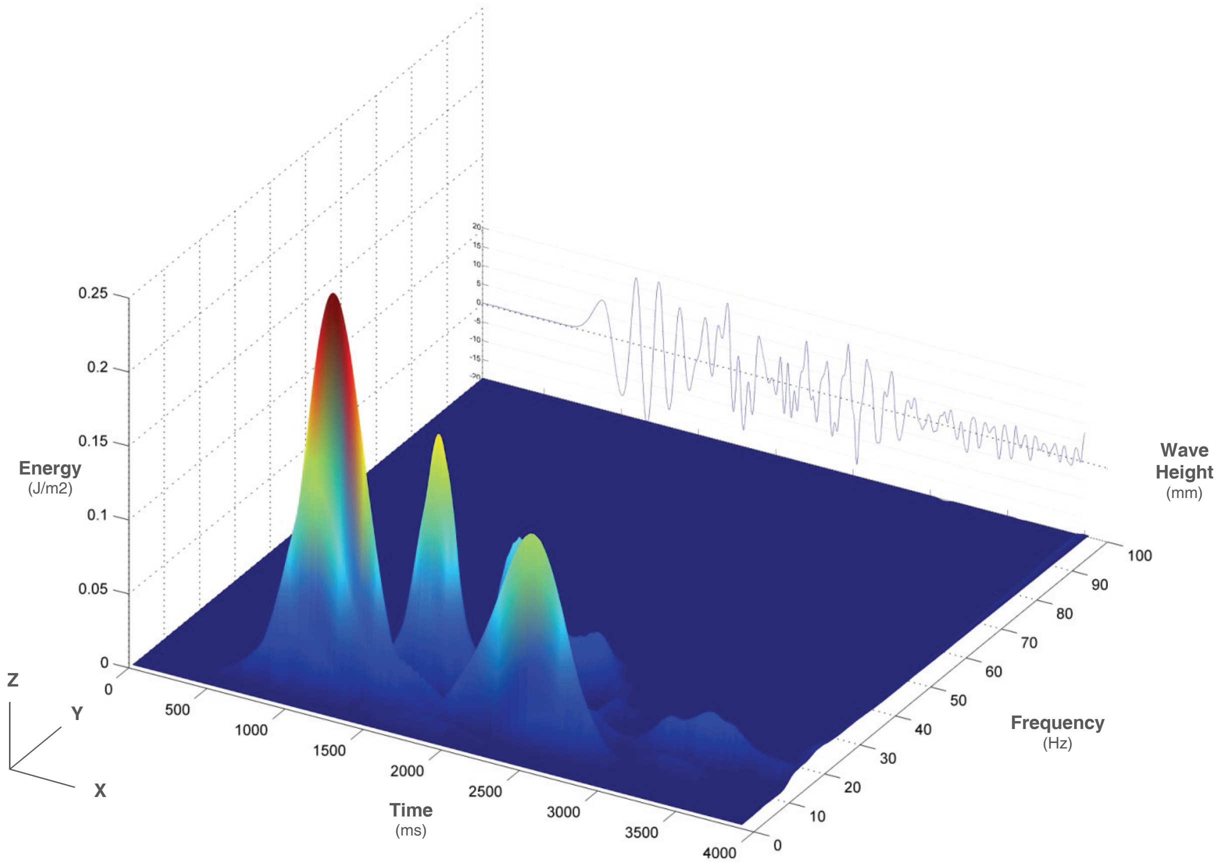

Figure 17 – Effect of Fr_h - 3D Plot - h = 400mm - Fr_h = 1.0

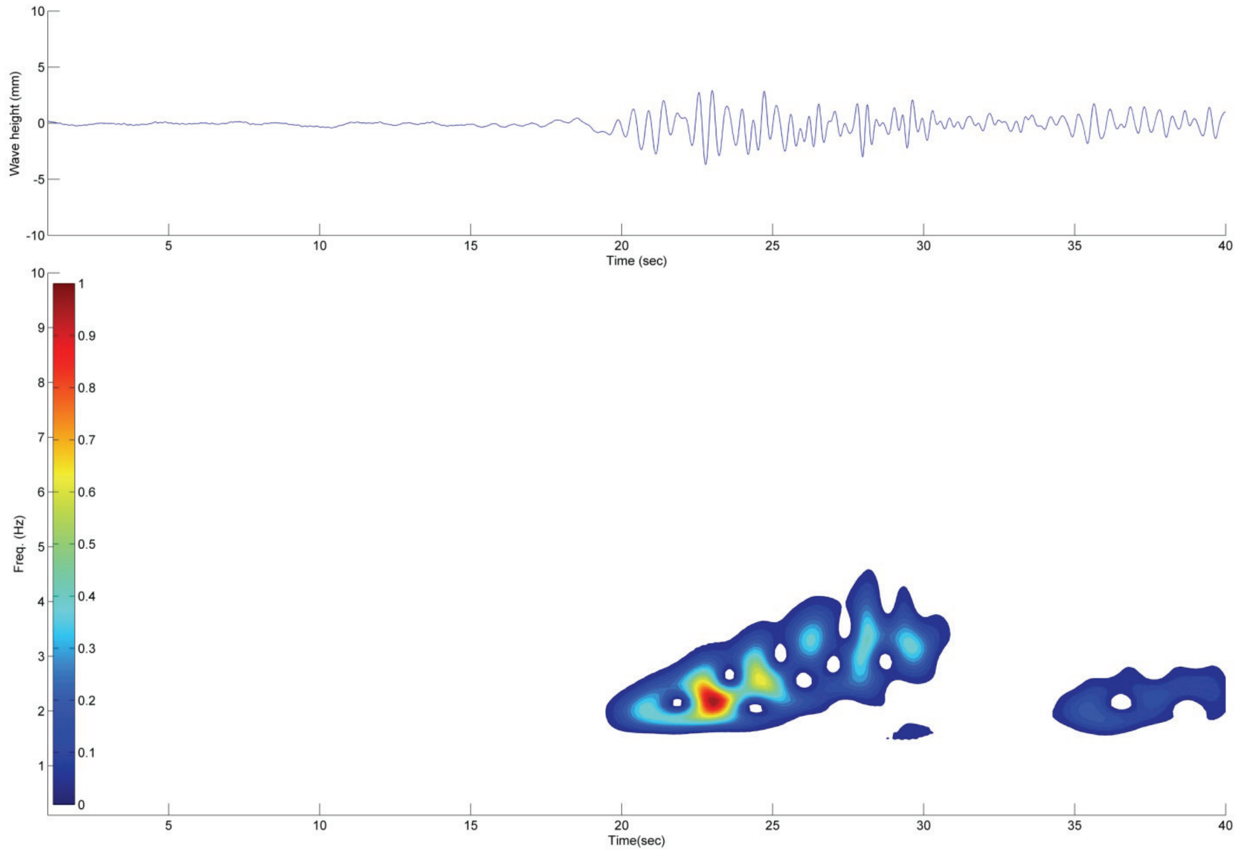

Figure 18 – Effect of Δ - 2D Plot – h = 400mm – Light Δ – Fr_h = 0.5

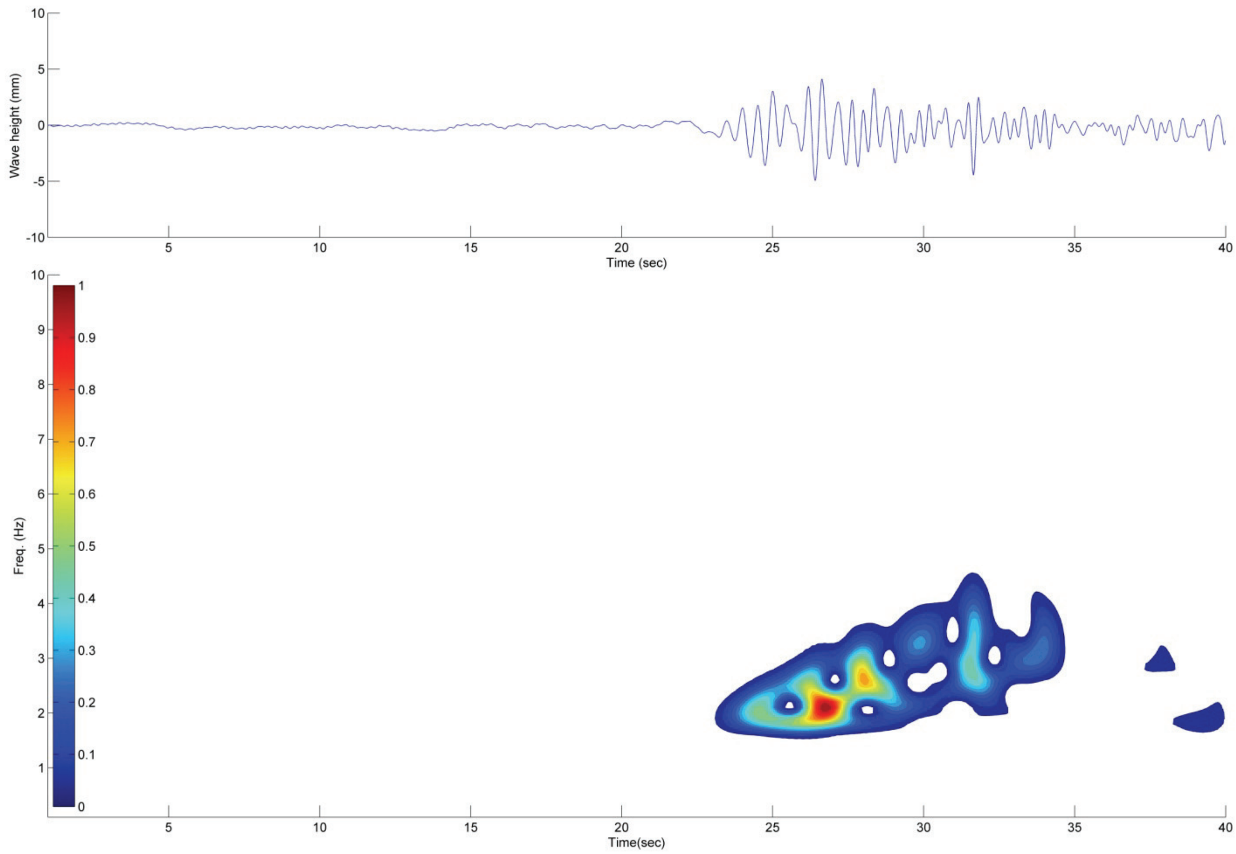

Figure 19 – Effect of Δ -2D Plot – h = 400mm – Heavy Δ – Fr_h = 0.5

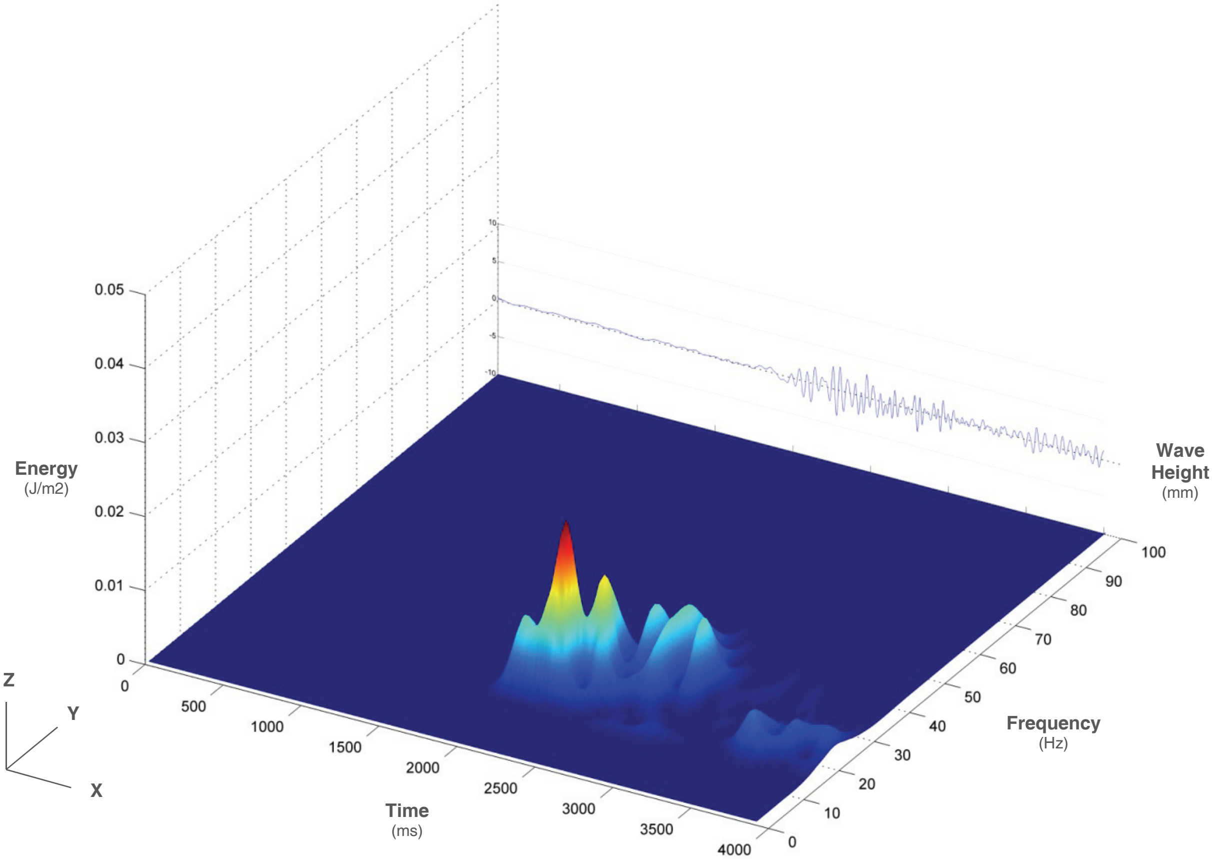

Figure 20 - Effect of Δ - 3D Plot - h = 400mm – Light Δ – Fr_h = 0.5

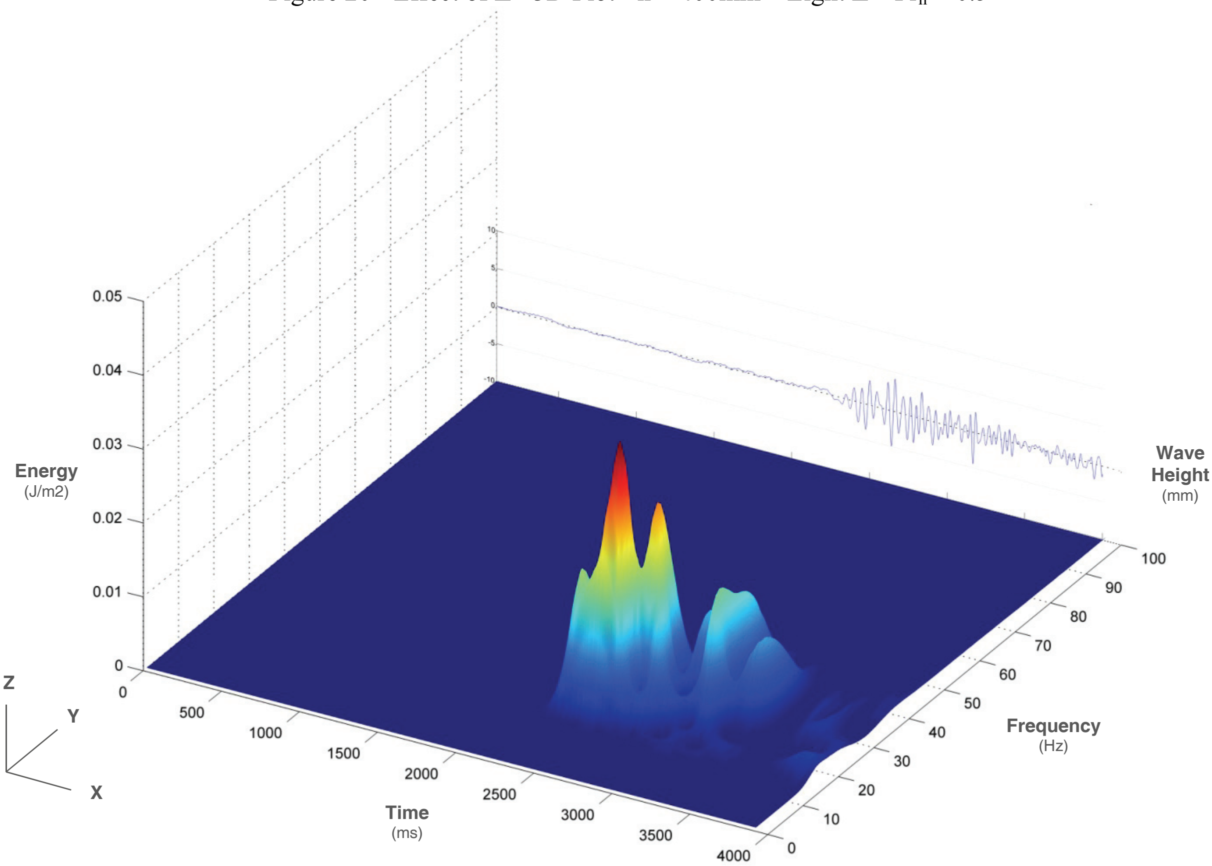

Figure 21 – Effect of Δ - 3D Plot - h = 400mm – Heavy Δ – Fr_h = 0.5

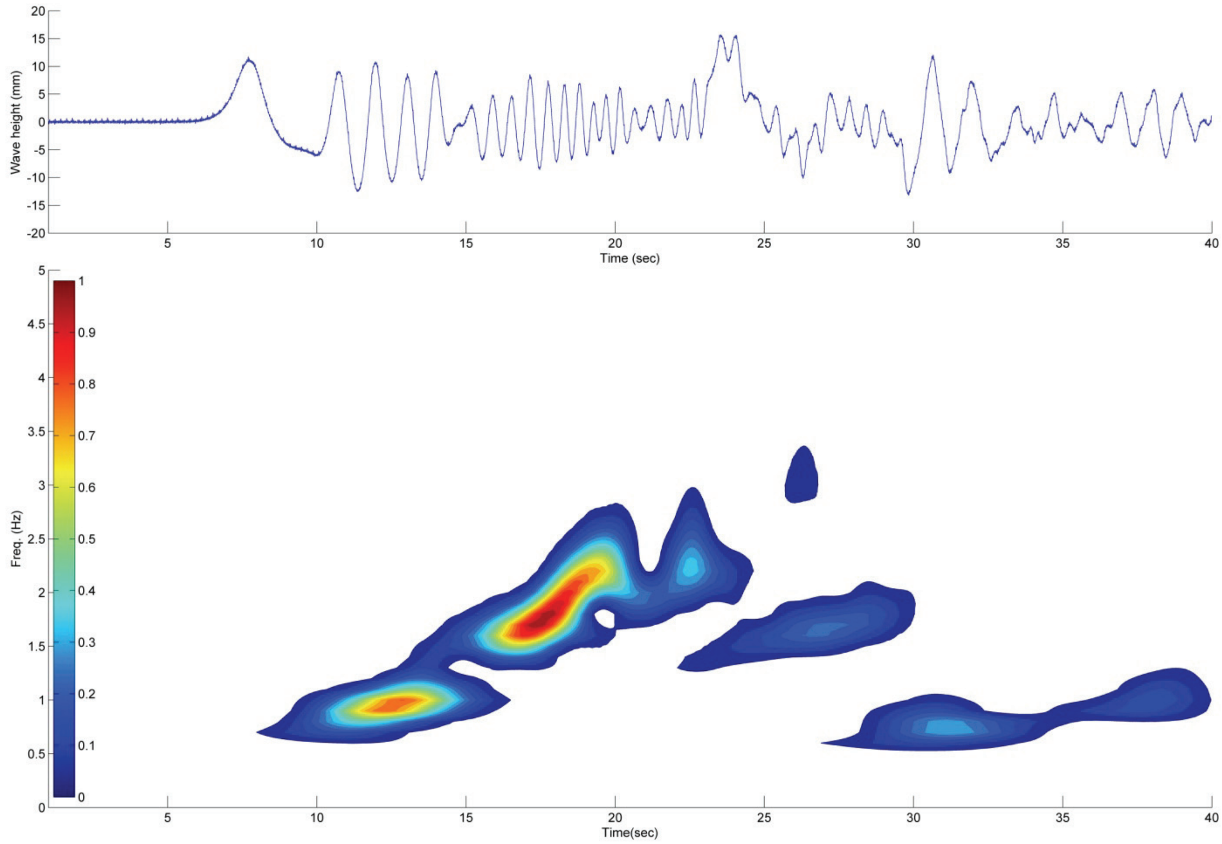

Figure 22 – 2D Plot – Hull = NPL+, h = 200mm, Fr_h = 1.0, Probe 13

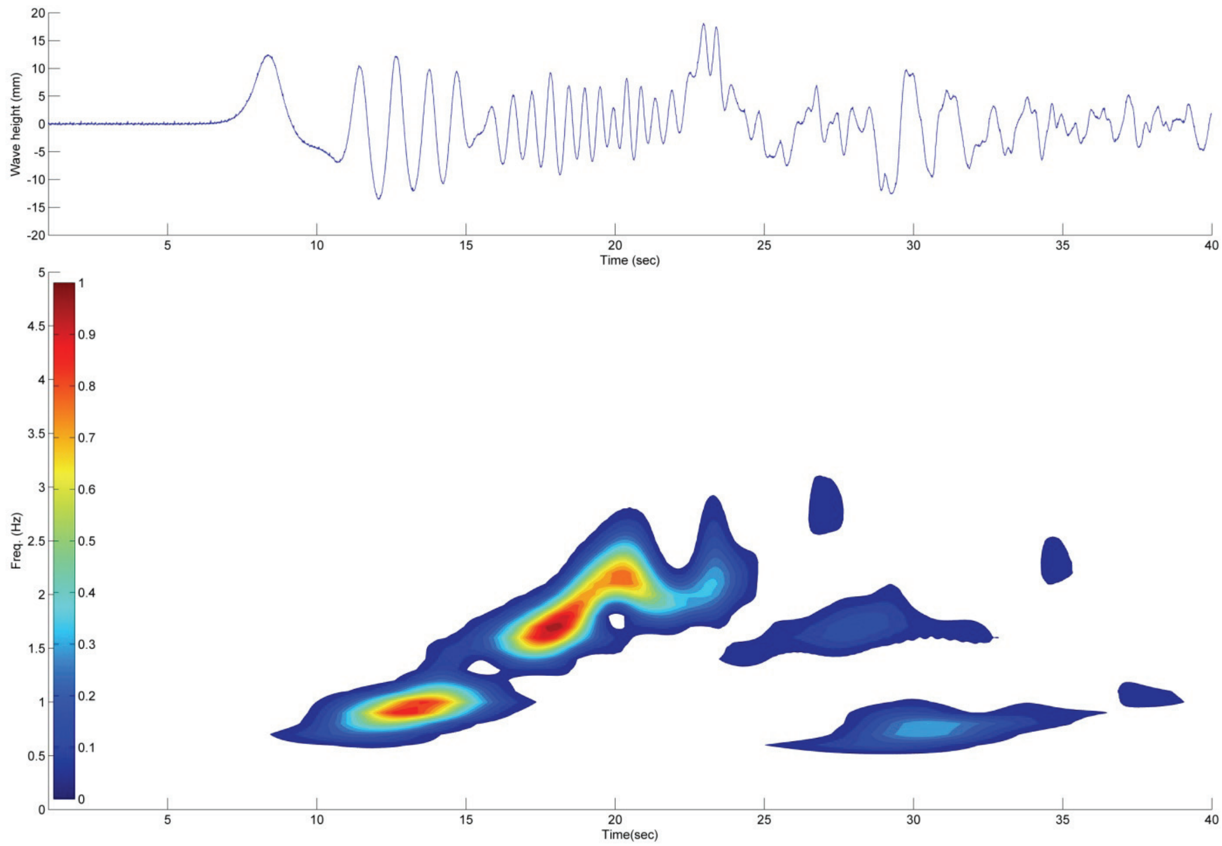

Figure 23 – 2D Plot – Hull = NPL+, h = 200mm, Fr_h = 1.0, Probe 14

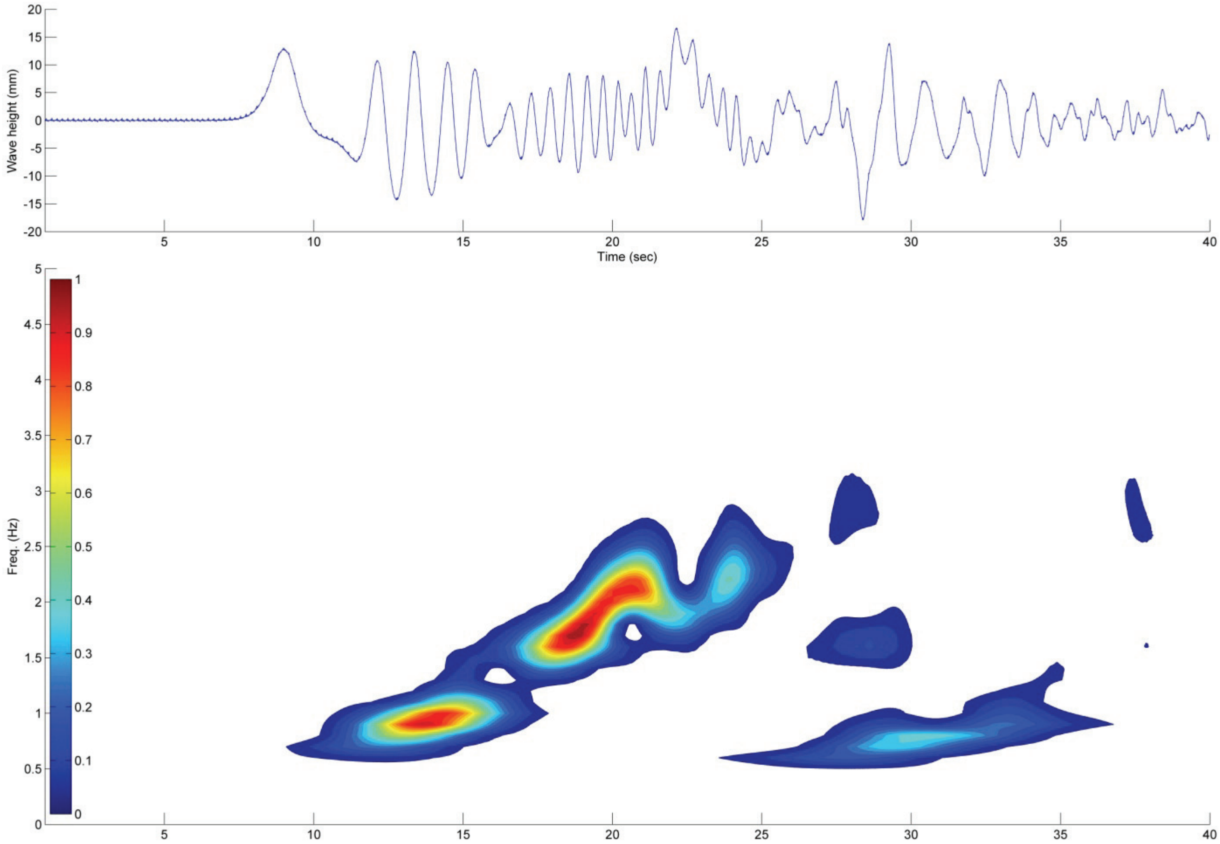

Figure 24 – 2D Plot – Hull = NPL+, h = 200mm, Fr_h = 1.0, Probe 11

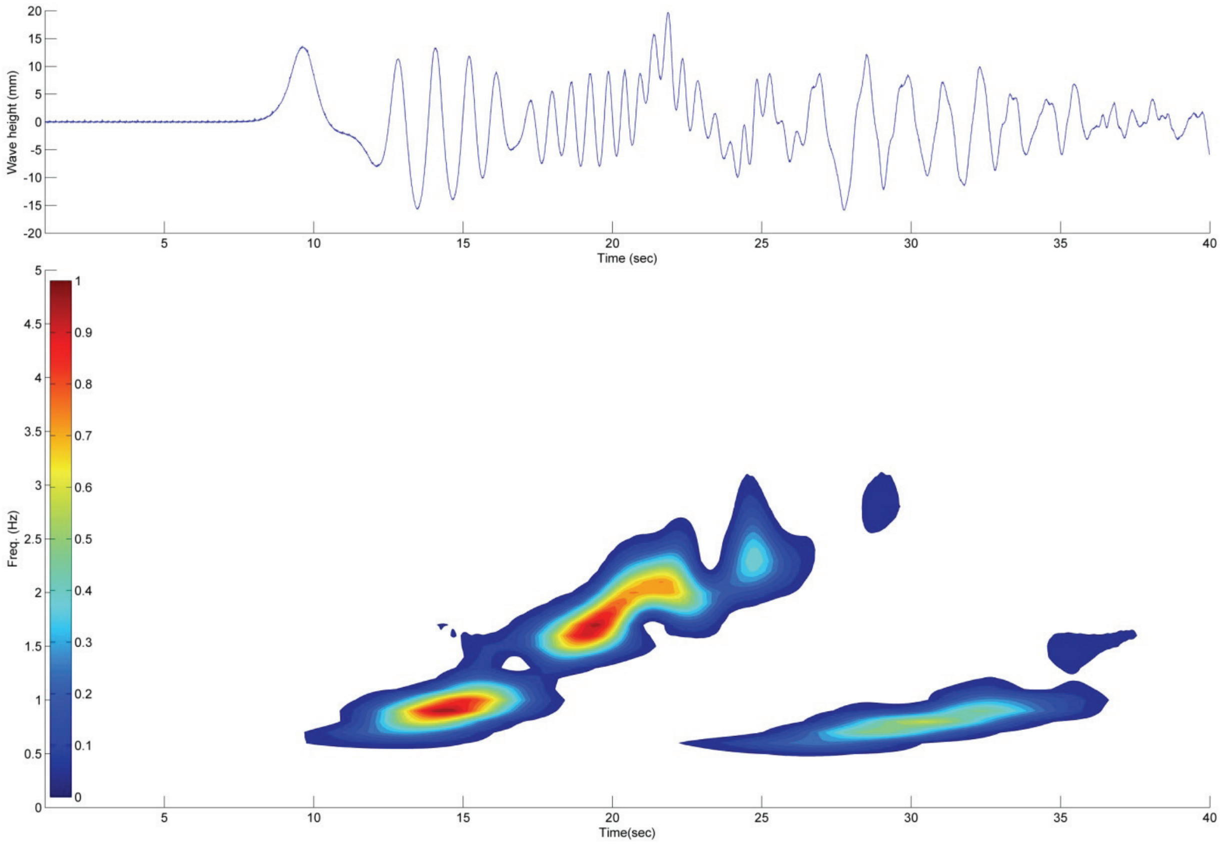

Figure 25 – 2D Plot – Hull = NPL+, h = 200mm, Fr_h = 1.0, Probe 15

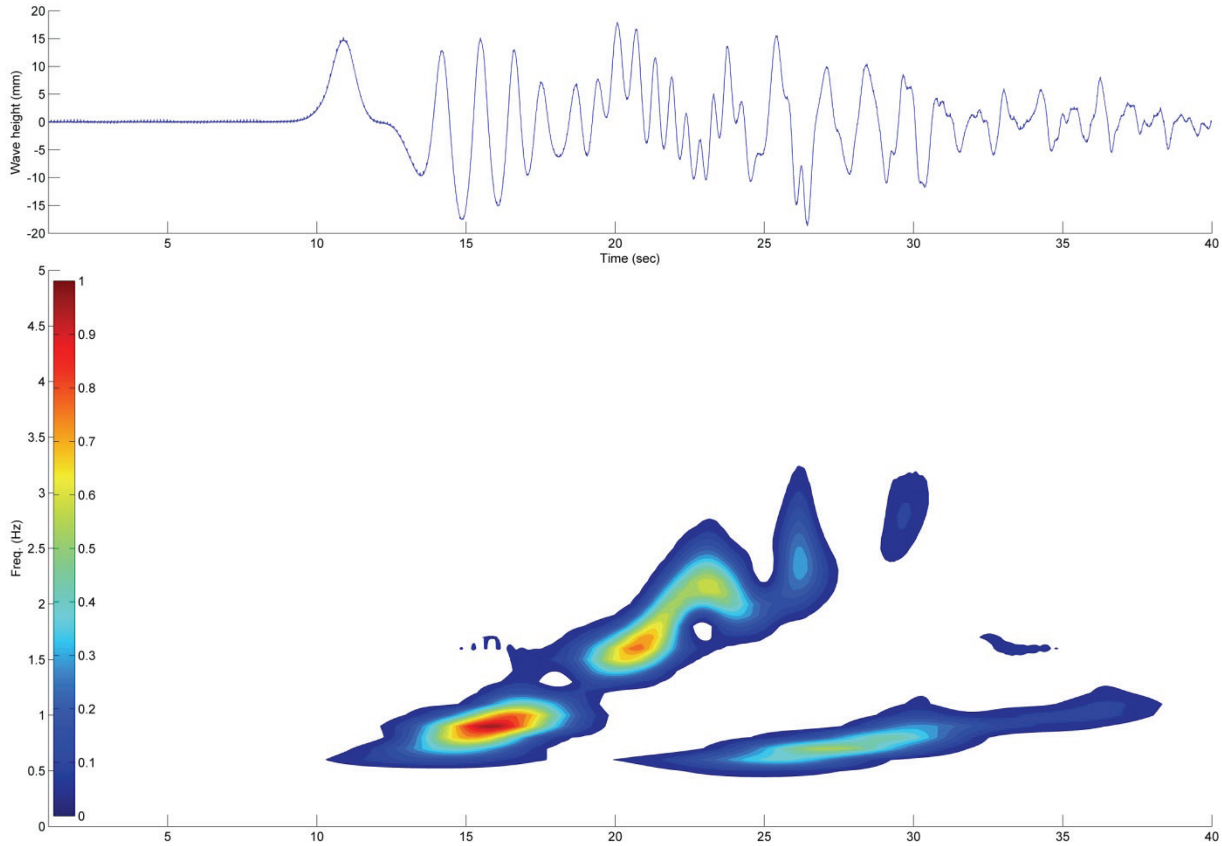

Figure 26 – 2D Plot – Hull = NPL+, h = 200mm, Fr_h = 1.0, Probe 16

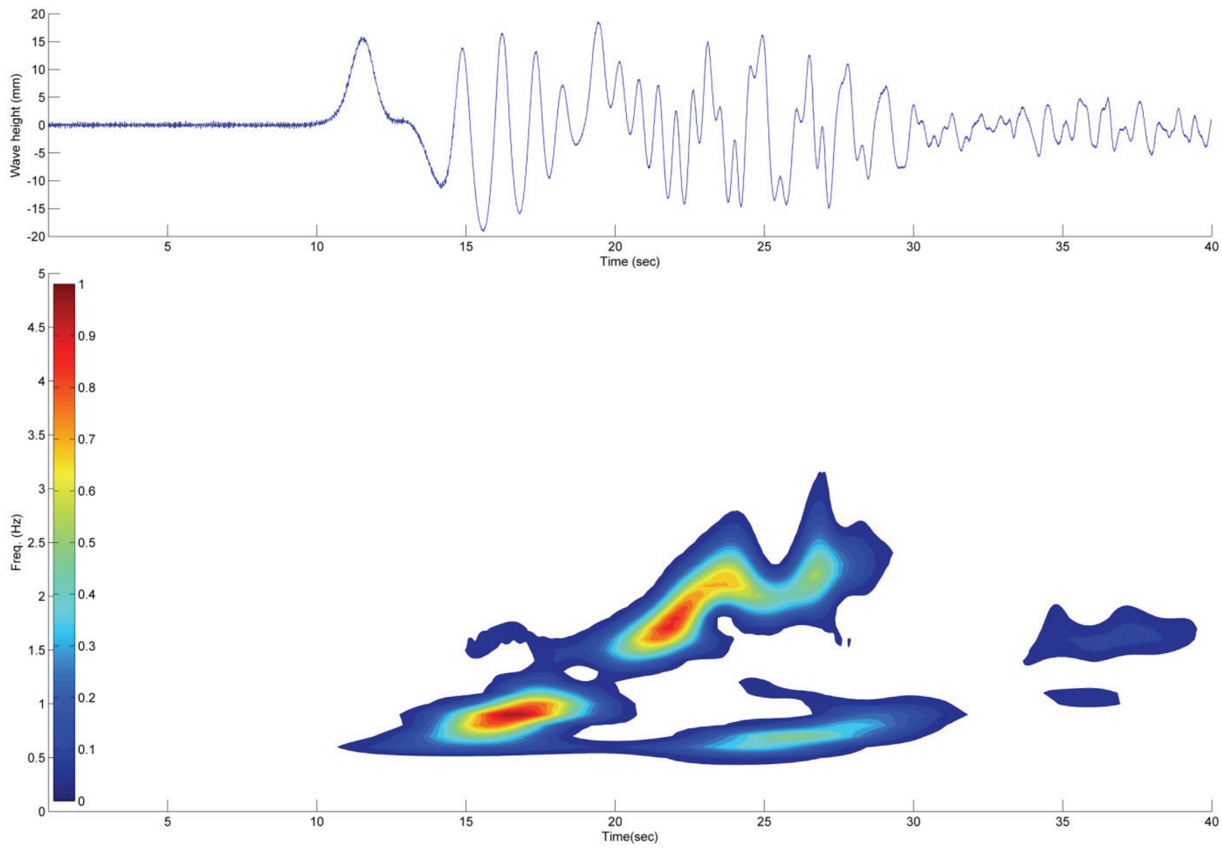

Figure 27 – 2D Plot – Hull = NPL+, h = 200mm, Fr_h = 1.0, Probe 17