NON LINEAR ANALYSIS OF SHIP MOTIONS AND LOADS IN LARGE AMPLITUDE WAVES

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SUMMARY

A non linear time domain formulation for ship motions and wave loads is presented and applied to the S175 containership. The paper describes the mathematical formulations and assumptions, with particular attention to the calculation of the hydrodynamic force in the time domain. In this formulation all the forces involved are non linear and time dependent. Hydrodynamic forces are calculated in the frequency domain and related to the time domain solution for each time step. Restoring and exciting forces are evaluated directly in time domain in a way of the hull wetted surface. The results are compared with linear strip theory and linear three dimensional Green function frequency domain seakeeping methodologies with the intent of validation. The comparison shows a satisfactory agreement in the range of small amplitude motions. A first approach to large amplitude motion analysis displays the importance of incorporating the non linear behaviour of motions and loads in the solution of the seakeeping problem.

NOMENCLATURE

a^{∞}_{ij}	Sectional added mass coefficient at infinite frequency (kg m^{-1})
A^{∞}_{ij}	Added mass coefficient at infinite frequency (kg)
B	Ship breadth (m)
b_{ij}	Sectional damping coefficient (kg s^{-1})
F^{DAM}_j	Damping force in j-th motion (N)
F^{D}_j	Hydrodynamic force in j-th motion (N)
F^{E}_j	Exciting force in j-th motion (N)
F^{H}_j	Fluid force in j-th motion (N)
f^{H}_j	Sectional fluid force in j-th motion (N m^{-1})
F^{IMP}_j	Impulsive force in j-th motion (N)
F^{LIFT}_j	Lift force in j-th motion (N)
Fn	Froude number
F^{R}_j	Restoring force in j-th motion (N)
g	Acceleration of gravity (m s^{-2})
G	Two dimensional Green function
k	Wave number (m^{-1})
K_{ij}	Kernel of memory effect function (kg m^{-1} s^{-2})
L_{BP}	Length between the perpendicular (m)
$\underline{M}$	Fluid momentum (kg m)
M_{ij}	Element of mass matrix (kg, kg m)
$\underline{n}$	Normal Vector
p	Pressure (N m^{-2})
S	Boundary surface of the fluid domain (m^2)
S_0	Transverse part of boundary surface (m^2)
S_{∞}	Far away part of boundary surface (m^2)
S_F	Free-surface part of boundary surface (m^2)
S_H	Body part of boundary surface (m^2)
T	Ship immersion (m)
$\underline{U}$	Ship velocity (m s^{-1})
U_n	Normal component of ship velocity (m s^{-1})
$\underline{V}$	Velocity inside the fluid (m s^{-1})
V_n	Normal component of fluid velocity (m s^{-1})
η_j	Displacement in j-th motion (m, rad)
λ	Wave length (m)
ρ	Density of water (kg m^{-3})
τ	Time integration variable (s)
φ	Velocity potential (m^2 s^{-1})
χ	Memory effect potential (m^2 s^{-1})
ψ	Impulsive potential (m^2 s^{-1})
ω	Wave frequency (rad s^{-1})

1 INTRODUCTION

The analysis of the seakeeping behaviour of ocean going vessels is one of the main tasks in ship design. Strip theories, depending on their complexity, can provide a fast and relatively accurate tool for the prediction of ship motions in waves at preliminary design stage. Theoretically rigorous approaches are widely used since Salvesen, Tuck and Faltinsen introduced their strip theory in the 70's [10] and can be classified at different levels as suggested by ISSC [5].

In the traditional form of strip theories the equations of motions are solved in the frequency domain considering the ship and the wave environment as a linear system.

The non linear components of the equations of motions are neglected under the hypothesis of small amplitude motions. However, their importance increases when wave elevation rises. For this reason it is thought that the analysis of ship behaviour under large amplitude waves may be investigated using more complex techniques.

During the last twenty years several nonlinear methods have been presented [1] [16] [15]. Those differ from each other depending upon their mathematical complexity and associated assumptions. The most complex non linear approaches attempt to solve the hydrodynamic problem in the time domain via employing the fully nonlinear boundary conditions. The most widely used methods are the one based on the mixed Eulerian-Lagrangian approach [7]. The latter require high computational cost and can be unstable due to wave breaking phenomena [16].

Other techniques have been introduced in order to avoid numerical instabilities due to wave breaking [1].These methods solve the hydrodynamic problem on a known free surface such as a calm water level or exciting wave pattern, but they are also computationally expensive and therefore they cannot be used for preliminary ship design or hull form optimisation.

With the aim of developing a fast seakeeping tool, which is still able to model the non linear part of the forces, the so-called blended methods have been proposed. This class of techniques combine linear and non linear assumptions. They consider solely non linear exciting and restoring forces [2] [4] and may also include non linear hydrodynamic forces [15]. In those cases where the effects of hull flexibility may play an important role hydroelasticity analysis can be considered [6] [14] with the aim to investigate the effects of spinging and whipping phenomena on global ship dynamic response.

The aim of this paper is to develop a tool which can be used for preliminary ship design and optimisation. The approach chosen is a two dimensional (strip theory) blended method. The ship is described as a rigid body with two degrees of freedom simulating heave and pitch. The radiation problem is treated non linearly and is solved in the frequency domain in way of the real wetted hull geometry using boundary elements. The solution is translated from frequency to the time domain using memory effect functions [3]. The non linear formulation of the hydrodynamic sectional forces enables the introduction of the impulsive and lift components, which are neglected in a linear approach. In order to ensure computational efficiency while simulating non linear effects the non linear exciting and restoring forces are calculated using the actual wetted hull portion for each time step. The reduced order seakeeping problem is solved numerically in the time domain using the fourth order Runge-Kutta method. The non linear hydrodynamic formulations are compared against two- and three- dimensional linear approaches for the case of the S-175 Container Ship.

2 MATHEMATICAL FORMULATION

This section provides a brief description of the main features of non linear seakeeping theory currently being developed for the purposes of this work. Particular emphasis is attributed to the non linear formulation of the hydrodynamic forces and to the formulation of the equations of motion.

The ship is considered as a rigid body system with two degrees of freedom simulating the heave and pitch motions. The ship is considered as a sum of cylindrical sections on which the forces are constant. The global forces are calculated via longitudinal integration along the body. The equations of motions are considered as follow:

$$\begin{cases} M_{33}\ddot{\eta}_3 + M_{53}\ddot{\eta}_5 = F_3^D + F_3^R + F_3^E \\ M_{55}\ddot{\eta}_5 + M_{35}\ddot{\eta}_3 = F_5^D + F_5^R + F_5^E \end{cases} \tag{1}$$

The external forces acting on the body are divided in three force categories: external hydrodynamic forces (F^D_j), restoring forces (F^R_j) as well as the combination of hydrostatic forces, weight effects and exciting forces (F^E_j).

2.1 HYDRODYNAMIC FORCE

Non linear hydrodynamic forces are calculated for each time step in a way of the actual wetted hull surface and the linearised free surface.

The hydrodynamic actions are calculated in accordance with potential flow analysis. Hence the fluid is considered to be inviscid, incompressible, uniform and irrotational. As suggested by strip theory assumptions, the hydrodynamic forces in this model are two dimensional and of constant magnitude along the strips. The sectional hydrodynamic forces are formulated along the principles of Newtonian dynamics, i.e. by assuming that the rate of change of momentum with time inside the fluid volume is equal and opposite to the sum of the external forces acting on the fluid volume.

$$\frac{D}{Dt}\underline{M}(t) = \underline{F_H}(t) \tag{2}$$

The velocity potential definition and the Gauss theorem are used to express the internal fluid momentum as a function of the velocity potential along the boundary surface of the fluid domain.

$$\underline{M}(t) = \iint_S \rho\varphi(y,z;t)\underline{n}ds \qquad where\ S = S_0 + S_H + S_F + S_\infty \tag{3}$$

The rate of change of momentum is expressed using the definition of its derivative and the properties of incompressible fluids

$$\frac{d}{dt}\underline{M}(t) = -\rho\iint_S \left[\left(\frac{p}{\rho} + gz\right)\underline{n} + \underline{V}\left(V_n - U_n\right)\right]ds \tag{4}$$

Equation (4) considers both the hydrodynamic and the hydrostatic contributions to fluid actions. By combining the momentum (see Equation (3)) and its time derivative (see Equation (4)) the assumption of potential flow leads to a more suitable formulation for the calculation of hydrodynamic forces as integral of the dynamic pressures along the wetted part of the hull.

$$\frac{d}{dt}\iint_S \rho\varphi\underline{n}ds = -\rho\iint_S \left[\left(\frac{p}{\rho} + gz\right)\underline{n} + \nabla\varphi\left(\frac{\partial\varphi}{\partial\underline{n}} - U_n\right)\right]ds \tag{5}$$

Equation (5) describes the rate of change of momentum by integrating the pressure variations along the boundary of the fluid domain.

The fluid domain surface *(S)* can be divided into four parts namely (a)S_∞ the far away boundary surface (b) S_H the body surface, (c) S_F the free surface and (d) S_0 the two transverse plane surfaces. Those delimit the fluid domain decomposition along the longitudinal axes and the far away boundary (S_∞). Considering the potential distribution and geometric properties of each boundary it is possible to simplify Equation (5) with the aim to obtain a formulation ample to calculate the fluid pressure along the body surface. The fluid force acting on each section is solved via integration along the hull surface of the fluid pressure, as described in Equation (6) below:

$$\iint_{S_H} p\underline{n}ds = -\rho\left(\frac{d}{dt} - U\frac{\partial}{\partial x}\right)\iint_{S_H} \varphi\underline{n}ds - \iint_{S_H} \rho gz\underline{n}ds \qquad (6)$$

The fluid force acting on the body is composed by the hydrodynamic and hydrostatic components, respectively the first and the second term in the right hand side of Equation (6).

The velocity potential is obtained by solving the boundary value problem. The velocity potential must satisfy the Laplace equation in the fluid domain and the boundary conditions and an infinite depth condition is assumed. The later leads to the following boundary value problem for the velocity potential φ.

$$\begin{cases} \nabla^2\varphi = 0 & \text{in the fluid} \\ \dfrac{d}{dt}\varphi + g\dfrac{\partial}{\partial z}\varphi = 0 & \text{on } S_F \\ \dfrac{\partial}{\partial \underline{n}}\varphi = V_n & \text{on } S_H \\ \nabla\varphi = 0 & \text{for } z \to \infty \\ \nabla\varphi = 0 & \text{for } |y| \to \infty \end{cases} \qquad (7)$$

The velocity potential used has a different formulation than the classical one given in Equation (7). In the time domain approach the velocity potential is composed of two components namely (a) the impulsive part related to an instant impulse of displacement and (b) the second part describing the fluid velocity due to wave radiation. The velocity potential is decomposed in two components (for this model) related to each of the modes of motion.

$$\varphi_j(x,y,z;t) = \psi_j(x,y,z;t)V_j(x;t) + \int_{-\infty}^{t} \chi(x,y,z;t-\tau)V_j(x;\tau)d\tau \quad j = 3,5 \qquad (8)$$

Where the sectional vertical velocity V_3 is defined as

$$V_3 = \dot{\eta}_3 - x\dot{\eta}_5 \qquad (9)$$

The boundary value problem of Equation (7) is not valid for this formulation of the velocity potential. The velocity potential φ is divided into two parts, and each boundary value problem is solved separately

$$\begin{cases} \nabla^2\psi_j = 0 & \text{in the fluid} \\ \dfrac{\partial}{\partial \underline{n}}\psi_j = n_j & \text{on } S_H \\ \psi = 0 & \text{on } z = 0 \end{cases} \qquad (10.1)$$

$$\begin{cases} \nabla^2\chi_j = 0 & \text{in the fluid} \\ \dfrac{\partial^2}{\partial \tau^2}\chi_j + g\dfrac{\partial}{\partial z}\chi_j = 0 & \text{on } z = 0 \\ \dfrac{\partial}{\partial \underline{n}}\chi_j = 0 & \text{on } S_H \\ \chi_j(0) = 0 & \\ \dfrac{\partial}{\partial \tau}\chi_j(0) = -g\dfrac{\partial}{\partial z}\psi_j & \text{on } z = 0 \end{cases} \qquad (10.2)$$

In Equations (10.1) and (10.2) the boundary conditions are more extensive than in the classical formulation of Equation (7). This is because via the subdivision of the velocity potential into two components, new conditions are required in order to satisfy the continuity of the velocity potential in time. The boundary conditions presented in Equations (10.1) and (10.2) assume that, to keep continuity in time, the elevation of the wave generated by the impulse of displacement in an instant t must be equal to the elevation of the radiated wave.

The solution of the two problems is not obtained directly in the time domain, but it is related to some well-known frequency domain approaches. The impulsive problem with its boundary condition is the same as the one corresponding to a floating body oscillating at an infinite frequency. Hence, the problem is solved using the same numerical procedure. The second potential is not solved directly in the time domain, but it is obtained using the inverse Fourier transform of the damping coefficient for the frequency domain [3] [14].

$$\rho\iint_{S_H} \psi_i n_j ds = a_{ij}^{\infty} \qquad (11.1)$$

$$\rho\iint_{S_H}\left[\int_{-\infty}^{t} \chi_j(x,y,z;t-\tau)V_j(x;\tau)d\tau\right]ds = \int_{-\infty}^{t} K_{ij}(x;t-\tau)V_j(x;\tau)d\tau \qquad (11.2)$$

$$K_{ij} = \frac{2}{\pi}\int_0^\infty B_{ij}(\omega)\cos(\omega t)d\omega \tag{11.3}$$

Where a_{ij}^∞ is the sectional added mass at infinite frequency and B_{ij} is the sectional frequency domain damping coefficient. Using Equations (6), (8), (10.1) and (10.2), the sectional hydrodynamic force acting on the hull can be formulated as

$$f_{Hij} = -\left(\frac{d}{dt} - U\frac{\partial}{\partial x}\right)\left[a_{ij}^\infty(x;t) + \int_{-\infty}^{t} K_{ij}(x;t-\tau)V_j(x;\tau)d\tau\right] \tag{12}$$

Considering the sectional hydrodynamic force nonlinear means that all the terms in Equation (12) are time dependent, and they must be derived with respect to time.

$$f_{Hj}(x;t) = \sum_i \left[-a_{ij}^\infty \frac{D}{Dt}V_j + U\frac{\partial}{\partial x}a_{ij}^\infty V_i - \frac{\partial}{\partial z}V_i^2 - \frac{D}{Dt}\int_{-\infty}^{t} K_{ij}(x;t-\tau)V_i(x;\tau)d\tau\right] \tag{13}$$

The non linear formulation is composed of four terms and it includes a lift and an impulsive component, respectively as shown in Equation (13).

2.2 HYDRODYNAMIC COEFFICIENTS

The hydrodynamic coefficients used in the time domain simulation (see Equations (10.1), (10.2)) are solved in the frequency domain and the velocity potential is calculated using the boundary element method. Since the hydrodynamic forces are non linear, the boundary value problem should be solved for the same section for different combination of immersions and heel angles.

The biggest drawback of the boundary value methods is the presence of irregular frequencies. Irregular frequencies are infinite set of frequencies in which the problem has no unique solution [12]. The presence of irregular frequencies can be easily avoided in a frequency domain analysis, since usually the first irregular frequency appears at a frequency higher than the range of interest. For the time domain the whole range of frequency may be important. In this work the direct method approach introduced by Sclavounos [11] has been used. This integral equation does not remove irregular frequencies completely, but reduces the range of the irregularities and consents to easily correct the hydrodynamic coefficients with no computational cost. The solution of the velocity potential is given by the following integral equation:

$$-\frac{1}{2}\varphi(\underline{x}) + \frac{1}{2\pi}\int_{S_H}\varphi(\underline{\xi})\frac{\partial G(\underline{\xi},\underline{x})}{\partial n_{\underline{\xi}}}d\xi = \frac{1}{2\pi}\int_{S_H} G(\underline{\xi},\underline{x})\frac{\partial \varphi(\underline{\xi})}{\partial n_{\underline{\xi}}}d\xi \tag{14}$$

Where $G(\underline{\xi},\underline{x})$ is the two dimensional Green's function used to describe the velocity potential, and is formulated as follow

$$G(\underline{x}_S,\underline{x}) = \ln(r) - \ln(r_1) - 2\oint_0^\infty \frac{1}{\mu - 1}e^{-\mu Z}\cos(\mu Y)d\mu + i2\pi e^{-Z}\cos(Y)$$

(15.1)

Where

$$\begin{aligned} r &= \sqrt{(y-y_s)^2 + (z-z_s)^2} \\ r_1 &= \sqrt{(y-y_s)^2 + (z+z_s)^2} \\ Y &= k\left|y-y_s\right| \\ Z &= -k(z+z_s) \end{aligned} \tag{15.2}$$

In Equations (15.1) and (15.2), x_S is the source point along the surface and x is the point on the hull surface where the velocity potential is calculated.

2.3 RESTORING AND EXCITING FORCES

The restoring and exciting forces are calculated directly in the time domain for each time step and related to the actual wetted hull surface in way of the calm water level.

The restoring force is the equivalent of the stiffness matrix in the linear seakeeping analysis carried out in the frequency domain. This force is given by the difference between the ship weight and the hull buoyancy. The exciting forces are composed by Froude-Krylov and the diffraction forces. They are calculated at each time instant using the strip theory approach described by Salvesen [10].

2.4 EQUATIONS OF MOTION

The equations of motion are numerically solved in the time domain with the Runge-Kutta fourth order method. The equations of motion in a manner suitable for the numerical solution are expressed as:

$$\begin{cases} \left(M_{33} + A_{33}^\infty\right)\ddot{\eta}_3 + \left(M_{53} + A_{53}^\infty\right)\ddot{\eta}_5 = \mathrm{F}_3^{\mathrm{E}} - \mathrm{F}_3^{\mathrm{Dam}} - \mathrm{F}_3^{\mathrm{Imp}} - \mathrm{F}_3^{\mathrm{Lift}} - \mathrm{F}_3^{\mathrm{R}} \\ \left(M_{55} + A_{55}^\infty\right)\ddot{\eta}_5 + \left(M_{35} + A_{35}^\infty\right)\ddot{\eta}_3 = \mathrm{M}_5^{\mathrm{E}} - M_5^{\mathrm{Dam}} - M_5^{\mathrm{Imp}} - M_5^{\mathrm{Lift}} - M_5^{\mathrm{R}} \end{cases} \tag{16}$$

All the forces involved are non linear and evaluated on the actual hull immersion at each time instant. The restoring and exciting forces are directly calculated in the time domain along the hull strip sections. The hydrodynamic forces are related to the frequency domain using an inverse direct Fourier transform, for the actual hull shape and for the calm water level for each time step. In order to reduce the computational time the hydrodynamic coefficients are calculated are stored before the simulation for different immersions and trim angles. The hydrodynamic coefficients are obtained through an extrapolation during the time domain analysis.

3 RESULTS

The method outlined in section 2 is applied to the prediction of the motions of the S175 container ship travelling in regular head waves at zero speed. The results of the non linear approach are compared against a linear frequency domain strip theory and a three dimensional Green function idealisation. The main particulars of the containership are shown in Table 1. The body plan of the S175 container ship is shown in Figure 1.

In two-dimensional linear and non linear strip based idealisations the ship is divided in 40 sections and mass distribution as shown in Figure 5. In the three dimensional Green function idealisation 789 pulsating source panels have been used. This idealisation ensured a relatively crude, yet adequate panel aspect ratio of the order of 2.10:1 as shown in Figure 2. The choice of 3D hydrodynamic mesh was decided after examining the dependence of hydrodynamic coefficients on the number of panels used.

The wave frequencies are in the range between 0.2 and 1.2 rad/sec. Wave amplitudes range from 0.1 meters to 6.0 metres with a ship immersion of 9.50 metres. The range of height of the simulated waves is formulated in a way that the response of the vessel can vary from a linear behaviour, for smaller amplitudes, to a non linear one, for biggest waves.

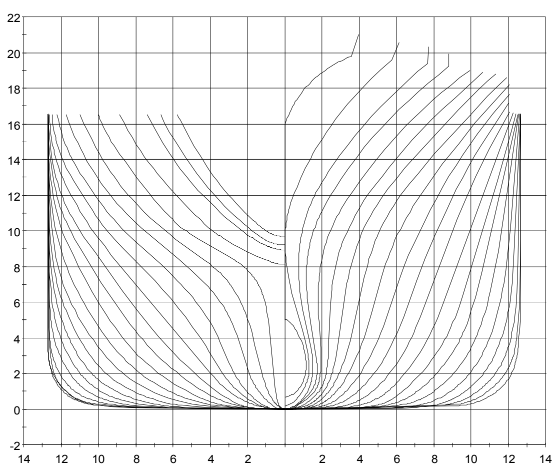

Figure 1: S175 Container ship

Table 1: Main particulars of the S175 container ship

Length between perpendiculars [m]	175.00
Beam [m]	25.40
Depth [m]	15.40
Draught [m]	9.50
Displacement [tonnes]	24792
Pitch radius of gyration [m]	43.75

In order to compare a nonlinear time domain code with a linear frequency domain method the simulations are conducted using small amplitude exciting waves. This condition is necessary to employ linear methods in the validation. It is considered that the response of a nonlinear system under small amplitude waves is not affected by any non linear behaviours and is analogous to the prediction of linear method.

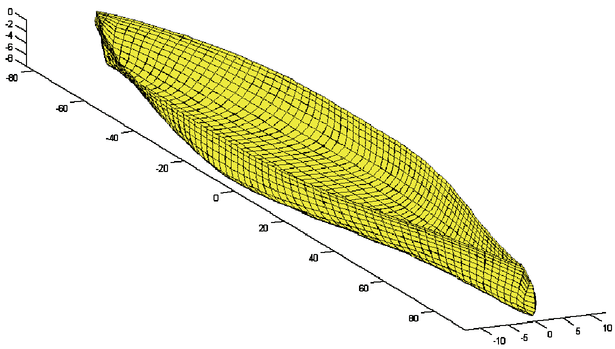

Figure 2: 3D hull idealisation of the S175 container ship used in the 3D Frequency domain method

The comparison for heave and pitch motion is described in Figures 3 and 4, where the non-dimensional responses are plotted against non-dimensionalised wave frequencies.

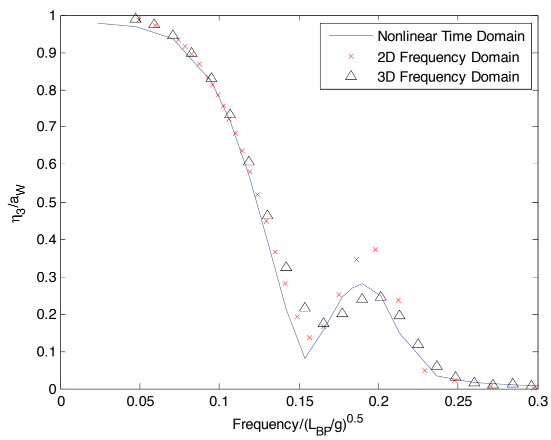

Figure 3: Comparison of heave response between nonlinear time domain (a_W = 0.05m) 2D and 3D linear frequency domain methods

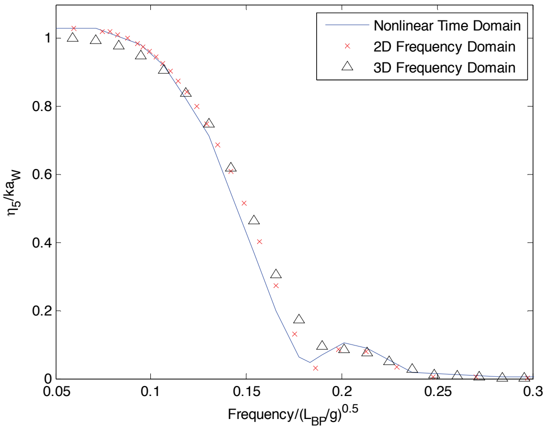

Figure 4: Comparison of pitch response between nonlinear time domain (a_W = 0.05m) 2D and 3D linear frequency domain methods

The prediction of vertical wave bending moment at amidship *x=87.5m* from AP is shown in Figure 5. The longitudinal mass distribution used in the calculation is described in Figure 5. The weight distribution is formulated in such a way that the design immersion of 9.50 metres in a way of amidships and a static trim of zero degrees is maintained during the simulations. The results are compared with a two dimensional and three dimensional linear frequency domain methods. In Figure 6 the results are non dimensionalised as shown below:

$$RVB = \frac{VBM}{\rho g a_w B L^2} \tag{17}$$

Where *RVB* is the non-dimensional wave bending moment, and *VBM* is the dimensional value for vertical wave bending moment.

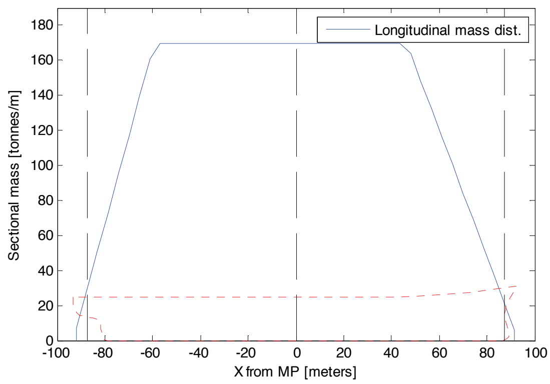

Figure 5: Longitudinal mass distribution for the S175 Containership

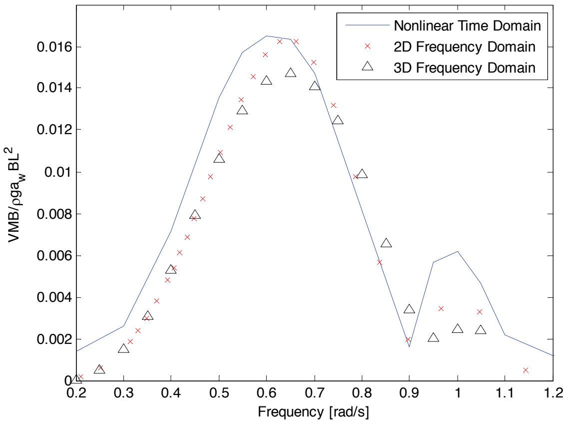

Figure 6: Vertical bending moment at x=87.5m from AP

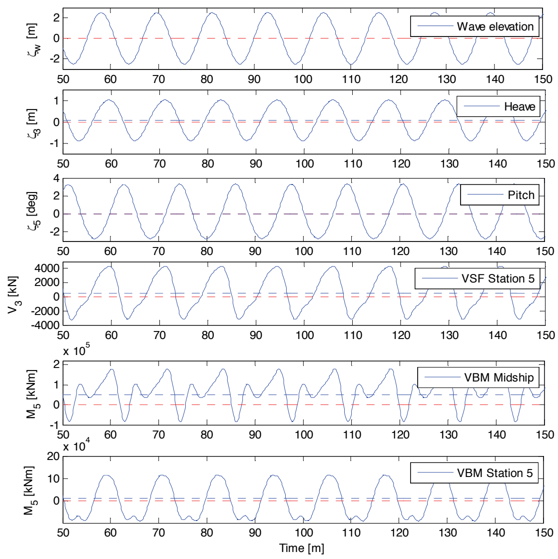

Figure 7: Time history of vertical responses for Fn=0.0 λ/L_{BP}=1.20 H_w/λ=1/42

Simulations for large amplitude waves are conducted with the intention of qualitatively assessing the nonlinear responses due to large amplitude motions. Figure 7 shows the time history of the vertical responses. At higher wave amplitudes the time histories show a non linear behaviour of the responses. The predicted motions and loads are non sinusoidal and not symmetrical.

4 CONCLUSIONS

A two dimensional non linear time domain method for the prediction of vertical motions and sea loads has been presented. In this formulation all the forces involved are non linear. The method has then been applied to the widely studied S175 containership. The results have been compared with commercial seakeeping codes for the purpose of validation.

The comparisons show a good agreement with the other methodologies for both the heave and pitch motions as well as in way of amidships vertical wave bending moment in regular head waves, at zero speed. Based on this limited investigation it is shown that the method under development is able to describe non linear effects due to the real hull geometry and due to the magnitude of motions. Nevertheless, more comparisons with theoretical prediction models and possibly experimental measurements are necessary in order to establish the range of validity of this method. It is believed that further parametric studies may emphasise the need for development of non linear methods, still within the potential flow domain, accounting for non linearities in radiation and diffraction potentials. Future work may concentrate on the further development of the current method towards a time domain hydroelastic model able to simulate the effects of symmetric (i.e. vertically induced) springing and whipping on global hull response.

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