TECHNICAL NOTE

ON THE APPLICATION OF THE EXTREME VALUE THEORY IN SHIP'S STRENGTH CALCULATIONS

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SUMMARY

A procedure is proposed for application of the extreme value theory (EVT) approach considering not only the maximal value of the corresponding random variable but also its probability of exceedance. It substantially reduces the probability of exceedance of any given limit value used in the case when traditional EVT is applied. Examples are provided to illustrate its application when records of the random process are available.

NOMENCLATURE

BM = Bending moment
C_B = Block coefficient
CDF = Cumulative Distribution Function
EVT = Extreme Value Theory
M_{SW} = Still water bending moment
M_t = Total bending moment
M_W = Wave bending moment
$M_{W,h}$ = Hogging wave bending moment
$M_{W,s}$ = Sagging wave bending moment
PDF = Probability Density Function
POE = Probability of Exceedance
POO = Probability of Occurrence
T = Time

1. INTRODUCTION

When records of a random process (e.g., obtained from full-scale measurement, model tests or numerically generated) are available, statistical analysis may be performed in two ways – either by application of the individual amplitude statistics or by extreme value statistics. In the former, all amplitudes are considered in the analysis while in the latter – only the maximal amplitudes in each time window.

When analyzing records of hull girder bending stresses/moments, the duration of each time window might be e.g. 5, 10, 20, 30 etc. minutes. If extreme value statistics is applied, only the maximal amplitude will be extracted from each time window. The probabilistic distribution type and probability of exceedance (POE) of the maximum amplitude in the corresponding time window is not considered as the present EVT stipulates. According to this theory (Ochi, 1989; Gumbel, 1958/2004, Kotz & Nadarajah, 2000), if M_n is the maximal variable among $X_1, X_2, \ldots, X_n$, i.e.

$$M_n(x) = \max\left(X_1,\ X_2,\ \ldots,\ X_n\right) \tag{1}$$

and $X_1, X_2, \ldots, X_n$ are independent and identically distributed variables, then the Cumulative Distribution Functions (CDF) F are:

$$F_{X_1}(x) = F_{X_2}(x) = \ldots F_{X_n}(x) = F_X(x) \tag{2}$$

The CDF of M_n will be

$$F_{M_n}(m) = P(X_1 \le m, X_2 \le m, .X_n \le m) = \left[F_X(m)\right]^n \Big\} \tag{3}$$

where n = number of observations within a given service life (e.g., number of cycles).

The EVT provides formulas (see, e.g., Ochi 1989) for calculation of the parameters of the newly derived extreme value distributions (Gumbel-, Frechet-, Weibull – type) using the information for the corresponding parent probabilistic distributions. This approach is the most frequently used due to its convenience, especially in design stages. As an example, the formulas given below for the CDF illustrates the way it can be done for asymptotic extreme value distribution type one (Ochi, 1989):

$$F_e\left(y_e\right) = \exp\left\{-\exp\left[-\alpha\left(y_e - \mu\right)\right]\right\} \tag{4}$$

Where μ = location parameter
α = scale parameter
y_e = corresponding random variant (e.g., M_W)

The parameters α and μ are calculated following the procedure described by (Ochi M, 1989).

The parameter μ is the probable extreme value expected to occur in "n" observations (e.g., "n" cycles). It can be evaluated from the parent CDF (e.g., M_W) for which the probability of exceedance (POE) of this value is 1/n, i.e.:

$$F(\mu) = 1 - 1/n \tag{5}$$

Where $F(\mu)$ = parent CDF of M_W calculated for $y_e = \mu$ (the design M_W in class societies' rules).

The other parameter α is calculated by the formula (Ochi, 1989):

$$\alpha = f(\mu) / [1 - F(\mu)] \tag{6}$$

where f = parent probability density function (PDF);

F = CDF (both functions are calculated for $y_e = \mu$)

The other approach is to use numerical methods when records of the random process are available. In this case, the basic steps in the application of the EVT are shown in Figure. 1.

Step 1: A new histogram is built that contains only the maximal amplitudes in each time window.

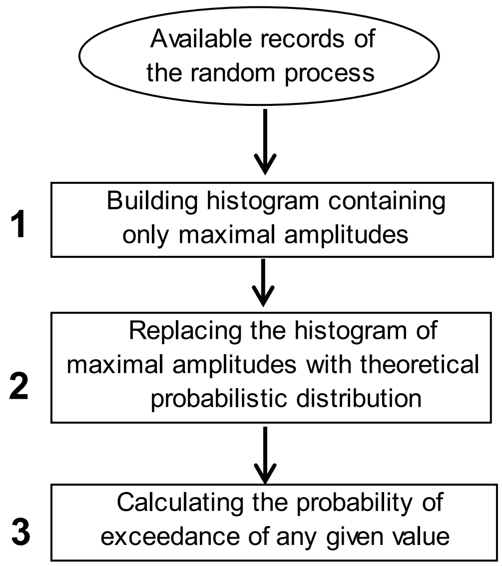

Figure. 1 Basic steps in EVT approach

Step 2: The histogram of all maximal amplitude is replaced by theoretical probabilistic distribution obtained by some of the available computer programs or analytical formulas. No doubt, the new histogram will be located much further towards the large values of the corresponding random variance under consideration relative to the original histogram built with the individual amplitude statistics.

Step 3: As one can expect, when the POE of the originally given permissible limit (used in the case when individual statistics are applied) is calculated, it is much higher than the originally calculated POE with the individual amplitude statistics. In some publications (e.g., Faulkner, Sadden 1979; Ochi 1973) on design wave bending moments given in classification societies' rules, its POE is calculated as 63.2% when extreme value statistics is applied. In mathematical terms, this number refers to the POE of the most probable extreme value. In a special case when the design wave bending moment is chosen such that its POE is 1/N (N is the number of cycles) it is treated as the most probable extreme value. Hence, the 63.2% POE of the most probable extreme value becomes 63.2% POE of the design M_W. Mathematically, the result is correct within the assumptions made in the EVT (i.e. when the random variables in each time window are independent and identically distributed and the design M_W is equal to the most probable extreme M_W). Whatever the interpretation of this 63.2%, if the POE of the design M_W in classification societies' rules is calculated by the traditional EVT following the above described procedure, then its POE will be much larger than if the individual amplitude statistics are used. Therefore, the proponents of the application of the EVT for assessing the hull girder bending moments propose increases of the design hull girder wave induced bending moments. Prior to doing that some aspects of the EVT application should be analyzed considering also the experience from real ship's operation. If the calculations with the EVT of the POE of this design hull girder bending moment are correct, ships would suffer more severe casualties than previously observed.

To avoid misinterpretations of the obtained numerical results with the EVT, it would be useful to address the above mentioned fundamental assumptions. One should either make some changes in the EVT or improve its application and interpretation.

2. PROPOSAL FOR A PROCEDURE CONSIDERING THE PROBABILITY OF OCCURRENCE OF THE MAXIMAL VALUE IN EACH TIME WINDOW

The basic steps in the proposed procedure are shown in Figure. 2. However, before commencing the explanation of each step in the proposed procedure, some information for the analysis of the available records of the random process should be given. In the paper, records of full scale measurements of a container ship are used. Comprehensive information for the multi-year full scale measurements can be found in (Yu H C et al 2006, 2008; Lee S J et al 2010). The strain gauges mounted on the hull girder of a container ship provide information for the recorded stresses. Based on the geometric properties of the hull girder at sections though the gauging' location, the records of the total bending moments (B.M.) are obtained. Since the study targets only vertical hull girder bending moment (VWBM), high-frequency load, horizontal and torsion bending moments are filtered.

The wave B.M. (M_W) and still water B.M. (M_{SW}) are extracted from the records of the total B.M. taking into consideration the ratio between wave-induced hogging B.M. ($M_{W,h}$) and sagging B.M. ($M_{W,s}$) as a function of block coefficient. For real ships, the sagging wave bending moment is always greater than hogging wave bending moment because the block coefficient (C_B) is always smaller than unity. This fact is reflected in all class rules. What causes this difference is the side structure flair in bow and stern part of the ship. The two bending moments will be equal only in the case when the block coefficient is equal to unity.

Abrahamsen and Vedeler (1957, 1958) derived the following formulas for the dimensionless coefficients, χ, for sagging and hogging wave bending moments:

$$\chi_{hog} = 1.55 C_B \qquad \chi_{sag} = 1.44 \frac{C_B + 0.8}{1.6} \tag{7}$$

where C_B = block coefficient

When the coefficients k are calculated, the corresponding $M_{W,h}$ and $M_{W,s}$ can be determined by the equation

$$M_{W,h} = \chi_{hog} \gamma B L^3 \qquad M_{W,s} = \chi_{sag} \gamma B L^3 \tag{8}$$

Where $\gamma = t/m^3$ is the specific weight of the sea water
B = ship's width and L = ship's length.

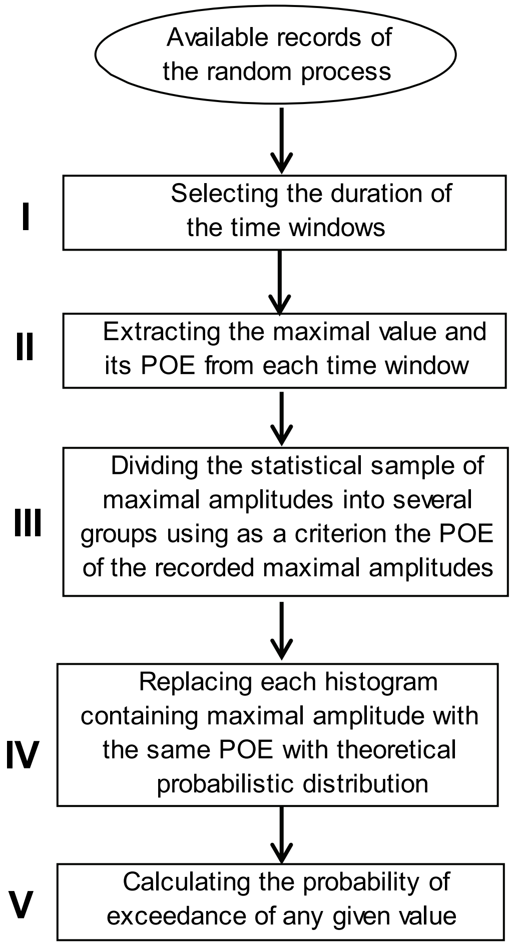

Figure. 2 Basic steps in the proposed procedure

Although Eq. (7) is derived for trochoidal waves, its application here is justified because the real sea wave is closer to trochoidal than to cosine wave. Based on Eq. (7), a new equation was derived to ensure equality of hogging and sagging wave bending moments for $C_B = 1$ and to meet the existing class rules for calculation of the design hogging and sagging wave bending moments:

$$\chi = \frac{M_{W,s}}{M_{W,h}} = 0.62 + \frac{1}{2.63 C_B} \tag{9}$$

Eq.(9) is illustrated in Figure. 3. The equation was used while subtracted from the records of the total bending moment data for hogging, sagging, and still water bending moments by the formulas:

$$M_{W,h} = \frac{M_{t,max} - M_{t,min}}{1+\chi}; \quad M_{W,s} = \chi M_{W,h} \tag{10}$$

$$M_{SW} = M_{t,max} - M_{W,h} = M_{t,min} + M_{W,s} \tag{11}$$

In almost all published research works, the still water bending moment has been calculated as the mean value of hogging and sagging wave bending moments. However, from the physical point of view, this is incorrect, especially for fast going ships as follows from Eq. (9) and Figure. 3. Thus, M_{SW} is not the mean value of $M_{t,max}$ and $M_{t,min}$ (see Figure. 4) but always above the mean.

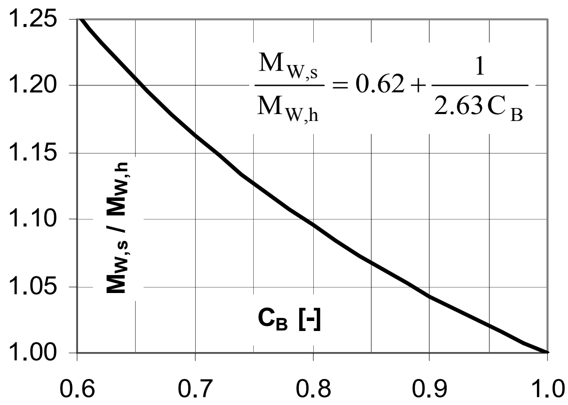

Figure. 3 Ratio between $M_{W,s}$ and $M_{W,h}$ vs. C_B

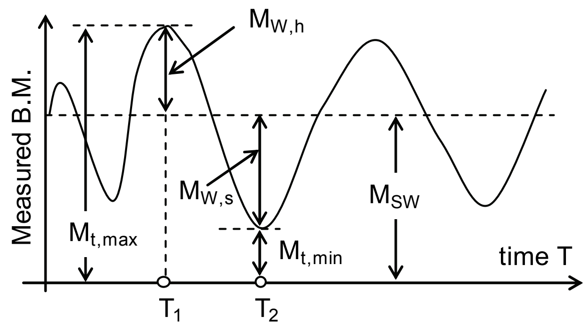

Figure. 4 Extraction of $M_{W,s}$, $M_{W,h}$ and M_{SW} from the filtered quasi-static vertical M_W

Step I: Selecting the duration of the time windows

In the examples here, the duration of the time windows was fixed to 20 and 60 minutes over one year and 3.5 years of full scale measurements. Of course, the time window can be of any other duration.

Step II: Extracting the maximal value and its POE from each time window

The calculations can be performed in two ways:

- Replacing the histogram built in each time window by theoretical distribution and calculating with it the POE of the maximal amplitude;
- Dividing unity by the number of cycles in each time window.

The former method is time consuming but allows for other analyses to be performed beyond the purpose of this particular study. The latter method is simple and fast and is recommended for cases when only hull girder bending moments are analyzed. The effect of the two methods on the final results was analyzed and preference was given to the latter approach. Thus, a new statistical sample is formed containing data for the maximal amplitudes in each time window with their corresponding POE.

Step III: Dividing the statistical sample of maximal $M_{W,h}$ into several groups using as a criterion the POE of the recorded maximal $M_{W,h,i}$.

It is not possible to calculate the Probability of Occurrence (POO) of any given value with the theory of probabilities. Only the POO within two given boundaries can be calculated. Therefore, several regions (with corresponding two boundaries) of the random variables are created and the probability of getting into each of them is calculated. The POO of the random variable within each group "i" is formulated as:

$$10^{-(i+0.5)} \leq \text{POE of max } M_{W,h,i} \leq 10^{-(i-0.5)} \quad (12)$$

where i = 1, 2, 3, 4, 5, etc.

Eq. (12) does not allow covering the range from zero to -0.50. For this case, the following equation is used:

$$0 \leq \text{POE of max } M_{W,h,0} \leq 10^{-0.5} \quad (13)$$

For the sake of brevity, the POE of the max $M_{W,h,i}$ is marked as:

$$\text{POE of max } M_{W,h,i} = P_i \quad (14)$$

Thus, in each group "i", the histogram of maximal $M_{W,h,i}$ contains only maximal $M_{W,h,i}$ with identical POE/POO. The data in each group "i" is used to build a histogram having as abscissa the maximal $M_{W,h,i}$ represented as a portion of the design $M_{W,h}$. Then, the histogram in each group "i" is replaced by theoretical PDF using the computer program EasyFit (the Kolmogorov-Smirnov criterion is utilized).

Figure. 5 - Figure. 8 represent the results for maximal hogging wave bending moments from four "i" groups (the contribution of the other six groups is close to zero in this particular case). The time window's duration is 60 minutes over one year full-scale measurements. The POE of the maximal $M_{W,h}$ is calculated as 1/N where N = number of cycles within one year. Figure. 9 represents the results when the POE of the maximal $M_{W,h}$ is neglected which corresponds to the procedure following the EVT presently applied for calculating the POE of the design $M_{W,h}$.

One should note here that the probabilistic distributions of maximal $M_{W,h}$ in each group are not identical. The POE of the design $M_{W,h}$ in each group, $P_{d,i}$, is calculated as:

$$P_{d,i} = P\left(\max M_{W,h,i} > \text{design } M_{W,h}\right) \quad (15)$$

It follows from Eq. (15) that the data in each time window were considered as independent but not as identical.

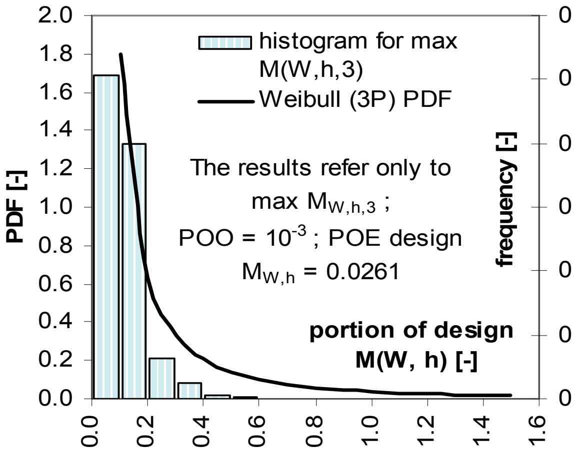

Figure. 5 POE of the design M_W when POO = 10^{-3} of the recorded max M_W is used

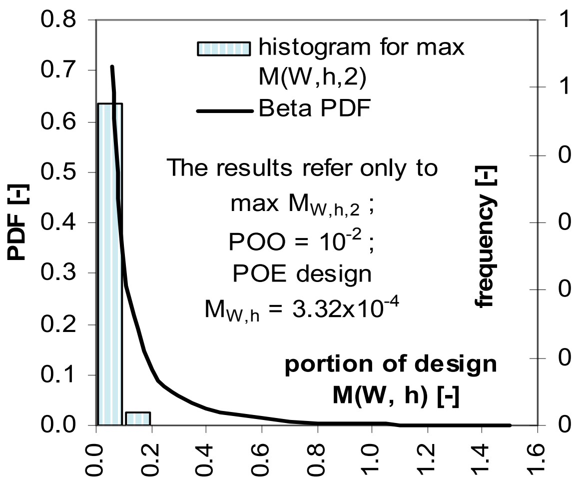

Figure. 6 POE of the design M_W when POO = 10^{-2} of the

recorded max M_W is used

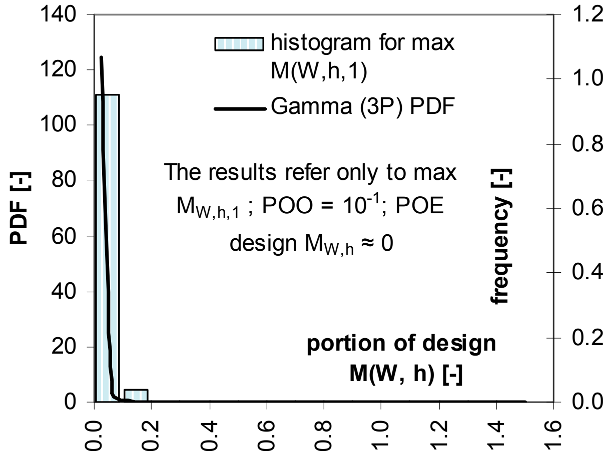

Figure. 7 POE of the design M_W when POO = 10^{-1} of the recorded max M_W is used

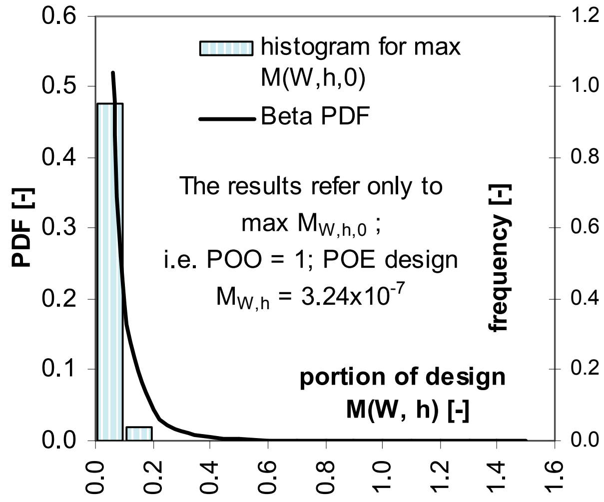

Figure. 8 POE of the design $M_{W,h}$ when POO = 1 of the recorded max $M_{W,h}$ is used

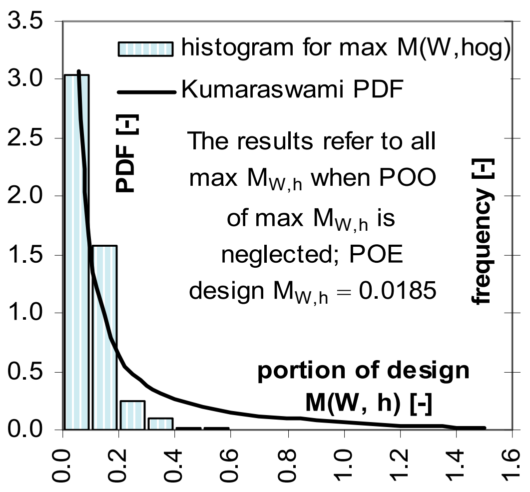

Figure. 9 POE of the design $M_{W,h}$ when the POO of the recorded max $M_{W,h}$ is neglected

Step IV: Replacing each histogram containing maximal amplitudes with the same POE with theoretical distribution

The POE of the design $M_{W,h}$ is calculated by the formula:

$$P_{d,t} = P\left(\max M_{W,h} > \text{design } M_{W,h}\right) \tag{16}$$

Step V: Calculating the POE of any given value

In the paper, the design M_W is taken as a given value. Because the probabilistic distributions of maximal $M_{W,h,i}$ in each group "i" are not identical, the POE of the design $M_{W,h}$ is calculated in the following way:

$$P = \sum_{i=0}^{\max i} P_{d,i} 10^{-i} \tag{17}$$

One should emphasize here that the design $M_{W,h}$ is calculated by the formulas given in class societies' rules, (e.g., ABS Rules, 2011) which is the most probable extreme value of M_W derived with the individual amplitude statistics.

The multiplier 10^{-i} serves as weight coefficient to take into consideration the conditional probability for the data "to get in the corresponding i-group" in which the POE of the design $M_{W,h}$ is calculated. The notation of the groups in the example for $M_{W,h}$ is:

Group 0 corresponds to POO = 1
Group 1 corresponds to POO = 0.1
Group 2 corresponds to POO = 0.01
Group 3 corresponds to POO = 0.001

The change of the final POE of the design $M_{W,h}$ is illustrated in Figure. 10 and Figure. 11.

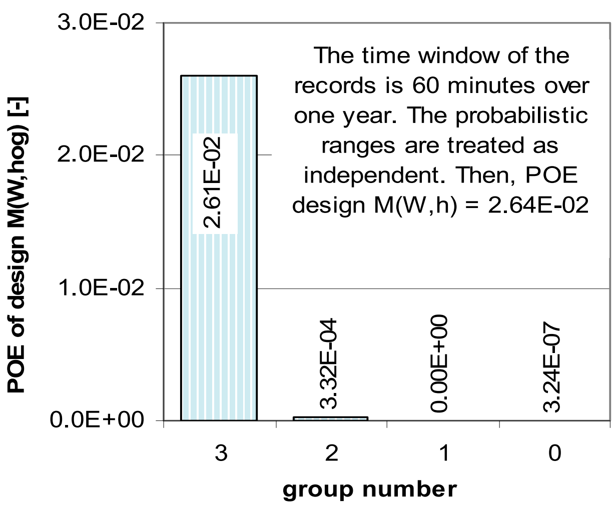

Figure. 10 POE of the design $M_{W,h}$ when the

probabilistic ranges are treated as independent

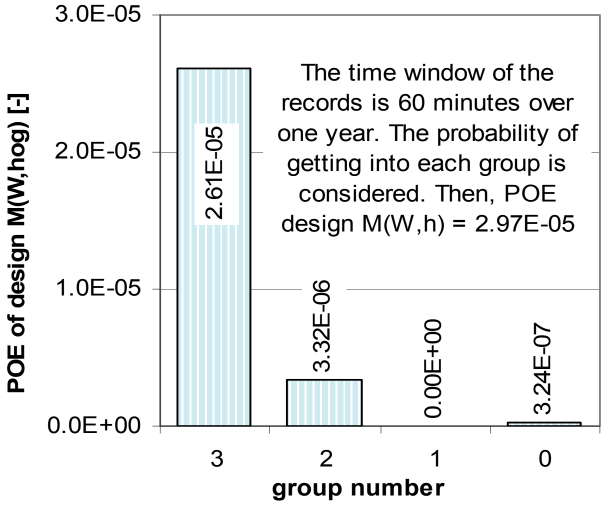

Figure. 11 POE of the design $M_{W,,h}$ considering the probability of getting into each probabilistic group

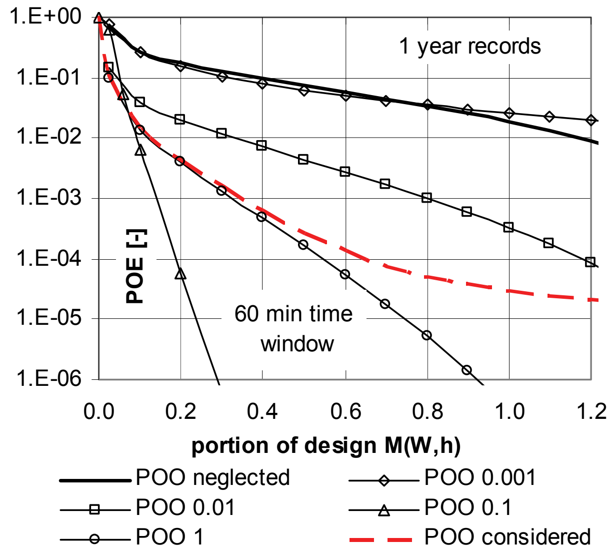

Figure. 12 POE of $M_{W,h}$ when the probabilistic ranges are treated as independent and POE of $M_{W,h}$ derived by conventional EVT and the proposed procedure

The final POE of the design Mw,h is calculated by Eq. (17). In this case P = 2.97E-05

When this result is compared with the result shown in Figure. 9, one can observe that the POE of the design $M_{W,h}$ obtained with the proposed procedure is around 620 times smaller than the POE of the same $M_{W,h}$ obtained by the traditional EVT (i.e., when the POO/POE of the maximal $M_{W,h}$ in each time window is neglected and the probabilistic distributions are assumed as independent and identical).

Although this result is valid only for this example, it clearly shows that the 63.2% POE of the design $M_{W,h}$ (derived for the special case where the design $M_{W,h}$ is chosen such that the POE is equal to 1/N) should be substantially reduced. If one assumes that the same ratio between the two results exists for the case when 63.2% POE of the design $M_{W,h}$ is calculated, the new POE would be around 0.10% instead of 63.2%. The graphs of the POE of any given $M_{W,h}$ are shown in Figure. 12 in log-scale. They illustrate the big difference between the POE of any given value, including design $M_{W,h}$, when the traditional EVT and the proposed procedure are applied. One can also observe the fact that, in this particular case, the maximum contributions come from Group 3 (i.e., POO = 0.001). In this group, the POE of the design $M_{W,h}$ is even greater than the POE of the design $M_{W,h}$ obtained by the conventional EVT. However, when Eq. (17) is applied, the contribution of Group 3 drops substantially.

Similar calculations were performed with time windows of 60 minutes for the total hull girder bending moment M_t over 3.5 years of records. The design M_t was calculated by the formula:

$$M_t = [\sigma] Z \tag{18}$$

where $[\sigma]$ = permissible total hull girder bending stress given in class societies' rules (e.g., ABS Rules, 2011), Z = hull girder section modulus (deck).

The results of the calculations confirmed the same trend as for the design $M_{W,h}$, i.e. the POE of the design M_t derived by the proposed procedure is much lower (around ten times lower) than the POE calculated by the traditional EVT approach.

Since the example refers to records from relatively moderate seaway which results in a very low POE of the design M_t, this obstructs the quantitative comparison between the results obtained by the traditional EVT and the proposed procedure. Considering the fact that the recorded M_t practically does not exceed 70 – 80% of the design M_t, numerical comparison between the results obtained by the two approaches was made for the POE of two limit values = 70% and 80% of the design M_t. When the former limit is used, the proposed procedure leads to around 550 times reduction of the POE. When the latter limit is used, the proposed procedure leads to around 220 times reduction of the POE.

Although the numerical results refer only to this particular case, a conclusion can be made that the reduction of the POE (calculated with the proposed procedure) of the design $M_{W,h}$ and design M_t is substantial relative to results obtained by the traditional EVT.

3. DISCUSSION

The key reason for the present discussion of the applicability of the EVT in shipbuilding is the calculated POE of the design M_W in class societies' rules by 63.2% by several researchers (e.g., Faulkner and Sadden 1979). From the mathematical point of view, this POE refers to the POE of the most probable extreme value. The proponents of EVT argue that, since the POE of the design M_W in class societies' rules is equal to unity divided by the number of load cycles within given time period (e.g., 25 years), it becomes equal to the most probable extreme value. Hence, they conclude that the design M_W in class societies' rules can be exceeded by 63.2%.

When the individual amplitude statistics are used (i.e., when all amplitudes are taken into consideration), the POE of the design M_W is 10^{-8}. The difference between the POE of the design M_W obtained by the individual amplitude statistics and extreme value statistics is tremendous but not surprising. The method for calculating the POE of M_W is significantly changed and as a result – the POE also changes significantly.

The mixture of two types of statistical analysis of M_W (i.e., individual amplitude and extreme value statistics) leads to situation where the design M_W is always interpreted as equal to the most probable extreme value of M_W . Thus, following the traditional EVT, it can be exceeded by 63.2% for any value of n (number of cycles). It means that the most probable extreme value of M_W will vary with the variation of number of cycles n while the design M_W in class societies rules is a fixed value for given ship's service life. Therefore, when EVT is applied, it is more reasonable to place on the derived probabilistic distribution of extreme M_W (derived by following the proposed procedure) the fixed number of class rules' design M_W.

In the design stages, there are no records of M_W to be used for the newly designed ship. The only option is to follow the conventional approach described in the Introduction, i.e. to use as accurate as possible parent distribution to calculate the needed parameters of the extreme value distribution by the formulas available e.g., in (Ochi, 1989). In these formulas, the parameter n (e.g. number of cycles) plays a governing role together with accuracy of the parent probabilistic distributions.

Many efforts are devoted to obtain as accurate as possible parent probabilistic distributions, to verify and calibrate them against records of full-scale measurements. Usually, the duration of full-scale measurements lasts around 3-4 years. Unavoidably, using the data from these measurements automatically implies the assumption that the future ship's operation (e.g., 25 years service life) will repeat the environment in which the full-scale measurements were carried out. Once the probabilistic distributions are verified, they can be used for any scenario of ship's operation in the future. The trouble is that it is almost impossible to predict accurately all details of the future ship's operation (operational regions, head angles, speed, Beaufort scale, etc.) despite the efforts and right intention. Hence, the prediction of number of load cycles n (see Eq. (5)) which the ship will be exposed to in the future also contains uncertainties.

The time windows' duration has a strong effect on the final results obtained with the present EVT. The same is valid for the proposed procedure as well. A parametric study is needed to explore the quantitative effect of different durations of the time-windows on the final POE of any given limit. However, in any case, taking into consideration the POE of the maximal values of $M_{W,h}$ or M_t will contribute to a more accurate calculation of the POE of any value of the design $M_{W,h}$ or M_t. This could result in better understanding classification societies' rules and may help avoiding possible misinterpretations.

Another issue is the sensitivity of the results when different criteria are used in the EasyFit computer program (or in any other computer program). In the example presented here, Kolmogorov-Smirnov criterion is used. When the Anderson – Darling criterion is used, the obtained numerical results are different from those given in the paper. To have an idea about the difference, one could refer to the POE of the design $M_{W,h}$ calculated using both criteria in the proposed procedure. In the example, the POE of the design $M_{W,h}$ is equal to 2.97E-05 when Kolmogorov – Smirnov criterion is used and equal to 7.52E-05 when Anderson – Darling criterion is used.

When the two criteria are used to calculate the POE of the design $M_{W,h}$ following the conventional EVT, the POE of the design $M_{W,h}$ is equal to 0.0185 when Kolmogorov – Smirnov criterion is used and equal to 0.3469 when Anderson – Darling criterion is used.

Obviously, more studies of the sensitivity of the numerical results are needed, e.g. - when different durations of the time windows and χ^2 fitness criterion are used. Whatever differences are obtained resulting from the use of different time windows and criteria for fitness, the two major issues in the proposed procedure will remain i.e. a) one should consider the POE of the recorded maximal B.M. in each time window; b) one should not use the design M_W in class societies' rules as the most probable extreme value to be applied in conventional EVT approach.

4. CONCLUSION

A procedure is proposed for application of the EVT in ship's strength calculations considering not only the maximal value of the corresponding random variable (within any time-window) but also its probability of exceedance. It substantially reduces the probability of exceedance of the class societies' design M_W (or any other given limit) when the EVT is applied. All steps in the proposed procedure are discussed and examples are provided to illustrate its application when records of the random process are available.

The most probable extreme value in extreme value theory depends on the number of observations (e.g., number of cycles of wave induced load) within a given service life. The design M_W in class rules is a fixed value which is determined with the individual amplitude statistics and has an extremely improbable probability of exceedance (around 10^{-8}). It does not follow any change of the most probable extreme value (derived by the extreme value theory). That is why one should not use the design M_W in class rules as equal to the most probable extreme value. The calculated 63.2% probability of exceedance of class rules by some authors is incorrect because the assumptions in the calculations is that the design M_W is always equal to the most probable extreme value.

Data from real random processes is not always available, especially at the design stage. Obviously, additional work should be done to develop a procedure that allows for taking into consideration the probability of occurrence of the maximal values of the random variable. After verification and calibration against real records, such a procedure could contribute to more realistic assessment of the structures' reliability.

Considering the sensitive nature of this issue as well as its importance to all involved in shipping, shipbuilding and ship repair, it deserves critical discussion by experts in the field in order to develop a reasonable procedure for practical application of the extreme value theory in ship strength calculations. Therefore, any constructive criticism or proposal for improvement of the proposed procedure described in this Technical Note is welcome.

5. REFERENCES

1. American Bureau of Shipping – Rules for Building and Classing Steel Vessels, 2011
2. Abrahamsen E, Vedeler G - The strength of large tankers-European Shipbuilding, 1957, No. 6, part 1, pp. 130-151; 1958, No. 1, part 2, pages 2-23
3. Faulkner D, Sadden J A – Towards a Unified Approach to Ship Structural Safety, Transactions of the Royal Institution of Naval Architects, 1979, vol. 121, pp. 1 - 28
4. Gumbel Emil Julius – Statistics of Extreme, Dover Publications, Inc., Mineola, New York, 1958/2004
5. Han C. Yu, J.W. Choi, G.I. Park, S.Y. Han, S.C. Tai – Full Scale Measurement of a Large Container Carrier on the Far East-Europe Route, SNAME Annual Meeting, Houston, U.S.A., 2008.
6. Han C. Yu, M.K. Ha, Jae-Woong Choi, James S. Tai - Design and Implementation of a Comprehensive Full-scale Measurement System for a Large Container Carrier, RINA Conference, London, UK, November 2006
7. Kotz Samuel, Nadarajah Saralees – Extreme Value Distributions. Theory and Applications, Imperial College Press, 2000
8. Mansour A E, Faulkner D – On Applying the Statistical Approach to Extreme Sea Loads and Ship Hull Strength, Transactions of INA, vol. 115, 1973, pp. 277-314
9. Ochi Michel K - Applied Probability and Stochastic Processes in Engineering and Physical Sciences, John Wiley & Sons, New York, Chichester, Brisbane, Toronto, 1989
10. Ochi Michel K – On Prediction of Extreme Values, International Ship Research, vol. 17, No. 1, March 1973, pp. 29-37
11. Seung Jae Lee, Han C. Yu, Sungeun (Peter) Kim, Mun-Keun Ha, Gun-Il Park, Jae-Woong Choi, Munsung Kim, James S. C. Tai – Analysis of full-scale hull girder loads of a container carrier and its simulation using a nonlinear seakeeping program, Proceedings of the 29th International Conference on Offshore Mechanics and Arctic Engineering, paper OMAE 2010 – 20661, June 6-11, 2010, Shanghai, China