QUANTIFICATION OF UNCERTAINTY IN MANOEUVRING CHARACTERISTICS FOR DESIGN OF UNDERWATER VEHICLES

Amit Ray, Indian Institute of Technology Delhi, India
Debabrata Sen, Indian Institute of Technology Kharagpur, India
SN Singh and **V Seshadri**, Indian Institute of Technology Delhi, India
(DOI No: 10.3940/rina.ijme.2010.a2.174)

SUMMARY

The prediction of manoeuvring characteristics of underwater vehicles during design involves approximations at various stages. This paper attempts to quantify some of the uncertainties involved in the manoeuvring characteristics of underwater vehicles. The first source of uncertainty is in idealization of mathematical model selected for trajectory simulation. This is illustrated for alternative mathematical models in trajectory simulation programs. Next, the values of the hydrodynamic coefficients (HDCs) in the equations of motion have their own levels of uncertainty, depending upon the methods used to determine them. The sensitivity of trajectory simulation results to uncertainty levels in various HDCs is examined. Finally, the level of uncertainty in full-scale measurements of manoeuvres of underwater vehicles is discussed and estimated. It emerges that the cumulative errors in the prediction process during design need to be reduced further, in order to maintain their levels of uncertainty below those of the validation process.

NOMENCLATURE

$\vec{F} = \{X,Y,Z\}$	External force on the body and its components along x, y and z directions
$\vec{M} = \{K,M,N\}$	Moment of the external force and its components along x, y and z directions
$\{x_G, y_G, z_G\}$	Coordinates of the body's center of gravity (CG) with respect to the body-fixed system *oxyz*
m	Mass of the vehicle
I_{ij}	Moments of inertia about the body-fixed system *oxyz*, where the indices i,j correspond to x,y,z coordinates.
u, v, w	Straight-line velocities of the body along x, y and z directions in the body-fixed frame of reference
p, q, r	Angular velocities of the body about x, y and z directions in the body-fixed frame of reference
g	Acceleration due to gravity

1. INTRODUCTION

The study of manoeuvring characteristics of underwater vehicles includes assessment of motion stability, controllability and trajectory simulation in six degrees of freedom. These have important operational implications for all types of underwater vehicles. There are a variety of approaches (empirical/analytical, experimental and numerical/computational techniques) for manoeuvring studies of underwater vehicles [1]. The problem of mathematical modelling of the motion of marine vehicles for trajectory simulation has been studied extensively over many decades [2-6]. The subject has gained renewed interest after the proliferation of Unmanned Underwater Vehicles (UUVs), which include Remotely Operated Vehicles (ROVs) and Autonomous Underwater Vehicles (AUVs) used for many commercial and scientific applications [7, 8].

Manoeuvring characteristics of underwater vehicles are difficult to predict accurately during initial stages of design. Frequently, recourse is made to empirical or semi-empirical methods for the purpose. Even after more detailed information is available from time-taking studies using tools like Computational Fluid Dynamics (CFD), or from expensive model tests, there are inherent uncertainties in the prediction process for manoeuvring qualities. All too often, attention is concentrated on particular aspects of the problem, such as model testing, CFD, control system design, system identification, or trajectory simulation. It is necessary to estimate the magnitude of uncertainty for the overall process of prediction of manoeuvring characteristics.

In order to quantify these uncertainties, it is necessary to understand the choices being made at each stage of the prediction process and their implications on the ultimate result. These aspects or stages for manoeuvring prediction / evaluation may be outlined as follows.

- Formulation of mathematical model for trajectory simulation
- Estimation of values of Hydrodynamic Coefficients (HDCs) for the body
- Simulation of trajectories for various manoeuvres
- Full-scale trials for standard manoeuvres and comparison with predictions

Each of these aspects has inherent uncertainties. The ultimate benchmark for evaluating any set of predictions should be comparing them with full-scale trial results for a set of standard manoeuvres. These are the only 'true' values against which the 'errors' in each stage of manoeuvrability prediction must be judged. However, not only are the full-scale measurements rarely conducted or reported, but the full-scale measurement process itself is beset with uncertainties. Therefore, it is important to define realistic limits for error bounds in each stage of manoeuvring prediction, in order to ensure practically reliable results.

This paper presents glimpses of the choices available to the designer / analyst at each of the stages for the manoeuvrability prediction process for underwater vehicles, ranging from manned submarines to UUVs. Cases are presented illustrating the effects of these decisions and the effects of variation in parameters on the trajectory simulation for typical underwater vehicles. Based on these simulations, the effect of variation in values of HDCs can be appreciated. Finally, the paper presents error bounds likely during full-scale measurements, which suggest the level of accuracy desirable for the preceding stages of manoeuvring prediction and analysis.

2. FOCUS OF THIS STUDY

Any study on the sensitivity of manoeuvring characteristics to the many variables in the modelling process needs to be limited to certain cases in order to keep the scope of the study within reasonable length. In this study, therefore, two underwater vehicle hull forms have been considered and various trajectory simulations have been undertaken for one particular type of definitive manoeuvre.

2.1 UNDERWATER VEHICLES CONSIDERED

Trajectory simulations were carried out for two underwater vehicle geometries in this study, which are typical of UUV and submarine forms, respectively:

- Axisymmetric body of revolution with cruciform control surfaces at stern [9, 10]. The length of the body considered is 10 metres and weight (*m.g*) is 196.5 kg.
- SUBOFF body with appendages (fin and cruciform control surfaces) as described in [11]. Length of the body is 4.26 m and weight (*m.g*) is 18.1 kg.

The dimensions and weights of the bodies mentioned above were used for the trajectory simulation studies. The vehicle mass properties and HDC values determined by model testing (Planar Motion Mechanism) and reported in [9, 11] were used in the trajectory simulation programs developed.

2.2 MANOEUVRE CONSIDERED

Definitive manoeuvres are carried out to characterise and compare the handling qualities of underwater vehicles. These include zigzag/overshoot, meander, spiral, pullout and turning circle manoeuvres [2, 12]. The depth-changing and depth-keeping abilities are particularly important for underwater vehicles due to the relatively narrow band of water (in depth) within which they require to operate. Therefore, the zigzag/overshoot manoeuvre in the vertical plane is considered for this study.

In the zigzag manoeuvre, starting from a level submerged course, the control surfaces are set at constant angle δ_1 as quickly and as smoothly as possible until the pitch angle (θ) becomes equal to the execute pitch angle (θ_e) decided for the manoeuvre. The planes are then deflected to an angle $-\delta_1$ until the original depth at the start of the manoeuvre is reached, or the execute pitch angle in the opposite sense ($-\theta_e$) is reached. The parameters of interest are the time to reach execute (t_e), time to check pitch (t_c), time to check depth (t_d), overshoot pitch angle (θ_O) and overshoot change of depth (z_O), as defined in Figure 1. These values will depend upon the specified plane deflection, execute pitch angle and the speed at which the manoeuvre is conducted.

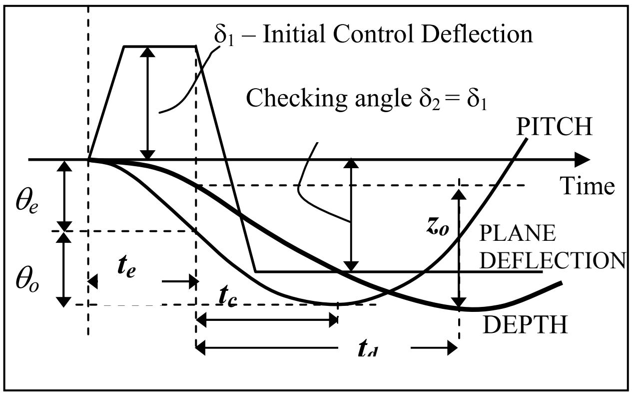

Figure 1 Definition of parameters for Zigzag manoeuvre in vertical plane [2]

The variation in values of the above parameters for standard 10/10 zigzag manoeuvre (i.e. δ_1=10 degrees, θ_e =10 degrees) and 15/5 zigzag at speed 5 knots was examined in this study for various mathematical models and HDC values.

3. MATHEMATICAL MODELS FOR TRAJECTORY SIMULATION

The motion of an underwater vehicle can be described in terms of Newton–Euler laws of motion, where the rate of change of momentum of a rigid body is equated to the external forces/ moments causing the change. For an underwater vehicle such as a submarine or an AUV, controlled motion is possible with six degrees of freedom (6-DOF).

3.1 RIGID BODY EQUATIONS OF MOTION

The trajectory of the body at any instant of time is described by its linear velocities *u, v, w*, and by its angular velocities *p, q, r,* in the body-fixed frame of reference, and by its position and orientation in an inertial frame of reference (Figure 2).

Let $\vec{F}$ denote the external force on the body, and $\vec{M}$ denote the moment of the external force about the origin *o* of the body-fixed system *oxyz*. Their components are denoted as, $\vec{F} = \{X,Y,Z\}$ and $\vec{M} = \{K,M,N\}$. The mass of the vehicle is *m* and the moments of inertia about the body-fixed system *oxyz* are denoted as I_{ij} where the indices *i,j* correspond to *x,y,z* coordinates. The coordinates of the body's center of gravity (CG) with respect to the body system *oxyz* are $\{x_G, y_G, z_G\}$.

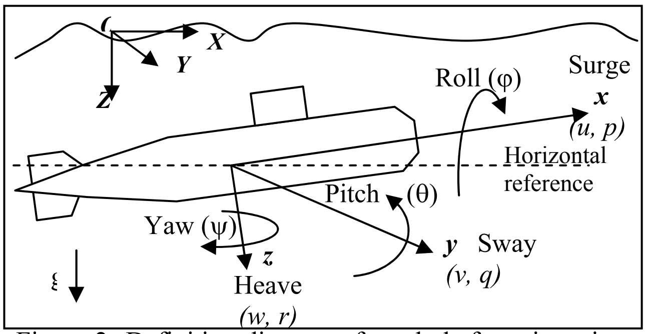

Figure 2 Definition diagram of symbols for orientation; linear and angular velocities

The governing 6-DOF equations of motion can be written as follows for the body-fixed frame of reference [5, 6, 13]:

Surge:

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X \tag{1}$$

Sway:

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] = Y \tag{2}$$

Heave:

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z \tag{3}$$

Roll:

$$\begin{aligned} &I_{xx}\dot{p} + (I_{zz} - I_{yy})qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} \\ &\quad + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K \end{aligned} \tag{4}$$

Pitch:

$$\begin{aligned} &I_{yy}\dot{q} + (I_{xx} - I_{zz})rp - (\dot{p} + qr)I_{yx} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz} \\ &\quad + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M \end{aligned} \tag{5}$$

Yaw:

$$\begin{aligned} &I_{zz}\dot{r} + (I_{yy} - I_{xx})pq - (\dot{q} + rp)I_{zy} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx} \\ &\quad + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N \end{aligned} \tag{6}$$

The above six dynamic motion equations can be written in matrix form [13] as:

$$[A]\{\dot{V}\} - \{F_I\} = \{F_E\} \tag{7}$$

where the external generalised forces are denoted by

$$\{F_E\} = \{X, Y, Z, K, M, N\}^T \tag{8}$$

and the terms on the left hand side of equation (7) are matrices of size 6x6, defined as follows.

The extended mass matrix (including added mass terms) is given by:

$$[A] = [\mathfrak{M}] + [\mathfrak{M}'] \tag{9}$$

where mass matrix is given by:

$$[\mathfrak{M}] = \begin{bmatrix} m & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m & my_G & -mx_G & 0 \\ 0 & -mz_G & my_G & I_{xx} & -I_{xy} & -I_{xz} \\ mz_G & 0 & -mx_G & -I_{yx} & I_{yy} & -I_{yz} \\ -my_G & mx_G & 0 & -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \tag{10}$$

and added mass matrix is taken as:

$$[\mathfrak{M}'] = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 & 0 & 0 & 0 \\ 0 & -Y_{\dot{v}} & 0 & -Y_{\dot{p}} & 0 & -Y_{\dot{r}} \\ 0 & 0 & -Z_{\dot{w}} & 0 & -Z_{\dot{q}} & 0 \\ 0 & -K_{\dot{v}} & 0 & -K_{\dot{p}} & 0 & -K_{\dot{r}} \\ 0 & 0 & -M_{\dot{w}} & 0 & -M_{\dot{q}} & 0 \\ 0 & -N_{\dot{v}} & 0 & -N_{\dot{p}} & 0 & -N_{\dot{r}} \end{bmatrix} \tag{11}$$

The subscript notation represents partial differentiation, so $X_{\dot{u}} = \partial X / \partial \dot{u}$, etc.

Acceleration vector is given by:

$$\{\dot{V}\} = \{\dot{u} \quad \dot{v} \quad \dot{w} \quad \dot{p} \quad \dot{q} \quad \dot{r}\}^T \tag{12}$$

The inertial force terms, independent of modelling of external hydrodynamic forces, are given by

$$\{F_I\} = \left\{ \begin{matrix} m[vr - wq + x_G(q^2 + r^2) - y_G pq - z_G pr \\ m[wp - ur + y_G(r^2 + p^2) - z_G qr - x_G qp \\ m[uq - vp + z_G(p^2 + q^2) - x_G rp - y_G rq \\ (I_{yy} - I_{zz})qr + I_{xz}pq + I_{yz}(q^2 - r^2) - I_{xy}pr - my_G(vp - uq) + mz_G(ur - wp) \\ (I_{zz} - I_{xx})rp + I_{yx}qr + I_{zx}(r^2 - p^2) - I_{yz}qp - mz_G(wq - vr) + mx_G(vp - uq) \\ (I_{xx} - I_{yy})pq + I_{zy}rp + I_{xy}(p^2 - q^2) - I_{zx}rq - mx_G(ur - wp) + my_G(wq - vr) \end{matrix} \right\} \tag{13}$$

Thus far, there are no approximations in the mathematical model, but the external forces and moments are yet to be modelled.

3.2 VARIANTS OF MATHEMATICAL MODEL

The external forces and moments due to hydrodynamic (and hydrostatic) loads are complicated functions of many factors, including water density, viscosity, surface tension, pressure, vapour pressure, and motions of the body. Rather than attempting to obtain hydrodynamic forces exactly from the Navier-Stokes equations, the components of the hydrodynamic forces and moments acting on the hull and on each appendage are usually treated as a function of the motion state variables, in terms of 'hydrodynamic coefficients' (HDCs) [2, 4, 5].

In general, the most important part of developing the dynamic motion equations is in expressing the external force and moment vector $\{F_E\}$ properly. These forces /moments may be split into physically meaningful components corresponding to hydrostatic (gravity-dependent) forces, fluid inertia (i.e. added mass, or acceleration-dependent terms), fluid damping forces (velocity-dependent terms), propulsion forces, and control surface forces, which are the most crucial for initiating any manoeuvre of the body.

The HDCs may be denoted using another form of subscripts, where subscripts denote the motion variable they are multiplied with. For example, for a small sway velocity v, the hydrodynamic force component arising from that motion is expressed as $Y_v v$, where Y_v is the HDC for the motion variable v in the equation of motion for the Y-force. All coefficients can be made non-dimensional using appropriate combinations of vessel length and initial axial velocity.

In order to generalise the various mathematical models available for expressing $\{F_E\}$ in terms of HDCs, we can consider $\{F_E\}$ as the product of various HDCs (as matrix [B] of size 6 x n) with respective motion variables (as matrix [C] of size n x 6), where n depends on the particular model and may typically vary from 6 to 15.

Thus,

$$\{F_E\} = diag\,([B].[C]) \qquad (14)$$

i.e. the vector $\{F_E\}$ is formed by the diagonal terms of the 6 x 6 matrix $[B].[C]$, which also includes terms corresponding to hydrostatic forces and control forces.

Various models developed for specific applications can be conveniently expressed in the above form. For example, only linear terms may be retained in some models, or cross-coupling between various motions may be neglected, or special terms may be added to account for cross-flow drag, or empirical corrective terms may be included.

Three mathematical models for external hydrodynamic force have been considered in this study, as described below.

3.2 (a) First Mathematical Model

For the first model [9, 13], we consider the following structure of matrices representing the external hydrodynamic forces.

$$[B_1] = \begin{bmatrix} X_{uu} & X_{vv} & X_{ww} & Z_{\dot{w}} & Z_{\dot{q}} & -Y_{\dot{r}} & -Y_{\dot{v}} & X_{\delta s\delta s} & X_{\delta r\delta r} \\ X_{\dot{u}} & Y_v & Y_{v|v|} & -Z_{\dot{w}} & -Z_{\dot{q}} & Y_r & Y_{\delta r} & 0 & 0 \\ Z_w & Z_{w|w|} & Y_{\dot{v}} & -X_{\dot{u}} & Z_q & Y_{\dot{r}} & Z_{\delta s} & 0 & 0 \\ K_{vw} & K_p & K_{v\dot{q}} & K_{qr} & K_{wr} & K_{\delta s} & K_{\delta r} & z_G W & 0 \\ M_{u\dot{w}} & M_w & M_{w|w|} & -N_{\dot{r}} & -Y_{\dot{r}} & -Z_{\dot{q}} & M_q & M_{\delta s} & -z_G W \\ N_{uv} & N_v & N_{v|v|} & Z_{\dot{q}} & M_{\dot{q}} & Y_{\dot{r}} & N_r & N_{\delta r} & 0 \end{bmatrix} \qquad (15)$$

where,

$$K_{vw} = Z_{\dot{w}} - Y_{\dot{v}} \qquad K_{vq} = Y_{\dot{r}} + Z_{\dot{q}}$$
$$K_{qr} = N_{\dot{r}} - M_{\dot{q}} \qquad K_{wr} = -Y_{\dot{r}} - Z_{\dot{q}} \qquad (16)$$
$$M_{uw} = X_{\dot{u}} - Z_{\dot{w}} \qquad N_{uv} = Y_{\dot{v}} - X_{\dot{u}}$$

The relations of (14) above hold good for any model, since the combination of some acceleration-dependent HDCs are merely being represented in terms of the state variables with which they are multiplied in the respective force / moment equation.

In the first model considered, we have

$$[C_1] = \begin{bmatrix} u^2 & ur & w & vw & uw & uv \\ v^2 & v & w|w| & p & w & v \\ w^2 & v|v| & vp & vq & w|w| & v|v| \\ wq & wp & uq & qr & rp & wp \\ q^2 & pq & q & wr & vp & pq \\ r^2 & r & pr & \delta s & uq & ur \\ vr & \delta r & \delta s & \delta r & q & r \\ (\delta s)^2 & 0 & 0 & c\theta.s\phi & \delta s & \delta r \\ (\delta r)^2 & 0 & 0 & 0 & s\theta & 0 \end{bmatrix} \qquad (17)$$

where $c\theta$ denotes $cos\theta$, $s\theta$ denotes $sin\theta$, and δr and δs denote rudder and stern plane deflection, respectively.

3.2 (b) Second Mathematical Model

The second model considered here is a partly linearised version of the first model, dropping squared velocity terms and retaining only those second order terms which are cross-products of velocity components. Thus, we have:

$$[B_2] = \begin{bmatrix} 0 & 0 & 0 & Z_{\dot{w}} & 0 & 0 & -Y_{\dot{v}} & 0 & 0 \\ X_{\dot{u}} & Y_v & 0 & -Z_{\dot{w}} & -Z_{\dot{q}} & Y_r & Y_{\delta r} & 0 & 0 \\ Z_w & 0 & Y_{\dot{v}} & -X_{\dot{u}} & Z_q & Y_{\dot{r}} & Z_{\delta s} & 0 & 0 \\ K_{vw} & K_p & K_{vq} & K_{qr} & K_{wr} & K_{\delta s} & K_{\delta r} & z_G W & 0 \\ M_{uw} & M_w & 0 & -N_{\dot{r}} & -Y_{\dot{r}} & -Z_{\dot{q}} & M_q & M_{\delta s} & -z_G W \\ N_{uv} & N_v & 0 & Z_{\dot{q}} & M_{\dot{q}} & Y_{\dot{r}} & N_r & N_{\delta r} & 0 \end{bmatrix} \qquad (18)$$

and

$$[C_2] = \begin{bmatrix} 0 & ur & w & vw & uw & uv \\ 0 & v & 0 & p & w & v \\ 0 & 0 & vp & vq & 0 & 0 \\ wq & wp & uq & qr & rp & wp \\ 0 & pq & q & wr & vp & pq \\ 0 & r & pr & \delta s & uq & ur \\ vr & \delta r & \delta s & \delta r & q & r \\ 0 & 0 & 0 & c\theta.s\phi & \delta s & \delta r \\ 0 & 0 & 0 & 0 & s\theta & 0 \end{bmatrix} \quad (19)$$

3.2 (c) Third Mathematical Model

The third model considered retains only strictly linear terms. Thus,

$$[B_3] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & Y_r & Y_{\delta r} & 0 & 0 \\ Z_w & 0 & 0 & 0 & Z_q & 0 & Z_{\delta s} & 0 & 0 \\ 0 & K_p & 0 & 0 & 0 & K_{\delta s} & K_{\delta r} & z_G W & 0 \\ 0 & M_w & 0 & 0 & 0 & 0 & M_q & M_{\delta s} & -z_G W \\ 0 & N_v & 0 & 0 & 0 & 0 & N_r & N_{\delta r} & 0 \end{bmatrix} \quad (20)$$

and

$$[C_3] = \begin{bmatrix} 0 & 0 & w & 0 & 0 & 0 \\ 0 & v & 0 & p & w & v \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q & 0 & 0 & 0 \\ 0 & r & 0 & \delta s & 0 & 0 \\ 0 & \delta r & \delta s & \delta r & q & r \\ 0 & 0 & 0 & c\theta.s\phi & \delta s & \delta r \\ 0 & 0 & 0 & 0 & s\theta & 0 \end{bmatrix} \quad (21)$$

Other mathematical models such as those given in [14, 15] can also be cast in this form, requiring additional HDCs to be evaluated / estimated. These models have retained many additional (cross-coupled and non-linear) HDC's, in addition to those obtained strictly by Taylor series expansions, based on the best-fit for measured forces in model testing.

4. TRAJECTORY SIMULATION

4.1 ALGORITHM FOR SIMULATION

The dynamic equations summarised in equation (7) can now be recast as:

$$\begin{aligned} \{\dot{u}\ \dot{v}\ \dot{w}\ \dot{p}\ \dot{q}\ \dot{r}\}^T &= [A]^{-1}\{\{F_E\}+\{F_I\}\} \\ &= [A]^{-1}\{\{diag[B].[C]+\{F_I\}\} \end{aligned} \quad (22)$$

The following transformation relations exist between the velocities in the body system *oxyz* and the inertial system $Ox_Oy_Oz_O$:

$$\begin{aligned} \dot{x}_0 &= c_2c_3\,u + (-c_1s_3 + s_1s_2c_3)v + (s_1s_3 + c_1s_2c_3)w \\ \dot{y}_0 &= c_2s_3u + (c_1c_3 + s_1s_2s_3)v + (-s_1c_3 + c_1s_2s_3)w \\ \dot{z}_0 &= -s_2u + s_1c_2v + c_1c_2w \end{aligned} \quad (23)$$

$$\begin{aligned} \dot{\phi} &= p + (s_1s_2/c_2)q + (c_1s_2/c_2)r \\ \dot{\theta} &= c_1q - s_1r \\ \dot{\psi} &= (s_1/c_2)q + (c_1/c_2)r \end{aligned} \quad (24)$$

where

$$\begin{aligned} s_1 &= \sin\phi; \quad s_2 = \sin\theta; \quad s_3 = \sin\psi \\ c_1 &= \sin\phi; \quad c_2 = \sin\theta; \quad c_3 = \sin\psi \end{aligned} \quad (25)$$

Equations (22, 23, 24) are thus 12 ordinary linear differential equation whose integration in time will determine the values of vehicle position X_O, Y_O, Z_O and orientation ϕ, θ, ψ with time. There are numerous integration schemes for integration of such ordinary linear differential equations, starting from simplest 1st order explicit schemes to high order implicit schemes such as Runge-Kutta schemes, Predictor-corrector schemes, etc. These (dynamic motion) equations are usually very robust and do not lead to numerical instability due to integration scheme if the time step size is sufficiently small.

By substituting the HDC values in the above equations of motion, the equations of motion can thus be integrated for known control inputs and speed, to calculate the accelerations, velocities and displacements (position) of the body as function of time. Thus, the trajectory of an underwater vessel can be simulated [5, 13].

4.2 TYPICAL RESULTS

A flexible trajectory simulation program was created, with scope for modifying the external force matrices [*B*] and [*C*], thus enabling the mathematical model to be readily changed.

Results of a 15/5 zigzag manoeuvre at 5 knots for the axisymmetric body and for SUBOFF geometry, both using the linear mathematical model (3.2(c) above) are shown in Figures 3 and 4. For the trajectory simulations shown in Figures 3 and 4, the main parameters of interest are shown in Table 1.

To examine the effect of varying the mathematical model, the same manoeuvre was simulated using different mathematical models (described in sub-section 3.2 above). Results are plotted for two of the models in Figure 5 (for axisymmetric body) and the parameter values obtained for all three mathematical models are given in Table 2.

Figure 3 Simulation of 15/5 Zigzag at 5 knots – Axisymmetric body

Figure 4 Simulation of 15/5 Zigzag at 5 knots – SUBOFF body

Parameter	Axisym-metric body	SUBOFF body
Overshoot pitch angle (θ_O); degrees	2.16	11.43
Overshoot change of depth (z_O); metres	2.81	26.34
Time to reach execute (t_e); seconds	6.12	18.96
Time to check pitch (t_c); seconds	9.16	45.48
Time to check depth (t_d); seconds	18.94	74.94

Table 1 Parameters estimated by trajectory simulation for 15/5 zigzag at 5 knots using strictly linear mathematical model

Figure 5 Effect of variation in mathematical model on trajectory simulation for axisymmetric body

Para-meters of Zigzag →	θ_O (deg)	z_O (m)	t_e (sec)	t_c (sec)	t_d (sec)
	(Difference from Model 3 values given in %)				
Model 1	2.40	2.91	6.28	9.44	19.14
	11.1%	*3.6%*	*2.6%*	*3.1%*	*1.1%*
Model 2	2.95	3.74	5.76	9.66	20.86
	36.6%	*33.1%*	*5.9%*	*5.5%*	*10.1%*
Model 3 (fully linear)	2.16	2.81	6.12	9.16	18.94

Table 2 Effect of variation in mathematical model on trajectory simulation results (15/5 zigzag at 5 kts) for axisymmetric body.

Examining the results shown in Table 2, it is seen that changing the mathematical model from strictly linear to non-linear terms causes variation in parameter values in the range of 1 to 37% for the manoeuvre and the body considered. In the sequel, the significance of this level of uncertainty will be examined in light of the other uncertainties in the manoeuvring prediction process.

5. VALUES OF HYDRODYNAMIC COEFFICIENTS

The main unknowns in the procedure of prediction of manoeuvring characteristics are the numerous HDC's. It is desirable that once the geometry of the vessel is defined in the early stages of the design, these derivatives are estimated by empirical methods based on similar vessels, or other methods, so that necessary checks can be made as to whether the design satisfies the various manoeuvring requirements.

As described in Section 3, the choice of mathematical model dictates which HDCs that need to be evaluated. Of the 36 first-order HDC's possible, many can be neglected depending on symmetry of the body [4, 16]. The model test data available in [11] includes only 21 linear HDCs. The number of second-order HDCs and various correction terms may vary from model to model. The mathematical models described in sub-sections 3.2(a) (Model 1) and 3.2(b) (Model 2) contain 55 and 24 HDCs, respectively.

The values of the hydrodynamic coefficients (HDCs) in the equations of motion have their own levels of uncertainty depending upon the model testing methods and facilities used to determine them. For the SUBOFF body, [11] the reported levels of uncertainty are listed in Table 3.

HDC	Estimated Uncertainty
Static HDCs: Z_w', M_w', Y_v', N_v'	4 – 5 %
Rotary HDCs: Z_q', M_q', Y_r', N_r'	10%
Control HDCs	6 – 10%
Added mass HDCs: $Z_{\dot{w}}'$, $M_{\dot{q}}'$, $Y_{\dot{v}}'$, $N_{\dot{r}}'$	7%

Table 3 Uncertainty estimated for various HDCs for SUBOFF body obtained by model testing [11]

The error level in one coefficient may be masked by errors in others, such that the margins of stability computed give misleading results, or such that the trajectories computed are similar for different combinations of HDC values.

5.1 SENSITIVITY STUDIES - BACKGROUND

Sensitivity studies have been carried out for HDCs of underwater vehicles in order to determine their relative importance and hence, the error bounds permissible for their values. In general, sensitivity has been defined as the ratio of the relative change in some output variable, O, and the relative change in an input parameter, I, each compared to nominal values (O_{nom}, I_{nom}). For each case the sensitivity, S, of the response to the variation in parameter can be calculated as [10]:

$$S = \frac{(O - O_{nom}) / O_{nom}}{(I - I_{nom}) / I_{nom}} \tag{26}$$

The influence of various hydrodynamic coefficients on the predicted manoeuvrability of submerged bodies has been examined [9, 10] and it is reported that for a submarine-like body, trajectories are most sensitive to linear damping coefficients.

For a bare hull axisymmetric body, the linear inertial coefficients were found to be the most significant. In another study [17], the sensitivity of geometrical characteristics of an AUV vis-à-vis its added mass HDCs have been explored. Sensitivity analysis using Genetic Algorithms [18] for various manoeuvres has revealed that difference in the model geometry caused HDCs to have different tendencies in sensitivity change. Also, the nature of sensitivity changes depending on the trials executed.

Considering the focus of this study on vertical plane manoeuvres, the most significant coefficients for axisymmetric body have been identified [9] as: $M_{\delta s}, Z_{\dot{w}}, Z_{\delta s}, M_{\dot{w}}, M_{\dot{q}}, M_q, Z_{\dot{q}}$. For submarine-like body, these are: $M_q, M_{\delta s}, Z_{\delta s}, Z_q, Z_{\dot{w}}, Z_w, Z_{\dot{q}}, M_{\dot{w}}, M_{\dot{q}}$. The difference in relative sensitivity has been attributed to the difference in the mathematical model used for the two bodies, but may also be the result of the difference in their geometries.

In order to ensure that the vertical plane trajectories do not vary by more than 10%, it is reported [9] that the HDCs that need to be determined within about 10% accuracy are $Z_{\dot{w}}, M_{\delta s}, Z_{\dot{q}}, Z_{\delta s}, M_{\dot{q}}, M_{\dot{w}}$ for axisymmetric body. Based on a study using Genetic Algorithm technique [18], M_w and M_q were found to be the two most significant HDCs.

5.2 SENSITIVITY STUDIES - AXISYMMETRIC BODY

For this study, the HDCs considered most significant for the axisymmetric body were varied, one at a time, by ±25 to 50% of their initial values and the trajectory simulation was repeated using the linear mathematical model (Model 3). Although these values are higher than the uncertainty levels estimated for model test data (Table 3), the aim was to explore the relative and cumulative effect of change in various HDCs on the trajectory simulation parameters for the zigzag manoeuvre in vertical plane. Further, as an extreme case, all the HDC values were simultaneously changed by ±50% from initial values and trajectory simulation repeated.

Results of these sensitivity studies are summarised in Table 4. The results are graphically shown in Figure 6, wherein the cumulative effect of changes in each HDC is presented for the five parameters of the vertical plane zigzag manoeuvre.

It is seen that the three most important HDCs for the case considered are $M_{\delta S}$ (change in pitching moment due to plane deflection), $Z_{\delta S}$ (change in heave force due to plane deflection) and M_q (change in pitching moment due to rate of change in pitch angle). Change in value of these HDCs by ±50% causes change in at least one of the vertical plane zigzag manoeuvre parameters by more than 50%. The marked influence of HDCs $M_{\delta S}$, $Z_{\delta S}$ and M_q are clearly evident from Figure 6.

Change in HDC by ↓		% Change in values of Zigzag Parameters				
		θ_O	z_O	t_e	t_c	t_d
$M_{\delta s}$	+50%	56.3	3.2	-23.9	-13.5	-18.0
	-25%	-48.5	-26.1	29.1	10.5	2.2
$Z_{\dot{w}}$	+50%	8.0	-4.6	-2.9	-1.5	-6.5
	-50%	-18.2	21.8	7.2	3.9	28.5
$Z_{\delta s}$	+50%	-50.6	-45.2	11.1	-6.6	-20.1
	-50%	68.7	55.5	-7.2	14.6	12.0
$Z_{\dot{q}}$	+50%	-2.9	-1.7	0.7	-0.2	-0.2
	-50%	3.0	1.8	-0.7	0.2	0.3
$M_{\dot{w}}$	+50%	-2.1	-2.2	-0.7	-1.1	-1.3
	-50%	0.5	1.4	0.3	0.7	1.0
$M_{\dot{q}}$	+50%	-6.9	15.3	21.2	19.7	18.9
	-50%	0.5	-26.5	-25.8	-26.9	-25.9
M_q	+50%	-38.3	-28.7	9.8	-3.1	-8.7
	-50%	78.2	110.8	-8.2	18.3	44.2
M_w	+50%	-26.5	-6.5	7.5	-0.7	10.6
	-50%	17.6	-5.5	-5.6	-2.4	-10.7
All	+50%	-31.9	-34.9	3.9	-7.0	-17.8
	-50%	163.6	197.5	-6.5	69.0	50.4

Table 4 Results of sensitivity studies of HDC values for vertical plane zigzag - axisymmetric body

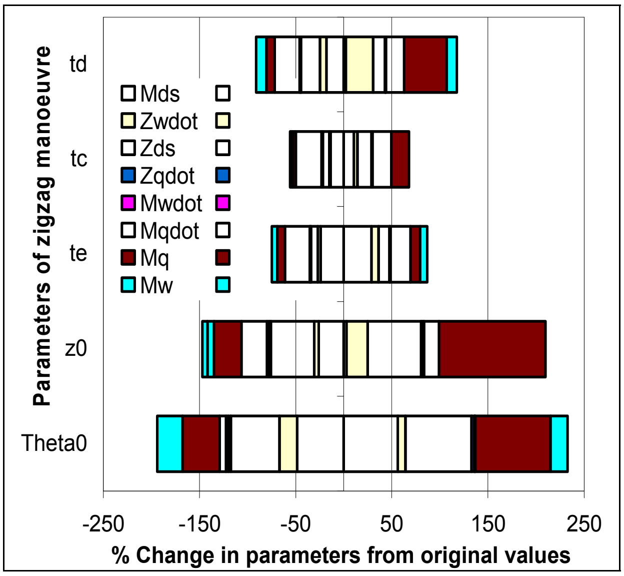

Figure 6 Graphical representation of results of sensitivity analysis showing percentage change in zigzag parameters due to ±50% change in HDCs

Figure 6 also shows that the total change in overshoot angle (θ_O) could be -200% to +225%, due to the sum of the effects of variation in each HDC by ±50%. However, as seen in Table 4, for simultaneous change of all HDC values by ±50%, simulated overshoot angle (θ_O) values change by -32% to +164%. For overshoot change of depth (z_O), around 200% increase is expected in case of simultaneous reduction in all listed HDC values by 50%, as well as in the case of the sum of changes in individual HDCs.

It is also seen that the parameters time to reach execute (t_e) and time to check pitch (t_c) and depth (t_d) are relatively less influenced (about ±100%) by change in HDC values, as compared to the parameters θ_O and z_O. The effect of change in trajectory of the body due to simultaneous change of ±50% in all listed HDCs is shown in Figure 7. The original response curves are in the middle, while quicker responses are for the case when HDCs were increased by 50%. Responses were delayed when all HDC values were reduced by 50%.

Figure 7 Change in trajectory simulation results for axisymmetric body due to change in HDCs by ±50%

5.3 SENSITIVITY STUDIES - SUBOFF BODY

These studies were repeated for the submarine-like SUBOFF body, varying all HDCs varied for axisymmetric body, as well as Z_q, again using the linear mathematical model (Model 3). Variation of HDC values by ±50% from model test values was considered. Results of these sensitivity studies are summarised in Table 5. Only those HDCs which had an effect of more than 2% on the zigzag parameters have been included in the Table.

The most significant HDCs for the SUBOFF body simulation emerge as $Z_{\dot{w}}, Z_{\delta s}, M_w$. It is thus seen that even for the same mathematical model, the body geometry has a significant influence on the relative importance of the HDCs.

The change in zigzag manoeuvre parameters due to HDC value variations does not exceed 21% when HDCs are varied one at a time. This is much lesser than the effect of variations in case of axisymmetric body. Even when all relevant HDCs are varied simultaneously by ±50% from original values, the variation in zigzag parameters is at most by 55% only. Although the magnitude of variation in actual terms is much greater, the percentage change is relatively less.

Change in HDC by ↓		% Change in Zigzag Parameters				
		θ_O	z_O	t_e	t_c	t_d
$M_{\delta s}$	+50%	-3.6	-7.7	-5.6	-5.6	-5.3
	-50%	4.4	9.4	6.6	6.5	6.1
$Z_{\dot{w}}$	+50%	15.7	19.1	5.3	11.5	11.4
	-50%	-17.4	-19.4	-6.4	-12.0	-11.4
$Z_{\delta s}$	+50%	17.3	0.3	-11.4	-6.6	-6.6
	-50%	-20.8	1.3	20.5	13.1	13.8
M_w	+50%	13.6	-2.4	-11.9	-9.5	-10.3
	-50%	-18.4	3.9	21.6	16.6	18.5
Z_q	+50%	-2.33	-2.58	0.11	-1.01	-0.93
	-50%	2.43	2.72	-0.11	1.01	0.96
All	+50%	54.9	16.0	-20.0	-5.9	-6.0
	-50%	-37.9	11.0	57.1	39.0	41.7

Table 5 Results of sensitivity studies of HDC values for vertical plane zigzag - SUBOFF body

The change in trajectory for simultaneous variation in HDCs by ±50% for SUBOFF body is shown in Figure 8. As in Figure 7, the quickest responses correspond to 50% greater HDC values.

Figure 8 Change in trajectory simulation results for SUBOFF body due to change in HDCs by ±50%

5.4 SENSITIVITY STUDIES FOR ±10% CHANGE IN HDC VALUES

Having gained some insight into the relative importance of HDC values for both the bodies, we now consider the effect of variation in the most significant HDCs by ±10%. This is the maximum uncertainty estimated in the model testing process (Table 3), and we explore the effect of this magnitude of variation on the zigzag parameters. Results obtained by trajectory simulation using fully linear mathematical model for the axisymmetric body are given in Table 6, and for the SUBOFF body in Table 7.

Change in HDC by ↓		% Change in values of Zigzag Parameters				
		θ_O	z_O	t_e	t_c	t_d
$M_{\delta s}$	+10%	13.9	2.8	-6.9	-3.3	-4.0
	-10%	-18.0	-6.9	8.5	3.1	2.7
$Z_{\dot{w}}$	+10%	1.0	-1.9	-1.0	-0.7	-2.1
	-10%	-3.7	1.0	0.7	0.2	2.1
$Z_{\delta s}$	+10%	-12.0	-11.0	1.6	-2.2	-4.1
	-10%	12.1	11.2	-1.6	2.4	3.6
$Z_{\dot{q}}$	+10%	-1.2	-0.7	0.0	-0.2	-0.2
	-10%	-0.4	-0.2	-0.3	-0.2	-0.1
$M_{\dot{w}}$	+10%	-1.4	-1.0	-0.3	-0.4	-0.4
	-10%	-0.2	0.1	0.0	0.0	0.1
$M_{\dot{q}}$	+10%	-2.4	3.0	4.2	4.1	3.9
	-10%	-0.2	-4.7	-4.9	-4.8	-4.6
M_q	+10%	-10.3	-8.3	1.6	-1.3	-2.9
	-10%	9.7	9.1	-2.0	1.3	3.2
M_w	+10%	-4.6	0.4	1.3	0.0	2.6
	-10%	3.8	-1.0	-1.3	-0.4	-2.5
All	+10%	-8.8	-11.0	1.0	-2.4	-5.3
	-10%	10.1	14.3	-1.3	3.1	6.3

Table 6 Results of sensitivity studies, with small change in HDC values, for vertical plane zigzag - axisymmetric body

Change in HDC by ↓		% Change in Zigzag Parameters				
		θ_O	z_O	t_e	t_c	t_d
$M_{\delta s}$	+10%	-0.68	-1.56	-1.16	-1.19	-1.07
	-10%	0.90	1.82	1.27	1.23	1.17
$Z_{\dot{w}}$	+10%	3.32	3.92	1.16	2.33	2.30
	-10%	-3.25	-3.80	-1.16	-2.37	-2.27
$Z_{\delta s}$	+10%	3.63	-0.05	-2.74	-1.72	-1.71
	-10%	-3.77	0.09	3.06	1.89	1.95
M_w	+10%	3.08	-0.48	-2.85	-2.24	-2.46
	-10%	-3.33	0.45	3.16	2.46	2.72
Z_q	+10%	-0.54	-0.59	0.00	-0.22	-0.21
	-10%	0.54	0.60	0.00	0.22	0.21
All	+10%	8.88	1.21	-5.38	-2.81	-2.96
	-10%	-8.34	-0.35	6.54	3.78	4.08

Table 7 Results of sensitivity studies, with small change in HDC values, for vertical plane zigzag - SUBOFF body

From Table 6 and 7, it emerges that the maximum change in trajectory parameters due to change in HDC values by ±10% is around 18% for the axisymmetric body and 9% for the SUBOFF body. Thus, considering an uncertainty of up to ±10% in HDC values estimated from model testing, the main parameters of the zigzag considered can be determined with a fair degree of confidence.

6. CUMULATIVE UNCERTAINTY IN MANOEUVRABILITY PREDICTION

The likely ranges of uncertainty due to choice of type of mathematical model (Section 4), and values of HDCs (Section 5) have been examined for two bodies. The effect of these variations on the parameters of the vertical plane zigzag manoeuvre has been explored by trajectory simulation. For the cases considered, we now attempt to assess the cumulative uncertainty in the parameters of the definitive manoeuvres due to the stages of the manoeuvring prediction process.

We consider the maximum variation in zigzag parameters due to choice of mathematical model (in trajectory simulation program) for the axisymmetric body, as listed in Table 2 as one source of uncertainty. The other source of uncertainty is due to variation in HDC values by ±10%, typical of model test values, as listed in Table 6 for axisymmetric body (Listed as case (a) in Table 8, with values taken from Table 6). Effect of variation of HDCs by ±50% is also listed (as case (b), with values taken from Table 4).

Summing these, we obtain the cumulative uncertainty in prediction for the axisymmetric body in Table 8 and for the SUBOFF body in Table 9. The range of total uncertainty may thus be estimated as 11% to 55% for ±10% variation of HDCs of axisymmetric body, for the manoeuvre considered. These values would be up to 45% for the SUBOFF body (using earlier results of Table 7). In case of ±50% variation of HDCs, the total uncertainty values would increase to up to 231% for axisymmetric body (and roughly up to 92% for SUBOFF body).

Parameters of Zigzag → / Source of Uncertainty ↓		θ_O	z_O	t_e	t_c	t_d
Mathematical model (Trajectory simulation)		37%	33%	6%	6%	10%
HDC value	(a) ±10 %	18%	14%	9%	5%	6%
	(b) ±50 %	164%	198%	29%	69%	50%
Total	(a)	55%	47%	15%	11%	16%
	(b)	201%	231%	35%	75%	60%

Table 8 Estimation of uncertainty in manoeuvring prediction process for 15/5 vertical plane zigzag at 5 knots – Axisymmetric body

Parameters of Zigzag → / Source of Uncertainty↓		θ_O	z_O	t_e	t_c	t_d
Mathematical model (Trajectory simulation)		6%	5%	1%	5%	7%
HDC value	(a) ±10%	9%	4%	7%	4%	4%
	(b) ±50%	55%	19%	57%	39%	42%
Total	(a)	15%	9%	8%	9%	11%
	(b)	61%	24%	58%	44%	49%

Table 9 Estimation of uncertainty in manoeuvring prediction process for 15/5 vertical plane zigzag at 5 knots – SUBOFF body

7. UNCERTAINTY IN FULL-SCALE TRIALS

For each stage of the trajectory simulation process described above (formulation of mathematical model, program for trajectory simulation and estimating values of HDCs), the ultimate check for the manoeuvrability prediction procedure is to compare the predicted trajectory with the measured values during full-scale trials. However, such comparisons are not only rare [2, 7, 19], but are also fraught with the uncertainties due to noise in the data obtained during full-scale trials, since laboratory conditions (say, of model testing for obtaining HDCs) cannot be replicated.

It is therefore necessary to quantify typical levels of uncertainty in the full-scale measurement process in order to arrive at realistic estimates of the accuracy required of the entire process of manoeuvrability prediction. This is applicable for large manned submarines as well as small UUVs.

7.1 SOURCES OF ERRORS

As per uncertainty analysis described in [11], the errors in measurement can be of two types: bias errors (which are constant throughout an experiment, affecting all measurements in the same sense) and precision errors (which are random scatter in results during an experiment).

Focusing our attention on zigzag manoeuvres in the vertical plane for underwater vehicles, we may identify the possible sources of bias errors in full-scale trials as:

- Errors in measuring instruments for depth, pitch angle and speed of the vehicle
- Set in inertial navigation or other motion / position-recording instruments
- Hydrostatic imbalance of the vehicle

- Changes in temperature of water with depth (affecting density and viscosity)
- Ocean currents at different depths

Sources of precision errors may be listed as follows:
- Errors in recording and processing data, depending on frequency of measurements
- Unknown and random sea water disturbances
- Errors due to variation in time for applying control forces

7.2 UNCERTAINTY ESTIMATES

Most of the above errors are impossible to quantify exactly, but the expected variations can be estimated, albeit somewhat crudely.

7.2(a) Instrumentation Bias

During full-scale sea trials on manned underwater vehicles reported in [19], the instrument resolution (least count) for on-board instrumentation of various parameters was as follows.

- Depth: 1 metre
- Pitch angle: 1 degree
- Time: 0.5 seconds
- Plane angle: 1 degree
- Speed: 0.5 knots

Based on the above values, the uncertainty in each of the parameters of interest in the zigzag manoeuvre may be quantified for typical assumed values, as shown in Table 10. The values used are those for the axisymmetric body parameters (Table 1) for the 15/5 zigzag.

Parameter	Typical Value	Instrument Uncertainty
Overshoot pitch angle (θ_O)	2 degrees	0.500
Overshoot change of depth (z_O)	3 metres	0.333
Time to reach execute (t_e)	6 seconds	0.083
Time to check pitch (t_c)	9 seconds	0.111
Time to check depth (t_d)	19 seconds	0.026

Table 10 Typical bias error estimated based on measuring instrument resolution (least count)

For UUVs, the values would change according to the resolution of the instruments installed, but may be estimated using the same approach.

7.2(b) Sea Water Density Variation

The variation in density of sea water of tropical waters may be from 1.015 to 1.030 tons/m^3. Change of vehicle depth in a region with sharp temperature gradient may cause imbalance between weight and buoyancy, which may be as much as 0.25% of the vehicle displacement. The effect of this magnitude of change during the course of a typical zigzag manoeuvre was found to be marginal. The uncertainty due to this source of error may be roughly estimated (on the higher side) as 0.003 for θ_O and 0.005 for z_O.

7.2(c) Errors in Data Recording

The value of precision error due to recording and processing data may be estimated in model testing by repeating a particular observation at different points of time. However, for sea trials, it is near-impossible to repeat the experiment in exactly the same fashion with the same environmental conditions to gauge the errors in the instrumentation or recording process. However, the values of error in data recording may be estimated to be of the order of the least count of the instruments for depth, pitch angle, time and speed. This suggests additional uncertainty values similar to those listed in Table 10.

Currents have not been mentioned explicitly, but their effects may be included in the environmental uncertainties affecting the data recording process. Currents may not affect the recorded motion parameters if the UUV measures speed through water, but could significantly bias the results if instead, its speed over the ground is measured using a Doppler velocity log.

7.2(d) Error in Control Force Application

Precision errors due to control force application result from inexact or asynchronous application of plane angles. Particularly in case of manned submarines, the time required to apply plane angle when ordered may vary from operator to operator and may not exactly coincide with the moment the execute pitch angle is reached. Typical variations are of the order of 2 seconds, leading to uncertainties in all time parameters of the zigzag manoeuvre by the same amount, and a variation in the overshoot angle by as much as 1-2 degrees, and variation in overshoot change of depth by 2-5 metres. These values are typical for low speeds (3 to 5 knots) and may be somewhat higher for greater speeds. The consequent uncertainties in the parameters of interest, for sample values listed in Table 10, may be (lower values): 0.50 for θ_O , 0.67 for z_O, 0.33 for t_e, 0.22 for t_c, and 0.11 for t_d.

In case of UUVs, this error due to human action would not be present. However, the uncertainty in time lag for actuation of control surfaces based on a logical input may need to be estimated for unmanned vehicles.

7.3 CUMULATIVE UNCERTAINTY IN FULL-SCALE MEASUREMENTS

In the worst case scenario, all the errors quantified above can occur simultaneously and hence the uncertainties estimated due to various sources of error would add up. Since all possible sources of error have not been quantified, this estimate is not necessarily conservative (i.e., the actual error may be even greater). Since all parameters of interest in the zigzag manoeuvre are measured directly in the full-scale trials, simple addition of the uncertainties due to the sources of error would suffice. Thus, summing up the estimated uncertainty values for various sources of error described above, the results are given in Table 11.

Parameter	Assumed Value	Total Uncertainty
Overshoot pitch angle (θ_O)	2 degrees	1.503
Overshoot change of depth (z_O)	3 metres	1.338
Time to reach execute (t_e)	6 seconds	0.500
Time to check pitch (t_c)	9 seconds	0.444
Time to check depth (t_d)	19 seconds	0.158

Table 11 Typical uncertainty levels estimated based on various sources of bias and precision errors for one set of parameter values

In case of an UUV, if the uncertainty due to error in control force application is ignored, the total uncertainty for the case considered would reduce to 1.00 for θ_O , 0.67 for z_O, 0.17 for t_e, 0.22 for t_c, and 0.05 for t_d.

It may be noted that these estimates are only meaningful for values of parameters close to those assumed as 'typical'. For different speeds, and certainly for different vehicles, these values would vary and accordingly the uncertainty levels would also change. In general, it is likely that for more unstable or larger bodies and / or greater speeds (or plane deflections), the values of parameters measured would be greater. In such a scenario, the associated uncertainties are likely to be lower.

To illustrate this aspect, the estimated total uncertainty for another set of parameters (for a case of higher speed or a more unstable vehicle or a greater control surface deflection) is shown in Table 12. The assumed values are similar to those obtained by simulation for the SUBOFF body in 15/5 zigzag (Table 1), which is a more unstable body than the axisymmetric body considered earlier.

In the uncertainty analysis described here, although some aspects pertain to manned submarines, the likely changes in case of UUVs have been mentioned. The typical manoeuvre parameter values quoted here also pertain to simulation of UUV-sized bodies. Hence the range of uncertainty reported is fairly representative of the entire gamut of underwater vehicles.

Parameter	Assumed Value	Total Uncertainty
Overshoot pitch angle (θ_O)	11 degrees	0.276
Overshoot change of depth (z_O)	26 metres	0.159
Time to reach execute (t_e)	19 seconds	0.158
Time to check pitch (t_c)	45 seconds	0.089
Time to check depth (t_d)	75 seconds	0.040

Table 12 Typical uncertainty levels estimated for another set of parameter values (for higher speeds / more unstable body/ greater plane deflection)

8. CONCLUSIONS

The process of manoeuvrability prediction of underwater vehicles is fraught with uncertainties at every stage. It is important to have an overall perspective of the subject in order to ensure that estimates at every stage are reasonably accurate, while also optimising the time and effort required in the process.

This study has considered two body geometries. A generalised trajectory simulation program was developed, using which simulations have been carried out using three mathematical models. Results have been presented for a typical 15/5 vertical plane zigzag manoeuvre at 5 knots. Conclusions have been drawn regarding the effect of various choices on the values of main parameters obtained from simulating this manoeuvre.

It was seen that choice of mathematical model (linear / non-linear) can influence the zigzag trajectory parameters by 1% to 37%, with greater effect on the pitch overshoot and depth overshoot, which are measures of the depth-keeping (rather than depth-changing) ability of the underwater vehicle.

Estimation of HDC values is usually the most time-consuming aspect of manoeuvrability prediction. Based on sensitivity analysis of the effect of HDC values on vertical plane zigzag parameters, it was seen that for the axisymmetric body considered, $M_{\delta S}$, $Z_{\delta S}$ and M_q are most significant. Variation in any one of these HDCs by ±50% causes more than 50% change in at least one of the zigzag parameters. However, for the SUBOFF body (which is more akin to a submarine form), the most significant HDCs emerge as $Z_{\dot{w}}, Z_{\delta S}, M_w$, for which individual ±50% change does not cause change of zigzag parameters by more than 21%. If all significant HDCs are varied by ±50% simultaneously, the zigzag parameters change by up to 55% for SUBOFF and up to almost 200% for the axisymmetric body. Thus, the sensitivity of HDCs depends not only on the mathematical model adopted, but also on the manoeuvre considered and the body geometry being studied.

Considering the experimental uncertainty (from model testing) as up to 10%, it emerges that the resultant uncertainty in zigzag parameters is approximately 18% for the axisymmetric body and 9% for the SUBOFF body. Summing up the possible uncertainties in the various stages of manoeuvrability prediction discussed, a figure of uncertainty of 11% to 55% may be suggested for the parameters of the manoeuvre considered in this study, for the axisymmetric body. For the SUBOFF body, this value would be approximately up to 45%.

However, if a variation of ±50% in the HDC values is assumed (which may be the case if prediction methods other than model tests, such as empirical or numerical methods, are used for HDC estimation), then the consequent range of uncertainty increases to approximately 231% for axisymmetric body and 92% for the SUBOFF body.

Uncertainty estimation for various sources of error has been discussed for full-scale sea trials, which are the ultimate benchmark for any prediction process. It is seen that the uncertainty levels in the measurement process depend on many unknown factors, and also on the values of manoeuvre parameters. Based on two typical cases, it is estimated that error magnitude of various parameters during full-scale measurements may vary from 5% to 150% for manned submarines and up to 100% in case of UUVs.

This range of uncertainty values in the measurement process appears greater than the cumulative uncertainty (11-55%) in the overall prediction process, considering 10% uncertainty in HDC values. However, in case the uncertainty in HDC values in higher (say ±50%), then the uncertainty levels of predicted zigzag parameters would be substantially greater than the uncertainty in full-scale measurements.

The quantitative results presented are for very specific cases of manoeuvre type, mathematical model, body geometry (typical of UUVs and submarines) and vehicle sizes (typical of UUVs). In addition, the following generic qualitative conclusions may be drawn:-

- The mathematical model used for trajectory simulation may contribute to a similar or even greater degree of uncertainty in results, compared to uncertainty due to variation in HDC values.
- The sensitivity (relative importance) of HDCs depends on the mathematical model, the manoeuvre considered, as well as the body geometry.
- Full-scale trials of the body performing open-loop definitive manoeuvres are the ultimate test of any manoeuvrability prediction process, but these trials have their own levels of uncertainty, affected by the manoeuvre performed.
- In real-life operations, uncertainties in the prediction process are often masked by the close-loop control system action of the operator or autopilot, unless the vehicle is highly unstable.

It needs to be emphasised that predictions need to be only as accurate as can be measured and verified. The quantification of uncertainties at various stages can offer insight into margins for improvement in each of the stages of manoeuvrability prediction during design. Meaningful targets can thus be set for accuracy of each aspect of the manoeuvrability design process.

Further studies may expand the approach adopted here to other definitive manoeuvres, other bodies and other mathematical models.

9. ACKNOWLEDGEMENT

This study was undertaken as part of a project supported by the Naval Research Board (NRB) of the Defence Research & Development Organisation (DRDO), India, which is gratefully acknowledged by the authors.

10. REFERENCES

1. RAY, A., SESHADRI, V., SINGH, S.N., SEN, D., 'Manoeuvring Studies of Underwater Vehicles – A Review', *Transactions of RINA*, 2008.
2. ARENTZEN, A. and MANDEL, P., 'Naval Architectural Aspects of Submarine Design', *SNAME Transactions*, 1960.
3. SPENCER, J.B., 'Stability and Control of Submarines', *Journal of the Royal Navy Scientific Service (JRNSS)*, Vol. 23, No. 3, 1967.
4. ABKOWITZ, M.A., *Stability and Motion Control of Ocean Vehicles*, MIT Press, 1969.
5. FOSSEN, T.I., *Guidance and Control of Ocean Vehicles*, John Wiley, 1994.
6. LEWANDOWSKI, E.M., *The Dynamics of Marine Craft*, World Scientific, 2004.
7. PRESTERO, T., 'Verification of a Six-Degree of Freedom Simulation Model for the REMUS Autonomous Underwater Vehicle', *M.S. Thesis, Massachusetts Institute Of Technology & Woods Hole Oceanographic Institution*, 2001.
8. NAHON, M., 'A Simplified Dynamics Model for Autonomous Underwater Vehicles', *The Journal of Ocean Technology*, Vol. 1, No. 1, 2006.
9. REDDY, D.N., 'Theoretical and Experimental Studies on Motion and Control of Submerged Bodies', *M.S. Thesis, Indian Institute of Technology Kharagpur*, 1996.

10. SEN, D., 'A Study on Sensitivity of Maneuverability Performance on the Hydrodynamic Coefficients for Submerged Bodies', *Journal of Ship Research*, Vol. 44 (3), pp. 186–196, Sep 2000.
11. RODDY, R. F. 'Investigation of the stability and control characteristics of several configurations of the DARPA SUBOFF model (DTRC Model 5470) from captive-model experiments', *David Taylor Research Center Report No.DTRC/SHD-1298-08*, September 1990.
12. LEWIS, E.V. (ed.), 'Principles of Naval Architecture, Vol. III', *SNAME*, 1989.
13. SEN, D., 'A Generalized Hydrodynamic Model for Underwater Bodies', *2nd Indian National Conference on Harbour & Ocean Engineering (INCHOE-97)*, Dec 1997.
14. GERTLER, M. and HAGEN, G.R., 'Standard Equations of Motion for Submarine Simulation', *Report 2510, Naval Ship Research and Development Center*, 1967
15. FELDMAN, J.P., 'DTNSRDC Revised Standard Submarine Equations of Motion'; *DTNSRDC Report SPD-0393-09*, June 1979.
16. PETERSON, R.S., 'Evaluation of Semi-Empirical Methods for Predicting Linear Static and Rotary Hydrodynamic Coefficients', *Naval Coastal Systems Center, NCSC TM-291-80*, June 1980.
17. PERRAULT, D., BOSE, N., O'YOUNG, S., WILLIAMS, C.D., 'Sensitivity of AUV added mass coefficients to variations in hull and control plane geometry', *Ocean Engineering*, Vol. 30, 2003.
18. YEOA, D.J., RHEE, K.P., 'Sensitivity analysis of submersibles' manoeuvrability and its application to the design of actuator inputs', *Ocean Engineering*, Vol. 33, 2006.
19. RAY, A., 'Manoeuvring Trials of Underwater Vehicles', *Journal of Ship Technology*, Vol.3, No. 2, July 2007.

