PROPELLER CAVITATION AND INDUCED VIBRATION

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(DOI No: 10.3940/rina.ijme.2011.a4.222)

SUMMARY

The demand to increase the efficiency of propellers has led to optimized propeller blade designs finding their way into the construction of high-powered commercial vessels, such as containers or LNG carriers and certain categories of passenger vessels, to mention but a few. It has become increasingly common to see the propeller tip rotate closer to the hull surface, sweeping the thick turbulent boundary layer attached to the hull, causing fluid structure interaction. At the same time, increasing the loading on marine propellers can lead to problems, such as noise, hull vibration, and cavitation. The degree above which, such phenomena as propeller cavitation can be the main perpetrators for intensive vibration during operation, their diagnosis and the solutions to mitigate this risk, such as the use of vortex generators, are discussed here, taking into account cost and longevity of the vessel as well as the involvement of classification rules.

NOMENCLATURE

q	the harmonic order (blade passing frequency)
ρ	fluid density(kg/m^3)
Z	No of blades
ω	angular velocity (rad/s)
R_p	distance from the field point (m)
V_{qz}	the qzth magnitude of the harmonic component of the cavity volume (m^3)
RMS	Root Mean Square

1 INTRODUCTION

The unsteady inflow to the propeller blade while passing through a non-uniform ship wake can cause alternating pressure profiles on the propeller blade surface. A decrease in blade pressure to a level below vapour pressure causes the water to boil locally on the propeller blade, so that the phenomenon of cavitation occurs.

Propeller cavitation, as a form of hydrodynamic instability, has been emerging as a serious problem for some types of commercial vessels, such as containers and LNG vessels for both the ballast and –less so- for the fully loaded condition, causing erosion of the propeller blade, pressure fluctuations on the stern hull, underwater noise and structural vibration. These problems highlight the need for detailed flow analysis based on both Computational Fluid Dynamic analyses (CFD) and reliable experimental measurements (model tank tests or cavitation tunnel tests) when designing the geometrical shapes of modern propellers. Literature survey shows that propeller cavitation studies are still very current research topics in most related technological institutes and universities. Experience has shown that a direct correlation between excitation forces predicted on the basis of model tank tests and pressures measured on full-scale ships is not always successful because the latter are themselves heavily influenced by the hull vibrations. In other words, one has to predict or measure not only the pressure fluctuations onto the hull but involve the “hull impedance” concept, see Gent W.V.[1], in order to obtain more accurate predictions. Furthermore, the strength of the cavitation depends on the total volume of cavitation and its dynamic behaviour, which, in turn, involves matching the timing of the actual pressure pattern with the pressure pattern observed in the model tank test. Any difference in the timing of these two patterns, i.e., between the tank test and in the real full-scale, can cause underestimation of the cavitation strength of the values of the maximum pressure pattern that the hull of the vessel is subjected to, see Vrijdag A et al, [2].

With respect to the CFD analysis of the wakefield, the prediction of the propeller pressure patterns depends very much on the “wake quality”. If the wake quality is good and predictable, this allows the designer to push further on propeller energy-saving design under the condition of appropriate control for propeller cavitation. If the wake quality is low, the risk of propeller-induced vibration will be higher and wake smoothing devices such as vortex generators or equalizing ducts may have to be adopted to remedy the problems, see Lee, S.K. et al [3], Liao, M, Wang, S, Propeller-Induced Hull Vibration – Analytical Methods, ABS Technical Papers 2006, Li, JY, et al [4].

There have been a number of recent cases cited by classification societies where high vibration levels at the aft end of newly built vessels have been observed when assessed against industry guidelines. Nearly all propellers can more or less be subjected to some degree of cavitation or hydrodynamic instability under various wakefield conditions. Although these phenomena are to be addressed at an early design stage of the propeller to ensure that its geometry minimizes such problems, there is, unfortunately, no clear design criteria or limits above which one can guarantee that propeller cavitation will not occur and will not cause severe vibrations during the operational life of the vessel, let alone the case of propeller blade self-erosion.

When a specification contract for a newly designed vessel is signed between two parties, such as shipyard and owner, most of the times, there is no specific requirement regarding the amount of acceptable propeller cavitation that the vessel can sustain. Classification societies have yet to introduce rules or limits, other than some guidance notes notifying clients about the potential risks and the effects of propeller cavitation. The only intervention from the ship owner/operator side can take place during the model tank tests, see Figure 1, whereby a report is established, where the blade area percentage of cavitation can be assessed together with a table of normalized pressure forces or pressure patterns from the propeller to the hull. The latter (pressure patterns) appear as amplitude versus propeller blade passing frequencies and its multiples. Assuming these frequency components are strong, vibration transmission to the hull and subsequent structures can cause resonances at the various structures' natural frequencies.

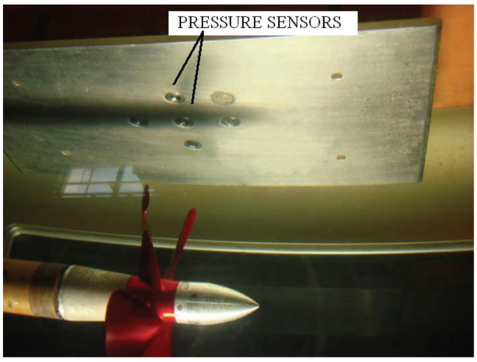
2 PROPELLER CAVITATION AND INDUCED VIBRATION MECHANISM

When cavitation bubbles collapse, broadband noise is generated. The noise is radiated to the surrounding marine environment and transferred onto the ship structure. The cavitation extent varies throughout one cycle or revolution of the propeller blade. Typically, the cavitation volume reaches its peak when the propeller blade is close to the hull, see Figure 2a. The fluctuating volume of the cavity then causes a pressure pulse, as the blades pass close to the surface of the hull, acting on the hull plating in the aft ship. Generally, the lower the volume rise as the propeller blade rotates, the lower the generated pressure differential across the blade. The cavitating blade contribution to the hull pressure field is therefore considered to derive from the suction side sheet and tip vortex cavities. Hence, these types of cavitation may collapse either on or off the blade. When cavitation bubbles collapse on the blade, these are related to the generation of the blade passing frequencies, while when "off the blade", they cause mainly white frequency noise. Thus, the frequency content of propeller cavitation contains a combination of impact and harmonic characteristics. See Figure 2b.

To estimate the effects of cavity volume variations on the cavitating pressures, modelling of the cavity volume on the blade as the propeller rotates in the wakefield is done through CFD. Numerical formulation of the magnitude of the pressure vector has been discussed in Breslin, JP et al [5] and an expression of the following form has been developed:

$$P_{cqz} = -\frac{\rho \cdot Z^3}{2\pi} \cdot \frac{(q \cdot \omega)^2}{R_P} \cdot \mathrm{Re}[V_{qz} e^{i \cdot q \cdot z \cdot \varphi}]$$

From this expression, it can be seen that the pressure magnitude of a specific harmonic order is inversely proportional to the distance R_p and demonstrates that the higher the clearance around the propeller the less the pressure pulse acting on the adjacent structure (hull or rudder, etc.) in a specific manner ($P_{cqz} \propto R_p^{-2.5}$ compared to $P_{cqz} \propto R_p^{-1}$). Comparing this with a similarly derived expression for a non-cavitating propeller, as derived by Breslin and Tsakonas [6], indicates that the pressure pulse from a non-cavitating propeller decays far more rapidly with distance, see Carlton, JS, [7].



CAVITATION TEST REPORT

Sid. nr. *Page No.* 1 (8)

CAVITATION TUNNEL TESTS WITH SUEZMAX TANKER WITH PSS

Uppgjord (även faktaansv. om annan) ***Prepared*** *(also subject resp. if other)*	**Titel** *Title*	**Dok Nr.** *Doc No*	
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Cavitation tunnel tests with Suezmax tanker with PSS

A cavitation tunnel test program has been performed on a model of the Suezmax tanker equipped with pre swirl stator. The ship model is designated and it was equipped with pre swirl stator and design propeller . The test program comprised cavitation observations, pressure pulse measurements and cavitation erosion test according to the loading conditions listed in Table 2. The test program performed is outlined in Table 1 and the loading conditions are presented in Table 2. The test set up in shown is shown in Figure 1.

Table 1: Test program

Series	Test
060	Blind hub test
061	Force measurement at atmospheric condition
064	Cavitation observations, photo and video documentation
065	Cavitation observations, photo and video documentation , run no. 1–3 Demonstration to the client run no. 4–6
066	Pressure pulse measurements: Ballast, NCR, cavitating condition, run no. 2 Ballast, NCR, atmospheric condition, run no. 8 Design, NCR, cavitating condition, run no. 3 Design, NCR, atmospheric condition, run no. 9

Figure 1: Cavitation Tunnel Tests and Typical Corresponding Report.

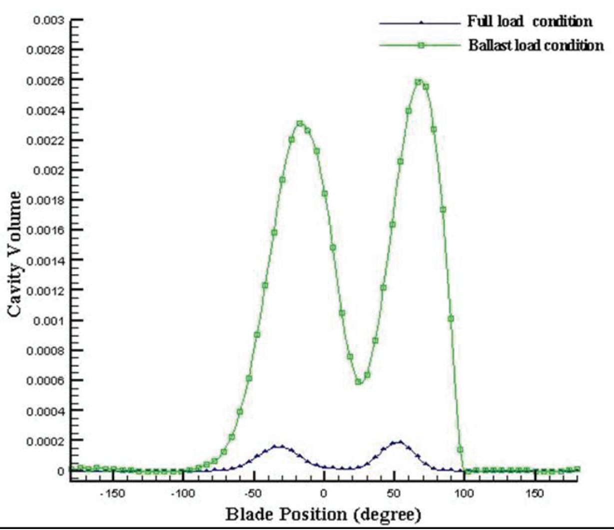

Figure 2a: Cavity volume variation through one revolution of the propeller.

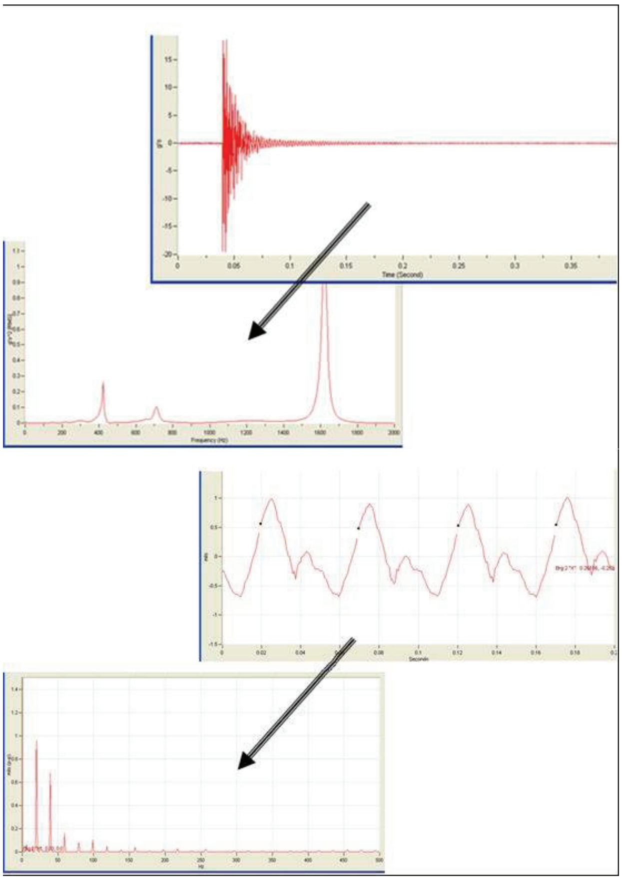

Figure 2b: Frequency content of the cavitation signal including harmonic, periodic and impact information.

3 CASE STUDY

ABS have witnessed a number of cases in the recent years, where it is too late or too costly for the vessel operator to request to have the propeller replaced due to cavitation and cavitation-induced vibration. Instead, the operator has to live with high vibration levels potentially until the propeller becomes eroded and commences stiffening up of the vibration affected structures. The first sign of potential propeller-cavitation-induced-vibration is manifested in the aft stern area of the vessel. Generally, vibration in the aft stern area of the vessel is due to frequencies coming from the wakefield, propeller or engine, exciting local structures ranging from the accommodation deck to aft mooring deck and lifeboat support structures. Once the excitation frequencies coincide with any of the structures' natural frequencies, a resonance such as a high amplitude vibration can occur leading to possible fatigue and failure.

Assuming the source of the problem is suspected to be hydrodynamic, then additional tests and observations need to be conducted, such as underwater pressure measurements in the area of the propeller, to establish potential blade overloading and pressure pattern analysis. In addition, underwater cameras or underwater stroboscopic cameras can be installed to observe the propeller interaction with the wakefield. The measurements need ideally to be accompanied by either model tank tests or additional Computational Fluid Dynamics (CFD) simulation of the wakefield for further comprehension of the problem. If a propeller cavitation problem is identified, then potential wake correction devices, such as flow fairings, stern fins, vortex generators, etc., can intervene into the wakefield pattern and correct or improve the flow in favour of the reduction of the propeller cavitation intensity albeit at the expense of some increase in the ship drag resistance. Changing the propeller from a four-bladed propeller to a propeller with larger blade areas or a propeller with five or six blades could reduce or eliminate the cavitation as the blade area load or blade area pressure would be reduced to lower levels. As a practical rule of thumb, a good safe loading for the propeller blades is about 6.4 lb/in^2 or 450 g/cm^2

3.1 STRUCTURAL RESPONSE

Once a problem such as general vibration in the aft part of the vessel is reported, then overall Root Mean Square (RMS) vibration levels are conducted to establish objective measurements against set criteria. Figure 3. shows the overall RMS vibration levels as originally measured on board an LNG carrier vessel with a four-bladed fixed pitch propeller under different service operating conditions. From the measurements, it can be seen that the problematic areas are the helideck, the aft mooring deck of the vessel and the steering gear room, where the levels exceed the overall RMS levels found in ISO 6954.

Concentrating on the areas of highest vibration level, a frequency analysis of the vibration signal at Maximum Continuous Rating, (MCR) speed is further carried out in the steering gear room and the helideck. The results shown in Figures 4a and 4b show the existence of a strong amplitude propeller blade passing frequency together with its higher multiples including up to the

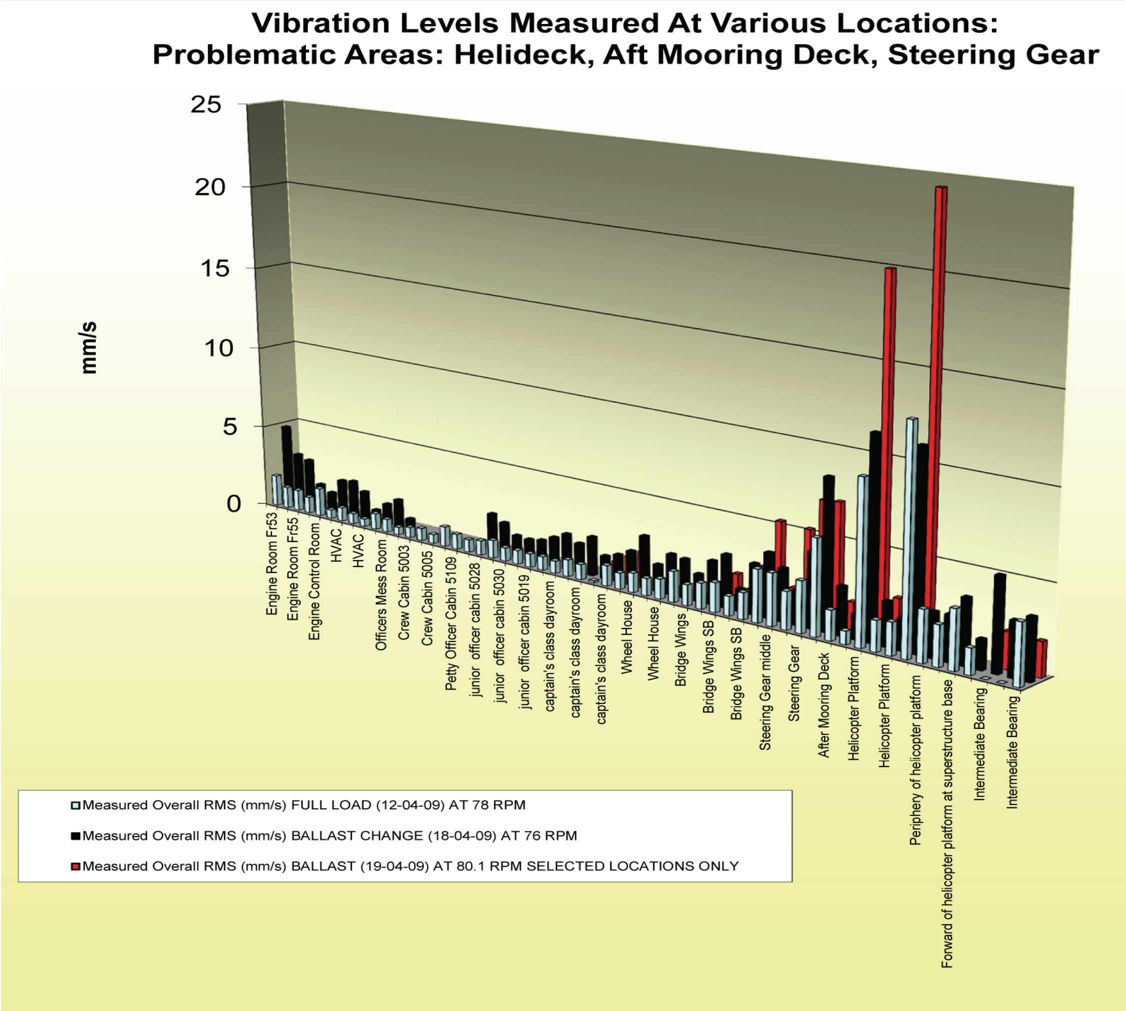

Figure 3: Overall RMS Vibration levels measured in various locations in the aft area of the vessel.

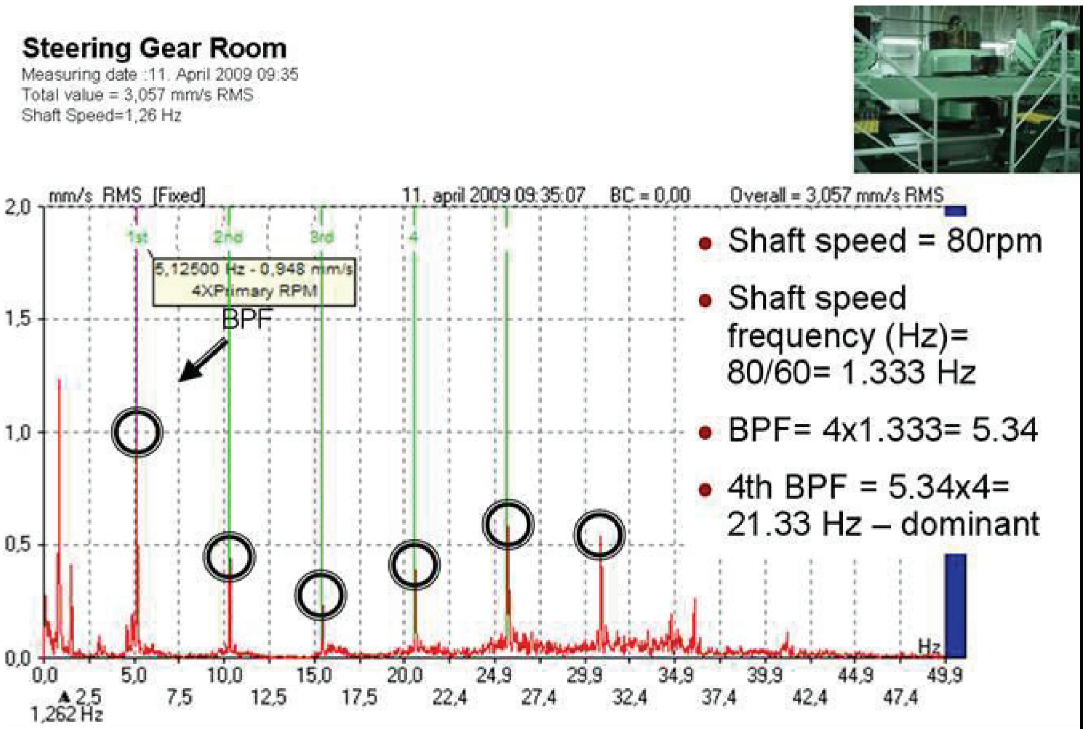

Figure 4a: Frequency Response Spectrum measured in the aft area of the vessel (steering gear room).

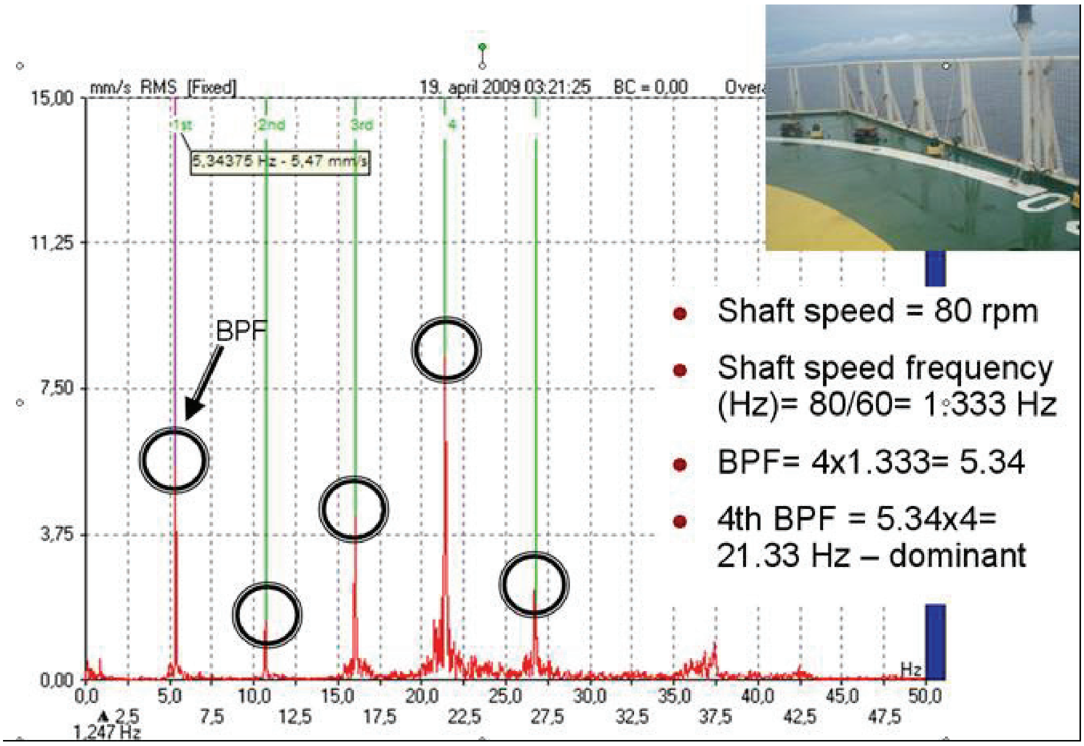

Figure 4b: Frequency Response Spectrum measured in the aft area of the vessel (helideck).

sixth. In some instances, the 3rd and the 4th Blade Passing Frequency (BPF) components show much higher amplitude than the first BPF. The diagnosis of this frequency spectrum pattern is unusual in vessels with non-cavitating propeller operation and this pattern points to a hydrodynamic type of vibration-related problem. In a typical frequency spectrum pattern of a non-cavitating propeller, one would observe that only the first blade passing frequency is of significant amplitude compared to its higher multiples, which demonstrate much lower insignificant amplitudes.

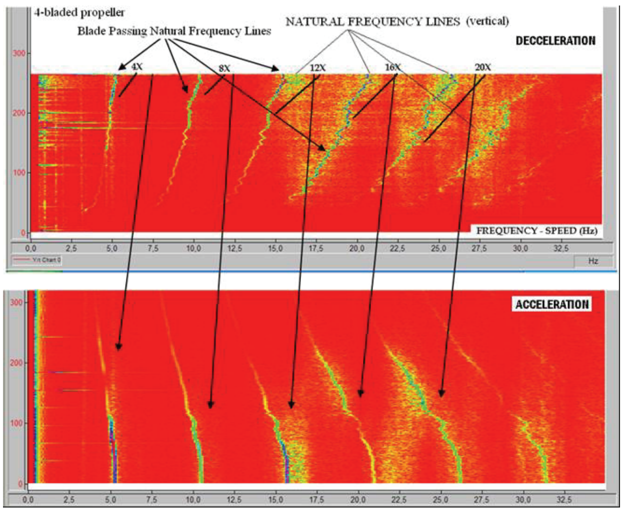

Figure 5: Spectrogram taken through run-up and run-down from the centre of the vessel's helideck at the aft area of the vessel.

In order to obtain a better understanding of the nature of the problem, a frequency sweep is requested to be performed, which, for a ship in operation, requires an acceleration from zero to MCR rpm and from MCR to 0 rpm. This would involve vessel's acceleration from 0 rpm to MCR rpm and from MCR rpm to 0 rpm, under different operating conditions (ballasted and fully loaded). A cascade plot of Frequency Response Spectra, usually called "spectrogram", is then recorded and analyzed in order to reveal possible resonances excited by the multitude of the propeller BPF's. Potential resonances are revealed as horizontal or vertical lines of increasing brightness when intersected by a harmonic "order", such as a Blade Passing Frequency component.

Frequency components intersecting with potential natural frequencies of the structure (faint yellow vertical lines can be seen in Figure 5). While the situation identifies a hydrodynamic problem stemming from the propeller, the natural frequencies of the affected structures still need to be determined. As it is not practical to perform modal tests through shaker testing on a vessel structure during a chartered voyage, it is decided instead to check the Operational Deflection Shapes (ODS) of the affected structures through vibration monitoring in order to get a better understanding of the modeshapes for potential reinforcement.


Figure 6a: Photos of the actual aft structure of the vessel subject to vibrations (helideck).


Figure 6b: Photos of the actual aft structure of the vessel subject to vibrations (aft mooring deck).

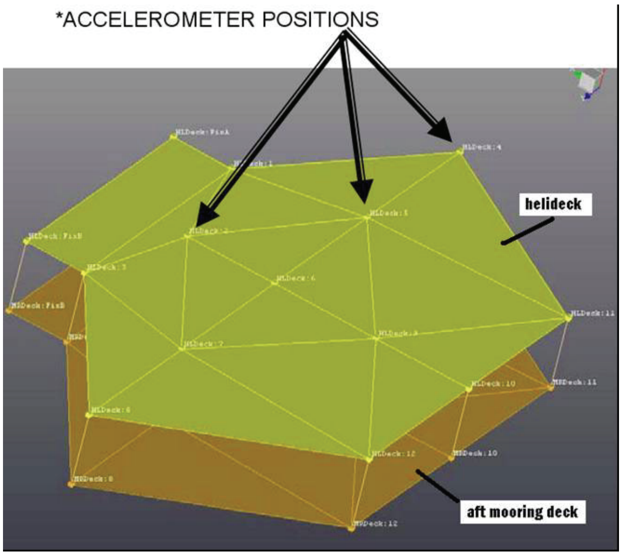

Figure 7: Mesh coordinate display of the positions of the accelerometers in the 2 levels of the aft structure of the vessel (helideck and aft mooring deck.

Twenty accelerometers are then positioned on the helideck surface area as well as in the aft mooring deck surface in order to simultaneously record and capture the actual operational deflection shape of the structure. Figures 6a and 6b show the surface areas where the accelerometers are to be positioned. The accelerometers' signals are analyzed using software and a specialized data acquisition system in order to obtain the operational deflection shapes. Figure 7 shows a computer mesh indicating the accelerometer positions, while Figures 8a and 8b show the analysed Operational Deflection Shapes (ODS) and cross spectrum in order to determine the energy content of the frequency.

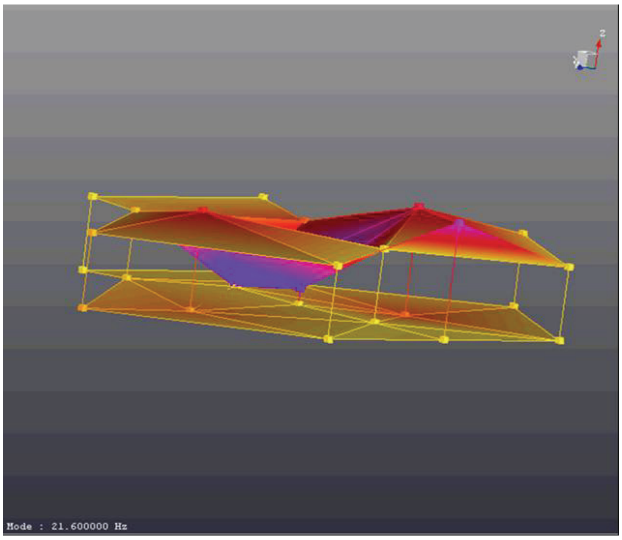

Figure 8a: Operational Deflection Shape (ODS) of the helideck and aft mooring deck at 21 Hz.

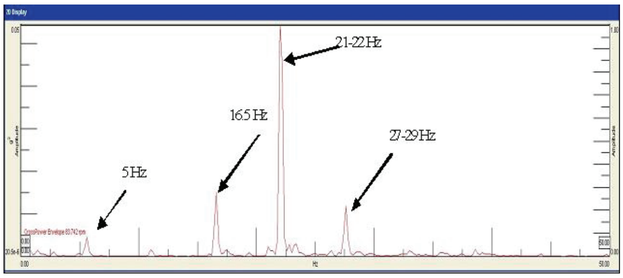

Figure 8b: Cross power spectrum showing the energy content of each mode, in g^2.

From the cross power spectrum and the animation of the ODS's, it is possible to observe the natural frequencies and the mode of vibration of the structure. From the modeshape, it is possible to understand the areas of the structure which could potentially need reinforcement. From the cross-spectrum (Figure 8b) at 21 Hz, it appears that there is a strong mode which involves substantial vibrational activity of the helideck and the aft mooring deck. Although 21 Hz is considerably greater than the first BPF of the vessel, this natural frequency gets excited by a strong 4^{th} propeller BPF, which due to a low damping of the steel structure, causes a high overall vibration level. The vibration had already caused cracks and failure on the supports of the helipad rails, broken landing lights, cracks on life boat support ladders and discomfort in the accommodation block during the first two years of the operational life of the vessel.

In addition to the hydrodynamic problem requiring further investigation, it is decided to proceed with local reinforcement of the helideck and aft mooring deck structure through Finite Element (FE) modelling and analysis of the local structure, see Figure 9a. Figure 9b shows a drawing of an FE model element plot depicting the upper part of the helideck structure together with the additional reinforcement supports. A simplified forced damped analysis is performed in order to ensure that both the modal strength is reduced and the natural frequencies move at higher values.

3.2 CAVITATION REDUCTION USING VORTEX GENERATORS (VG's)

In order to mitigate the uncomfortable vibration level in steering deck and after mooring deck as indicated earlier (see Figure 3), propeller excitation forces are required to be further reduced through wake flow improvement in front of the propeller. In this specific case, it is found that the main cause of the uncomfortable vibration on stern structures stems from adverse stern wake quality in front of propeller. Instead of the most common cause, usually

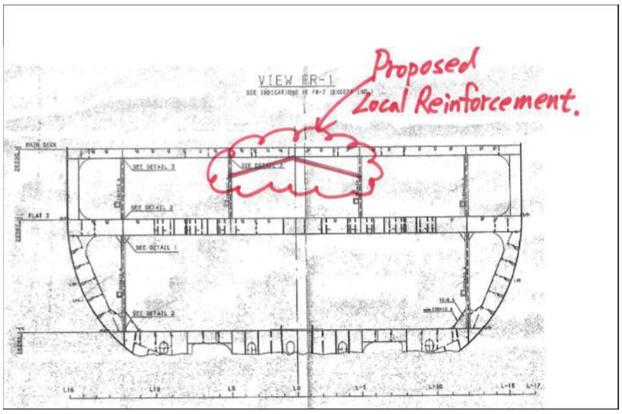

Figure 9a: Proposed local reinforcement plan to change the natural frequency of the helideck and aft mooring deck of the structure.

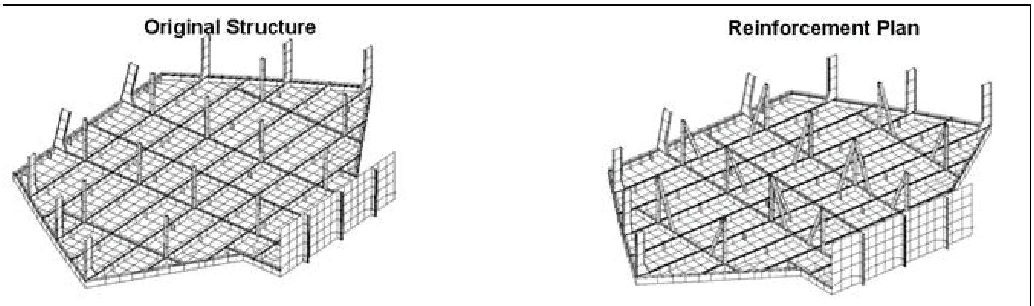

Figure 9b: FE Model of the local reinforcement to change the natural frequency of the helideck and aft mooring deck.

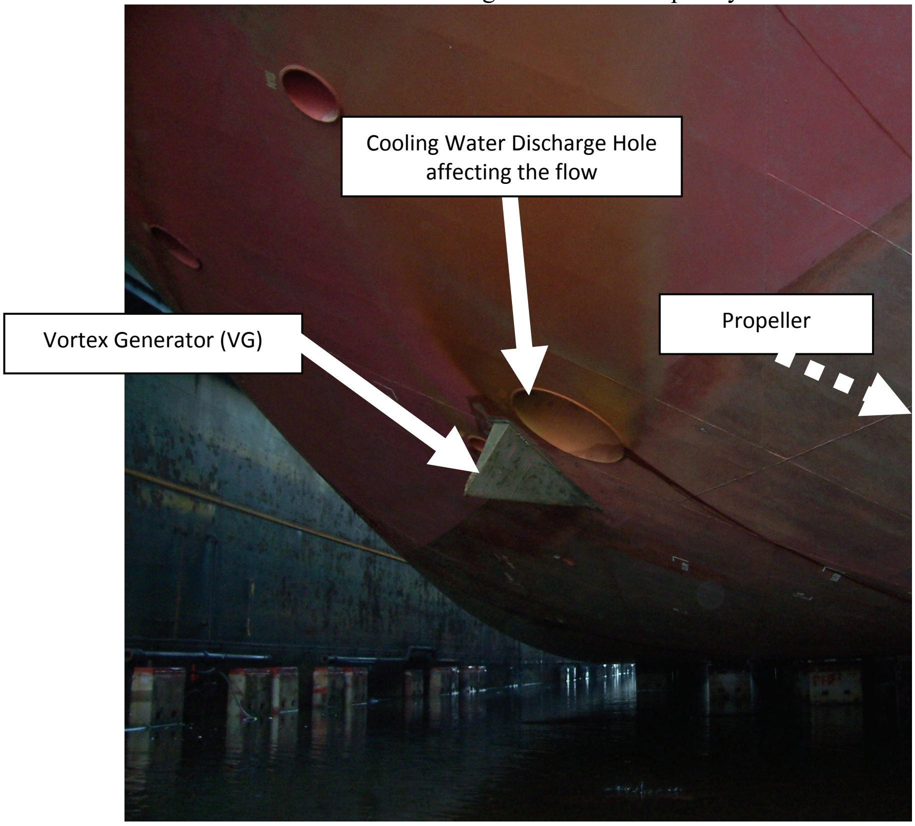

Figure 10: Cooling water discharging hole in front of propeller.

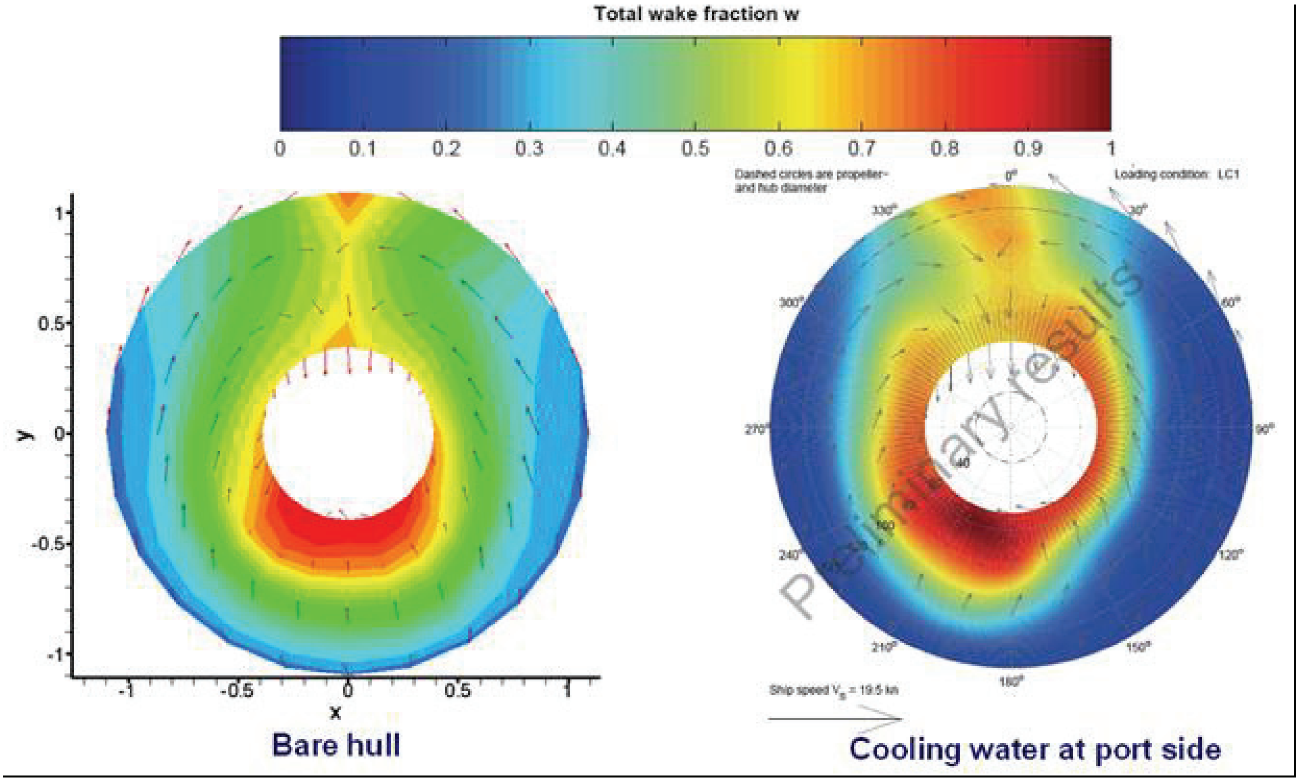

Figure 11: Wake field quality (CFD) – without cooling water jet versus. with cooling water jet effect at design draught condition.

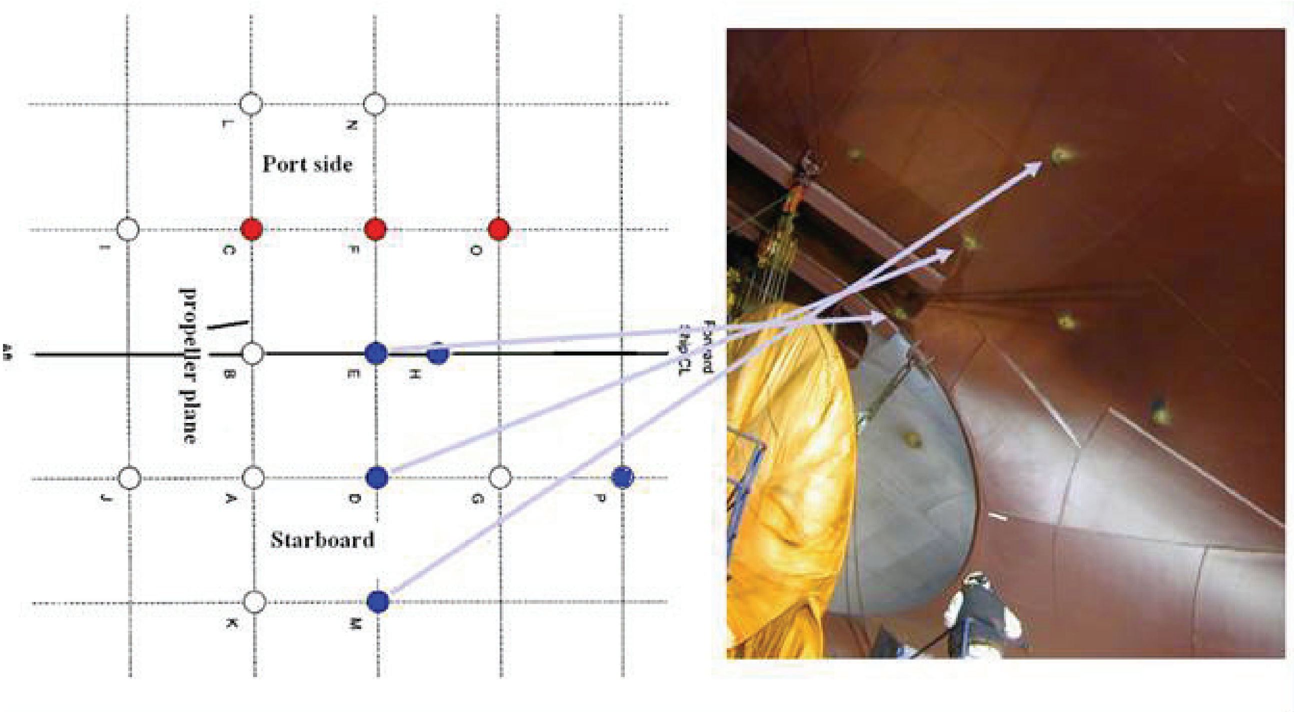

Figure 12: Locations of pressure sensors for sea-trial hull pressure measurement.

found to be bad wake quality due to bad stern shape design, in this instance it is found from the model tank test wake measurements that the port side cooling water discharging jet at the upstream of propeller (see Figure 10), is the potential root cause of the adverse wake quality. To demonstrate the effect of the cooling water jet on the wake quality, measured wakes with and without cooling water jet at port side are CFD computed and plotted in Figure 11. As seen, the cooling water jet (with a velocity of 5 m/s) causes a higher extent of slow velocity area at 12 o'clock position and a stronger swirling flow at port side. Hence, it is expected that more cavitation on propeller blade surface will occur and higher hull pressure impulses will be created compared to the original wake field without the cooling water jet scenario.

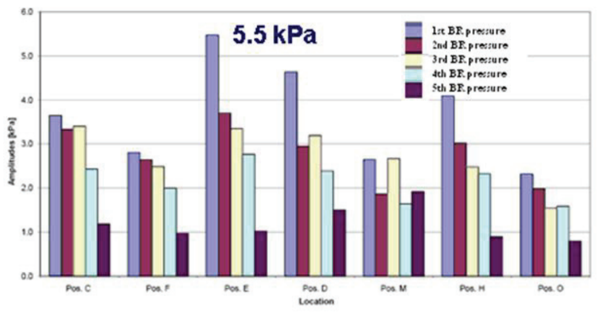

Figure 13: Measured pressure amplitudes from eight pressure sensors – 84 rpm at a speed of 21 knots (full scale measurements).

To improve the wake flow for reducing the propeller-induced excitation, a pair of VGs (Vortex Generators) were designed based on experience and were installed at the port and starboard sides of the actual hull, see Figure 10. To evaluate the effectiveness of the experience-based design, sea-trial measurements for hull pressure were performed using eight pressure sensors installed in the stern areas, see Figure 12. During the sea-trials, the propeller operated in the range of 80-84 rpm with ship speed around 21 knots. To have a general idea of the hull pressure impulse intensity, the measured pressures at 84 rpm are plotted in Figure 13. From the measurement, it is understood that the maximum pressure of 5.5 kPa is above the usual levels found in similar sized LNG carriers.

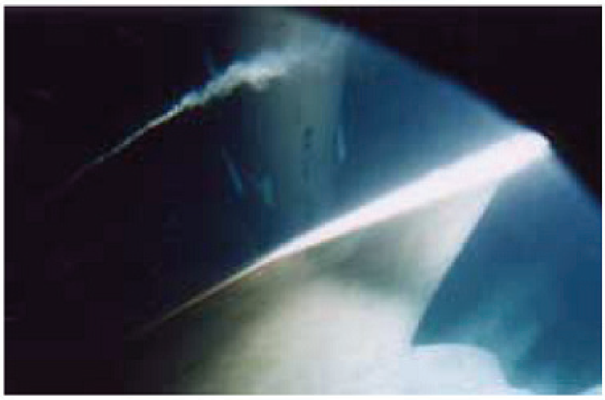

Figure 14: Typical tip vortex cavitation bursting from the actual video recording.

Furthermore, from the video records based on boroscopic camera for propeller cavitation observations, it is found that strong bursting of tip vortex cavitation, Figure 14, does occur causing strong pressure pulses transmitted to the hull, especially for higher blade passing frequency (BPF) pressure components of the 2nd, 3rd and 4th order, as measured by sensor E, see Figure 13.

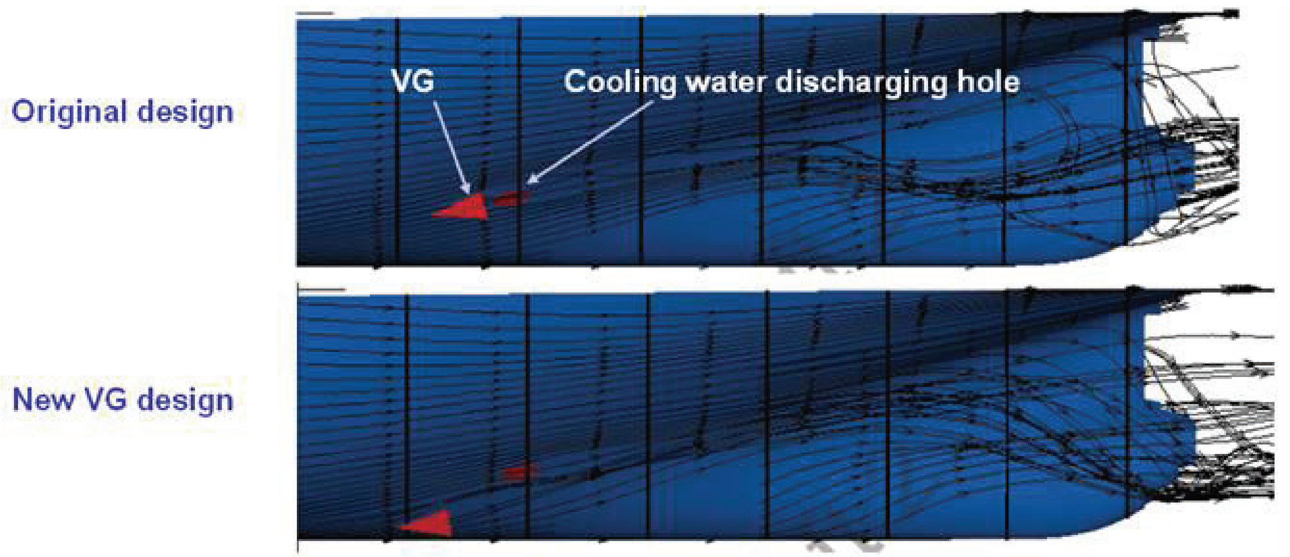

Figure 15: Location of port side VG – original location versus. new location, CFD study.

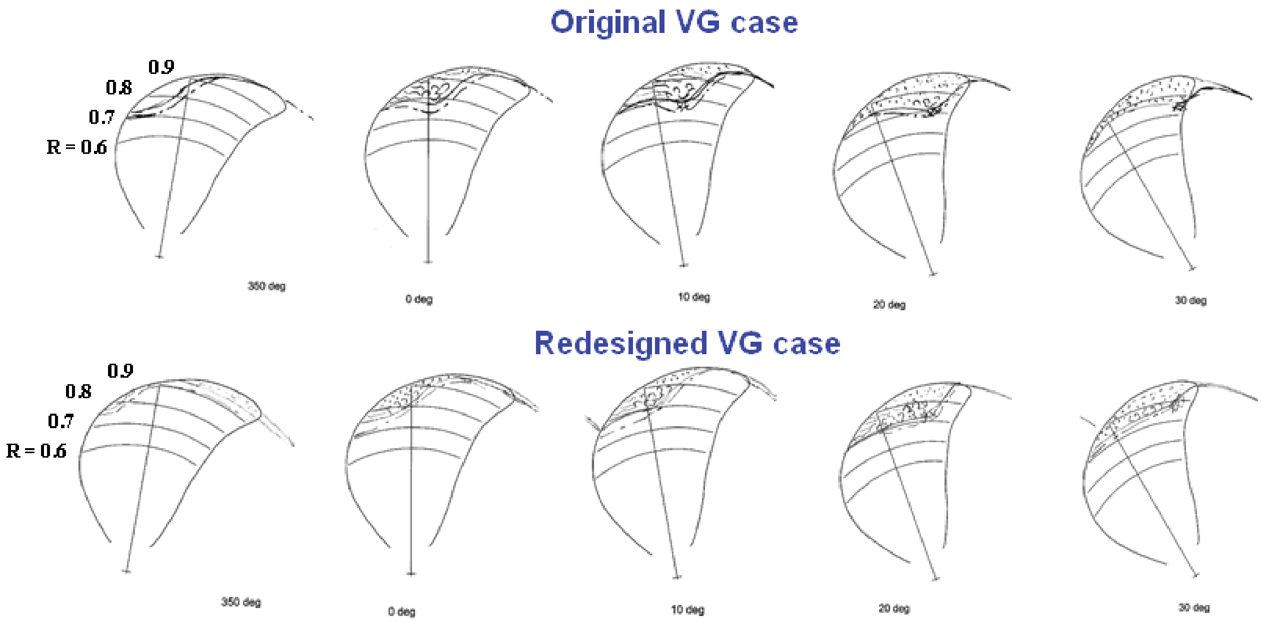

Figure 16: Blade cavity patterns – original VG design versus. new VG design, based on video recording images from the cavitation tunnel tests.

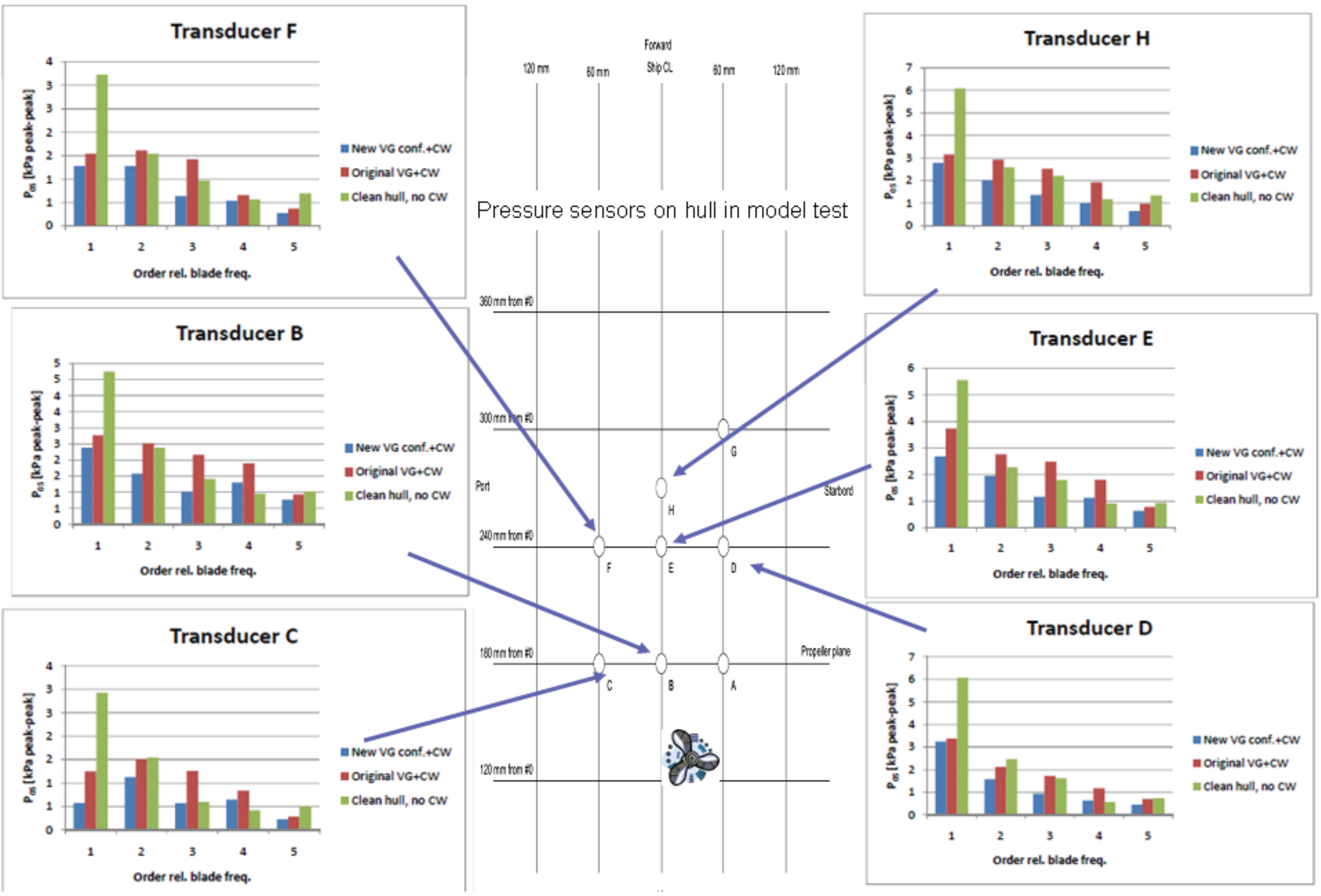

Figure 17: Propeller induced hull pressure (peak-to-peak) and the corresponding locations. Measurements are from cavitation model tank tests.

Transducer	A	B	C	D	E	F	G	**H**
Original VG	3.151	3.268	1.583	3.782	4.156	1.825	4.064	**3.829**
New VG	2.912	2.303	1.041	3.269	2.955	1.433	3.382	**2.600**
% reduction	**7.5**	**29.5**	**34.2**	**13.6**	**28.9**	**21.5**	**16.8**	**32.1**

Table 1: Tabular comparison of pressure reduction between original VG and new VG design as per Figure 17 data.

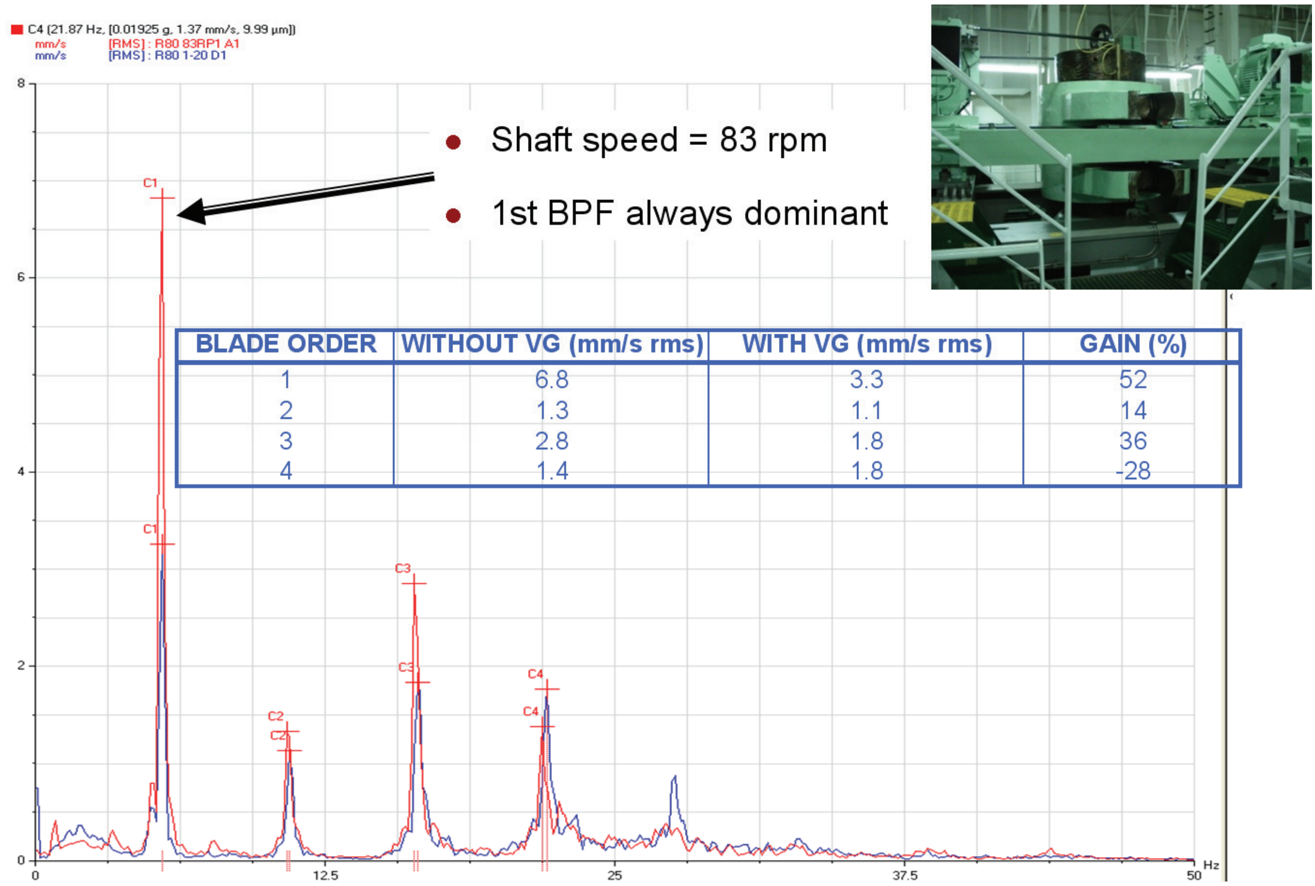

Figure 18: BPF components reduced by up to 52% with the installation of Vortex Generators. Measurement in the Steering Gear Room.

In order to optimize the VG design, further CFD investigation was carried out. Through systematic flow field simulations and comparisons, a new VG location on port side was proposed (see Figure 15). Compared to the original design, the new VG was moved down and forward away from the cooling water discharging hole. Also, as indicated in Figure 15, due to the effect of new VG location adjustment, streamlines of the cooling water jet were flown away from the critical 12 o'clock position of the propeller disk and the flow went downward and further away from the propeller disk, at approximately 6 o'clock of the propeller tip area.

Finally, to evaluate the actual performance of the latest design, cavitation tunnel tests were carried out to observe the propeller cavitation pattern and to measure magnitude of the hull pressure impulse. The similarity law for propeller cavitation inception and growth is considered to have been met and the propeller cavitation number between cavitation tunnel test and sea trials was considered equivalent.

Figure 16 shows the comparison results of blade cavity patterns between the original VG and new VG designs, as observed from the cavitation tunnel test. As seen, cavitation volume for the new VG design reduces at the critical area, at around 12 o'clock position (0^{o} degree). Since cavitation volume is closely related to wake quality, cavitation decrease implies that the wake quality is improved by the new VG location adjustment.

To further evaluate the hull pressure improvement due to the new VG design, the peak-to-peak hull pressure magnitudes for the bare hull (without VG), as well as the original VG design and the new VG design, all under the same cooling water discharging jet effect, are plotted in Figure 17. As seen in the figure, comparing to "the without VG case" (the green bars in the figure), large pressure reductions of more than 50% are obtained for the 1st BPF pressure in the two VG "add-on" cases. However, it should be noted that the original VG design (the red bars) causes higher hull pressures at high blade passing frequency components (2nd, 3rd, 4th and 5th blade rate pressure). For the new VG design (the blue bars in the figure), it is found that further reduction of the pressure can be achieved for all BPF pressures compared to the original VG and the "without VG" cases. Table 1 summarizes the % reduction of the hull pressure, based on the model test results for the original and the new VG design cases. As seen from the table, the hull pressure reduction due to the new VG design is in the range of 7.5% - 34%. On average, the hull pressure pulsation can be reduced by about 23% due to the new optimised VG design. This shows that experience does not always help when it comes to empirically positioning Vortex Generators in order to cure a propeller cavitation problem. Instead, only comprehensive CFD analysis can assist in improving the wake quality.

It is eventually noted that the BPF pressure patterns observed during these measurements reflect the Fast Fourier Transform (FFT) pattern of the vibration measurements analyzed earlier, in terms of frequency content. However, the two patterns (FFT pressure and FFT vibration) differ in that the higher order vibration BPF components (above 1 x BPF) excite certain natural frequencies of structures and get magnified at relatively high amplitude magnitudes. This becomes the cause of the intense vibration. Consequently, the corresponding frequency responses could be additionally used as a diagnostic tool to detect onsets or even advanced cases of propeller cavitation.

From the vibration measurements (see Figure 18), one can see a frequency response overlay between the original (without VG's) situation and the last VG (optimised) design. It can be seen that the vibration is substantially reduced with the successful installation of the new VG's.

4 CONLUSIONS

Analysis of the vibration signal at locations in the aft area of the vessel through frequency spectrum plots and spectrograms shows a dominant blade passing frequency component and its multiples, occasionally at higher amplitudes than the main blade passing frequency. This is a diagnostic signature of a propeller hydrodynamic problem and, in particular, a problem of propeller cavitation. The frequency content of the signal can cause excitation of natural frequencies of structures that were not designed to be excited at more than two or three times the propeller blade passing frequency. Thus, the existence of these higher BPF multiples can cause undesirable, unpredicted vibration on board the vessel. Assuming it is too costly or impractical to replace the propeller during vessel's operational service with a better propeller design with reduced cavitation, interventions on the wakefield through flow-improving devices such as vortex generators can reduce the cavitation intensity and hence the induced vibration on the hull. Comprehensive CFD analysis, accompanied by model tank tests where possible, rather than pure experience, is necessary for a successful and effective VG installation. Where, after wakefield flow improvements, vibration problems are not completely eliminated, then stiffening of the subject structures should be attempted to further reduce the vibration. The stiffening process, in order to be effective, should involve both measurements through vibration monitoring and/or modal tests, always correlated with results from FE analysis.

5 ACKNOWLEDGMENTS

The authors would like to acknowledge the invaluable assistance of AK Seah (ABS Vice President, Corporate Environmental Solutions), Peter Tang-Jensen (ABS Senior Vice President, Corporate Technology) and Dr Jer-Fang Wu (ABS Singapore Offshore Technology) for their contribution and for facilitating the coordination among the authors. Last but not least, the authors would like to acknowledge the invaluable contribution of Davor Sverko (Senior Principal Engineer, Corporate Marine Technology) with respect to the technical content and the measurements, as well as Wärtsilä Danmark A/S.

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