APPLICATION OF VARIOUS SPECTRAL FATIGUE ANALYSIS METHODS FOR SHIP STRUCTURES

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SUMMARY

The Spectral Fatigue Analysis (SFA) is a comprehensive fatigue life assessment method. The SFA is performed by following a systematic process at the onset of hydrodynamic analysis, structural analysis, and spectral analysis. Hydrodynamic analysis and finite element based structural analysis are numerically intense stages, and require a substantial amount of computational time and resources. In the present paper, some simplifications are imposed on individual stages to perform the SFA analysis in a practical time scale but not compromising on the underlying theoretical assumptions. Three distinct methods (semi-analytical formulation, 2D strip method, and 3D panel method) have been used to compute the wave-induced loads while the structural responses are obtained using the beam theory based formulations (in case of semi-analytical and 2D strip method) and finite element analysis (in case of 3D panel method). Fatigue damages are calculated using these methods at the selected locations of a bulk carrier and results are compared with each other. It has been shown that the first two methods (semi-analytical and 2D strip based methods) are quick and efficient and can be used in initial design assessment or identifying the fatigue prone locations. The third method is realistic and accurate and can be used in case of a comprehensive assessment of the design.

1. INTRODUCTION

In the last two decades, fatigue assessment gained significant interest from the shipping industry. Many classification societies published guidelines to carry out the detailed fatigue analysis for both, ships and offshore structures. Side shell cracks were observed and found responsible for many reported bulk carriers accidents (IMO, 1995) which lead to a serious concern from the shipping fraternity to accept prevailing fatigue assessment criteria. Not only bulk carriers but many other ship accidents were also recorded with the cracks in the hull structure within the short period of commencement into operation in which fatigue was considered to be an important contributor for these structural damages. It has been recognised that even though fatigue damage does not result in complete structural failure but the estimated cost of repair and likelihood of marine pollution are relatively high. Apart from past experiences with ship structural failures and damages, there were other reasons which contributed to the inclusion of fatigue assessment as new criteria to be considered in the initial design stage. Some of these reasons are as follows:

- Optimise hull structure to improve the strength-to-weight ratio by introducing new materials such as Aluminium and high tensile steel.
- Rise in number of ageing ships with lack of maintenance.
- Growing concern towards the safety of ship, human and environment.

Fatigue failure of material occurs due to cyclic loads. The fatigue damage is estimated by calculating the cyclic loads encountered by the material to its fatigue capacity. The fatigue capacity of a material is represented by the S-N curves. S-N curve shows the relationship between the stress ranges and the number of constant amplitude load cycles to failure. The cyclic loads can be calculated using the following methods:

- Simplified fatigue method,
- Deterministic fatigue method
- Spectral fatigue method.

In the simplified fatigue method, the dominant loads are calculated by empirical formulas provided in the rules of ship classification society. The long-term distribution of stress range is characterized by Weibull distribution. In spite of being a simple practical method, this method does not account for specific ship details and specific environmental/operating conditions. In case of deterministic method, a sea state is simply characterized by a deterministic wave height and wave period. But deterministic method does not consider the spectral energy corresponding to sea state and this method is limited to special marine structures and specific operating conditions (ABS, 2018).

The limitations of simplified and deterministic fatigue methods are addressed by the spectral-based approach. In this method, the hydrodynamic loads are computed using an advanced seakeeping program. Various techniques of computing ship motions and loads can be found in (Hirdaris *et al.*, 2014). The hydro-structural interaction (one way) becomes complex in this method as the number of panels are different in hydro and Finite Element (FE) model. In this regard, Ma, *et al.*, (2012) and Li *et al.*, (2014) have discussed the important aspects of pressure mapping on FE model. Stress Transfer Functions (STFs) are computed by performing structural analysis. Thereafter, stress range distributions are calculated using the spectral method as shown in Figure 1. Fatigue damage is calculated by taking the ratio of the stress range distribution to fatigue capacity of material. Cumulative fatigue damage is calculated using the Palmgren-Miner rule. In spectral analysis, the short-term stress range distribution is defined by Rayleigh distribution on the assumption of the sea state being a random, narrow banded, and stationary process.

This paper shows the application of various loads (three methods) and structural responses (two methods) determination methods to identify the most appropriate and robust method for the assessment of fatigue life of ship structures. More details are given in subsequent sections. The fatigue life calculation is performed for the butt welded plate joints located at deck and side shell of mid-ship section of a 170000 DWT bulk carrier.

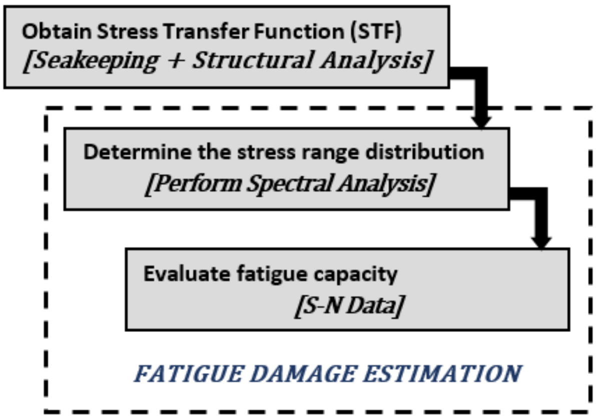

Figure 1: Schematic diagram for spectral fatigue assessment

In this paper, the ship has been assumed as a rigid body. The effect of high frequency loads such as whipping and springing are neglected. The influence of high frequency loads in the design of large ship structures can be referred in (Hirdaris *et al.*, (2010), (2016)).

2. APPROACHES

Spectral fatigue analysis primarily consists of four different stages: Computation of hydrodynamic loads, structural analysis, determination of stress distribution, and fatigue damage calculation as shown in Figure 1. The STF can be obtained using the various methods as shown in Table 1. Based on the combination of hydrodynamic load and structural analysis method, fatigue assessment methods (FAM) can be categorized as follows:

1. FAM-1: Vertical Bending moment Response Amplitude Operator (RAOs) are obtained using Semi-analytical formulation and STFs are calculated by using the Euler-Bernoulli beam theory based formulations.
2. FAM-2: Global loads (Vertical and Horizontal Bending moment RAOs) are computed using a 2D strip theory based program. Euler-Bernoulli beam theory based formulations are used to determine the STFs.
3. FAM-3: 3D panel method based seakeeping program is used to compute wave-induced pressure and ship motions. The effect of global and local loads are incorporated in FE analysis. STFs are calculated at the selected structural locations.

Finite element analysis based FAM-3 is considered the most comprehensive method to compute the structural responses i.e. (STFs). The wave loads are computed using a 3D seakeeping program. The hydrodynamic hull pressure and motions are applied to the FE model. This method needs substantial computation resources and time.

Table 1. Summary of the fatigue assessment methods

Method ID	**Load computation Method**	**Structural analysis method**	**Hydrodynamic load effect**
FAM-1	Semi-analytical formulation	Beam theory	Vertical Bending Moment
FAM-2	2D Strip theory	Beam theory	Vertical and Horizontal Bending moment
FAM-3	3D Panel method	Finite element analysis	Dynamic Pressure and ship motions

Wave load can be computed using the Semi-analytical formulation (Jensen and Mansour, 2002) or using a 2D strip theory based program. These load computation methods are used in the FAM-1 and FAM-2 respectively while the structural responses are determined by the beam theory based formulations as shown in Table 1. The above three methods are used to obtain the STFs. The individual fatigue analysis methods as listed in Table 1 are referred by the method IDs subsequently.

3. STRESS TRANSFER FUNCTION (STF)

Stress Transfer Functions (STF) represent the stress response of specified wave frequency and heading for unit wave amplitude at a structural location. Once STFs are known, spectral analysis is performed for each bin (set of significant wave height and period) of the scatter diagram. To compute the STFs, the detailed procedures of hydrodynamic analyses and structural response analyses are given in appended sections.

3.1 HYDRODYNAMIC LOAD

During the voyage, vessel encounters ocean waves from all directions which induce oscillatory loads. Details of adopted methods to obtain the load RAOs are described below.

3.1 (a) Vertical Bending Moment (VBM) calculation using Semi-Analytical Close-from Formulation

In this method, semi-analytical close-form formulations are used to calculate frequency response functions for the

wave-induced VBM. The formulations are valid for mono-hull ships (Jensen and Mansour, 2002).
Inputs required by the closed-form expression are restricted to the main dimensions i.e. length, breadth, draught, block coefficient and waterplane area, vessel's speed, and wave headings. Then Eqn. (1) makes it simple to obtain the wave-induced VBM transfer functions as an alternative to numerical computation.

$$\frac{\Phi_M}{\rho g B L^2} = \kappa \frac{1-kT}{(k_e L)^2}\left[1-\cos\left(\frac{k_e L}{2}\right)-\frac{k_e L}{4}\sin\left(\frac{k_e L}{2}\right)\right]F_V(F_n)F_C(C_b) \quad (1)$$

$$\kappa = \exp(-k_e T) \qquad k_e = |k\cos\theta|$$

where, V is the forward speed, θ is the heading angle (180° corresponding to head sea), B and T are the breadth and draught, k is the wavenumber, ω is the wave frequency ($\omega^2 = kg$).
$F_C(C_b)$, $F_V(F_n)$ are the correction factors for the block coefficient ($C_b \geq 0.6$) and speed ($F_n < 0.3$) respectively.

3.1 (b) VBM and Horizontal Bending Moment (HBM) calculation using Strip Theory

In this method, the wave loads are computed by integrating the two-dimensional loads on the cross-sections of an un-restrained ship over the ship length (Salvesen et al, 1970). The dynamic loads (VBM and HBM) at any section are the difference between the inertia force I_j, and the sum of the external forces acting on the portion of the hull. If the external forces are separated as static restoring force/moment R_j, the exciting force/moment E_j, and hydrodynamic force/moment due to body motion D_j, then load Eqn. can be written as below:

$$V_j = I_j - R_j - E_j - D_j \qquad (2)$$

Where, j = load index (j = 5 for VBM and 6 for HBM)

Here, the inertia force is expressed in terms of the sectional inertia force. Hydrostatic moments are linear and computed by considering the actual variation of the individual sectional draft and thus accounting for the vessel motions. Since there is no resorting in a horizontal plane, therefore $R_6 = 0$. For excitation forces, incident Froude-Krylov and diffraction components are evaluated. The hydrodynamic moments are caused due to body motion. So, the D_j term in Eqn. (2) consists of sectional added mass and damping. All the terms of the dynamic load equation suggest that the solution of the motion equation is a prerequisite. While computing dynamic loads, a critical test for consistent treatment of forces and moments is to be conducted i.e. by ensuring both dynamic VBM and HBM must be equal to zero at the aft and forward of the ship. This condition needs to be satisfied through careful consideration of several details such as hydrostatic balancing of forces and moments.

3.1 (c) Pressure and Motions computation using 3D PANEL Method

In this method, the hydrodynamic loads are computed using commercial software (ANYS AQWA). This is a zero-speed Green's function based program. The numerical solution of zero speed boundary value problem is corrected for low to moderate forward speed in regular waves for different wave headings to determine the wave-induced motions and loads on the slender conventional ships.

A representative figure of a hydrodynamic model is shown in Figure 2. It displays the 3D panels below mean water line for a bulk carrier in homogeneous loading condition. A total of 5020 panels are used in the hydrodynamic model. Similarly, the hydrodynamic models are prepared for other loading conditions.

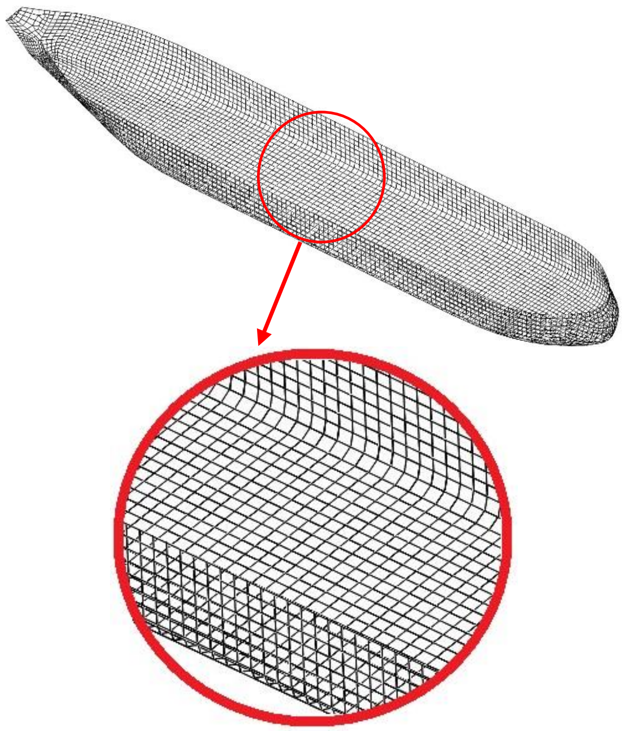

Figure 2: Panel distribution ship geometry of bulk carrier in homogeneous loading condition

3.2 STRUCTURAL RESPONSES (SR)

Structural response functions have been evaluated by two methods which are explained in sections 3.2(a) and 3.2(b).

3.2 (a) SR using Beam Theory

The ship is assumed as a rigid beam, therefore, Euler-Bernoulli beam theory formulations can be used to compute the stress transfer functions. Beam theory formulations are applied in the case of FAM-1 and FAM-2. The stress RAOs are calculated as follows:

$$RAO_{\sigma,h} = \frac{y}{I_{ZZ}} RAO_{M,H} \qquad (3)$$

$$RAO_{\sigma,v} = \frac{(z-z_0)}{I_{yy}} RAO_{M,V} \qquad (4)$$

where, $RAO_{M,V}$ and $RAO_{M,H}$ are the VBM and HBM RAOs respectively, z is the vertical distance of the structural location from the baseline. y is the horizontal distance of the structural location from the centerline. z_0 is the distance of the neutral axis from the baseline. I_{yy}, I_{zz} are the second moment of area about y and z-axis respectively at mid-ship section. The combined transfer function is derived as:

$$RAO_\sigma = \begin{bmatrix} (RAO_{\sigma,v})^2 + (RAO_{\sigma,h})^2 + \\ 2.RAO_{\sigma,v}.RAO_{\sigma,h}.\cos(\varepsilon_v - \varepsilon_h) \end{bmatrix}^{1/2} \quad (5)$$

where, ε_v and ε_h denote the phase of the stress process due to vertical and horizontal hull girder bending respectively. STF can be obtained using Eqn. (6)

$$H_\sigma(\omega|\theta) = RAO_\sigma \quad (6)$$

Semi-analytical close-form load calculation method (section 3.1(a)) provides the VBM RAOs. Therefore, STFs are calculated using Eqn. (4), (5), and (6) contain the effect of VBM alone.

3.2 (b) SR using FE Analysis:

The computed loads and motions discussed in section 3.1(c) are transferred to the FE model. Following steps are followed for FE analysis, for further details refer (Parihar *et al*., 2019).

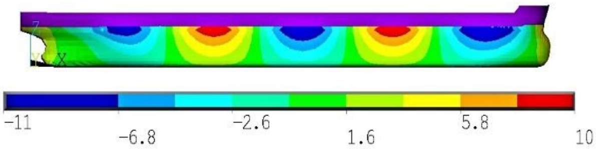

Figure 3: Distribution of real component of complex pressure in frequency domain (homogeneous condition, head.=180˚)

i. 3D FE modelling of ship
ii. Very fine mesh at the selected locations
iii. Distribution of lightship weight, cargo weight, ballast water, bunkering, and other appendages on FE model
iv. Apply ship motions and hull pressure (e.g. in Figure 3) on the FE model
v. Balancing check of the structural model
vi. Solution of the numerical problem
vii. Extraction of hot-spot stress (STF). Extrapolation of stress is performed as per the IIW recommendation (IIW, 2008))
viii. The process listed from (iii) to (vii) are repeated for 27 frequencies and two parts of frequency (real and imaginary), 12 headings, for each loading condition, at a given speed. The total number of 2592 (27×2×12×4×1) FE analyses are performed.

4. SPECTRAL ANALYSIS

The spectral analysis is performed as follows:

4.1 OPERATING CONDITIONS

Ship operating conditions are required which includes:

1. Voyage information or percentage of time spent in each wave environment (p_{ij}).
2. Fraction of time spent in a loading condition (p_l).
3. Probability of encountering a wave heading (p_k).
4. Vessel's speeds

The candidate ship is built to operate for worldwide locations except the North Atlantic. Four standard loading conditions, homogeneous, alternate, normal ballast, and heavy ballast are considered. The fractions of time spend in each loading condition of the bulk carrier are taken as per rule (IRS, 2020). An equal probability of ship heading with respect to the direction of the wave is considered. The vessel's speed is assumed as 75% of the service speed (IRS, 2018).

4.2 WAVE ENVIRONMENT

The wave data is represented in the form of a scatter diagram. It contains the probability of occurrence of different sea states which is defined by the significant wave heights (H_S) and the zero-crossing periods (T_Z). For each combination of H_S and T_Z, the probability of occurrence is found by dividing the observation for a sea state by the total number of observations. The Pierson-Moskowitz (PM) spectrum is used to describe the short-term sea states (see Eqn. (7)).

$$S_\xi(\omega|H_S,T_Z,\theta) = \frac{H_S^2}{4\pi}\left(\frac{2\pi}{T_Z}\right)^4 \omega^{-5} \exp\left(-\frac{1}{\pi}\left(\frac{2\pi}{T_Z}\right)^4 \omega^{-4}\right) \quad (7)$$

4.3 SPECTRAL MOMENTS

Stress response spectrum can be obtained using the Eqn. (6) and (7) as shown below:

$$S_\sigma(\omega|H_S,T_Z,\theta) = |H_\sigma(\omega|\theta)|^2.S_\xi(\omega|H_S,T_Z) \quad (8)$$

3D irregular seaway can be modeled using the spreading function. Cosine-squared spreading is assumed from +90 to –90 degrees on either side of the selected dominant wave heading. Spectral moments for each short-term sea state are computed using Eqn. (9):

$$m_n = \int_0^\infty \int_{\theta'=\theta-90}^{\theta'=\theta+90} \left(\frac{2}{\pi}\right) cos^2\theta'.(\omega_e^n S_\sigma).d\theta'.d\omega \quad (9)$$

$$\omega_e = \omega - \frac{V\omega^2}{g} cos\theta'$$

where θ' is the spreading angle between a wave component and the dominant wave direction.

5. FATIGUE DAMAGE

S-N curve gives the material fatigue failure at constant amplitude stress. Ships are subjected to variable amplitude load cycles. Therefore, the Palmgren-Miner rule is employed where variable amplitude cyclic loads are divided into the block of stresses. It is assumed that the total cumulative damage of a structural element is a linear summation of the damage in each stress block, and can be given by the Eqn. (10):

$$D = \sum_{i=1}^{n_t} \frac{n_i}{N_i} \tag{10}$$

where, n_i is the number of cycles of constant amplitude stress ranges, N_i is the total number of cycles to failure under a constant amplitude stress range. n_t represents the total number of stress blocks.

Stress range corresponding to 10^{-2} probability level has been considered in the analysis. Appropriate factors for correction of mean stress, thickness, and material are taken in the analysis (IRS, 2020). A factor of 0.85 is considered to account for the exclusion of harbor conditions.

5.1 SPECTRAL APPROACH BASED ON SHORT TERM RESPONSE

The stress range is normally expressed in terms of probability density functions for different short-term intervals corresponding to the individual cells of the wave scatter diagram. Linear addition of short-term damages sustained over all the sea states gives the total damage. So, the total fatigue damage accumulated over operational service life (T_D = 25 years) can be estimated by accounting for all sea states encountered with the different wave directions and loading conditions.

The fatigue damage expression (IRS, 2018) is based on short term Rayleigh distribution within each short term sea state for a loading condition, and the two-slope S-N curve can be defined as:

$$D = \nu_0 T_0 . \sum_{\substack{i=1 \\ j=1}}^{\substack{All\ Seastates \\ All\ headings}} \nu_{ij} \left(\frac{\left(2\sqrt{2m_{0ij}}\right)^{m_1}}{K_2} \Gamma\left(1 + \frac{m_1}{2}; \left(\frac{S_0}{2\sqrt{2m_{0ij}}}\right)^2\right) + \frac{\left(2\sqrt{2m_{0ij}}\right)^{m_2}}{K_3} \gamma\left(1 + \frac{m_2}{2}; \left(\frac{S_0}{2\sqrt{2m_{0ij}}}\right)^2\right)\right)$$

(11)

where,

ν_0 = long-term average zero-up-crossing-frequency (Hz)
T_0 = Design life in seconds.
K_2, m_1 = S-N fatigue parameters for N < 10^7 cycles
K_3, m_2 = S-N fatigue parameters for N > 10^7 cycles
γ = Incomplete Gamma function
Γ = Complementary Incomplete Gamma function
ν_{ij} = the relative number of stress cycles in short-term condition *i*, loading condition *j*
S_0 =Stress range in *S-N* curve, where the change of slope occurs
m_{0ij} = zero spectral moments of stress response process in short-term condition *i* and loading condition *j*.

6. TOTAL FATIGUE DAMAGE

Total fatigue damage is taken as a sum of damage occurred in all loading condition. The combined fatigue damage (IRS, 2020) is represented by Eqn. (12).

$$D = 0.25\,D_1 + 0.25\,D_2 + 0.2\,D_3 + 0.3\,D_4 \tag{12}$$

where, that D_1, D_2, D_3 and D_4 are the damages occurred in homogeneous, alternate, normal ballast and heavy ballast loading condition respectively.

7. NUMERICAL COMPUTATION

Fatigue damage assessment is carried out at the transverse butt-welded plates. The two locations (DK1 and DK2) at the deck and one location at the side shell (SS1) are selected. These locations predominantly experience global loads and therefore suitable for comparing the various fatigue assessment methods which are considered in the analysis.

Table 2. Ship particulars

Ship type	Bulk carrier
Length overall [m], L_{oa}	287.50
LBP [m], L_{BP}	279.00
Breadth (moulded) [m], *B*	45.00
Depth (moulded) [m], *D*	24.10
Scantling Draught [m], T_{sc}	18.49
Max Service speed [knots], V_s	14.60

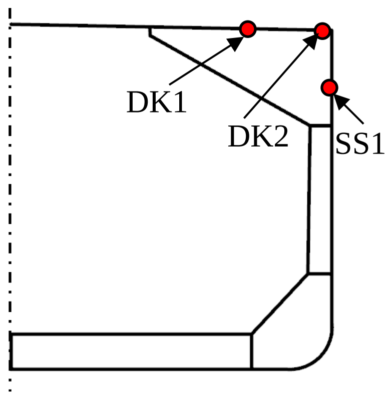

Figure 4: Representative mid-ship section showing the butt-welded plate joints with ids (DK1, DK2, and SS1)

Ship particulars are shown in Table 2. Figure 4 shows the representative mid-ship section indicating the butt welded joint locations. The following wave parameters are considered in the analyses:

- Frequency: λ/L = 0.2 ~ 5.0
- Wave headings: 0~330 (step of 30 deg.)
- Speed profile (75% of the service speed)

The fatigue damage is calculated as described in Sections 3 and 4. The nominal stress approach is used to determine the fatigue damage of all transverse butt-welded joints. The S-N curve 'D' class is selected, it incorporates an axial misalignment of 10% in plate thicknesses (UK HSE, 1990; IACS, 1999).

The fatigue damages calculated using three methods are shown in Figure 5 to Figure 7. Combined damage is calculated using Eqn. (12). These damage values are compared and shown in Figure 8.

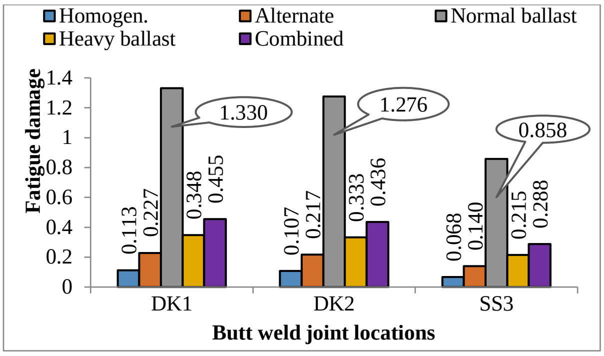

Figure 5. Fatigue damage using FAM-1

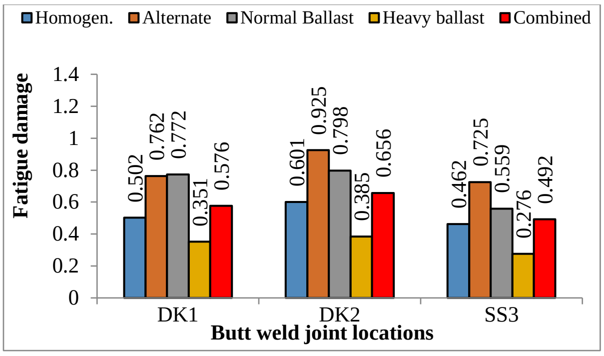

Figure 6. Fatigue damage using FAM-2

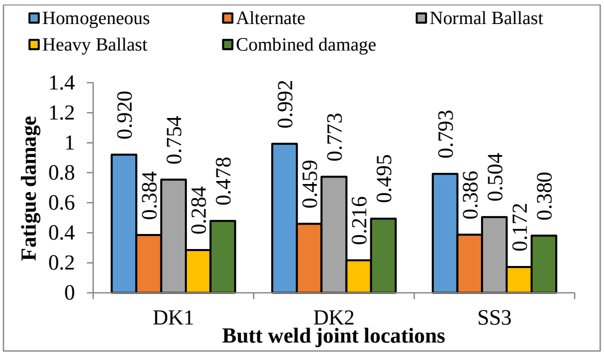

Figure 7: Fatigue damage using FAM-3

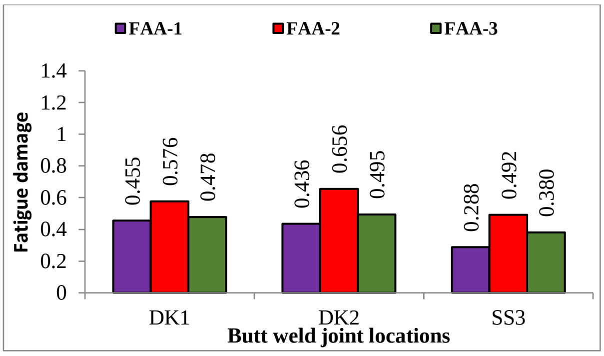

Figure 8: Total fatigue damage at given locations using FAM-1, FAM-2, and FAM-3 methods

8. CONCLUSION

In this paper, the spectral fatigue analysis has been carried out using three methods to estimate the fatigue damage at butt-welded joints located in the deck and side shell structure of a bulk carrier. These methods are comprised of distinct loads computation methods and structural analysis methods.

A large scatter in fatigue damage values can be noticed across all the loading conditions which are quite obvious primarily due to the difference in considered hydrodynamic load effects. It can be seen that the inclusion of HBM and torsional moment (TM) affects fatigue damage results. FAM-1 completely misses the effect of HBM and TM. In this case, DK1 becomes the most critical location. FAM-2 is considered the HBM but TM is ignored, while, in the case of FAM-3 includes all the load effects. The location DK2 becomes critical in the case of FAM-2 and FAM-3 as HBM accounted in these methods. In the case of ships with large deck openings and low torsional rigidity, it is necessary to consider the effect of the HBM and TM in oblique wave headings.

FAM-1 and FAM-2 may serve as an initial level of SFA to sort out the number of critical cases for detailed 3D hydro-structural analysis or to predict the critical locations which require immediate attention during the design phase or before carrying out the ship structural survey. The fatigue assessment using FAM-1 and FAM-2 can reduce the time and effort significantly compare to comprehensive analysis FAM-3. However, fatigue predictions based on FAM-3 methods cannot be ignored for the more realistic results.

9. ACKNOWLEDGEMENT

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