A NOVEL APPROACH TO ROBUST CONCEPT DEVELOPMENT FOR OFFSHORE FLOATING SYSTEMS

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SUMMARY

This paper describes a new approach to the application of robust design optimisation to the development and selection of floating systems during concept and early-stage engineering. The basis of the approach is to combine a genetic algorithm with an efficient non-intrusive statistical model to allow both uncertainties in modelling (intrinsic uncertainty) and requirements (extrinsic uncertainty) to be accounted for within a Robust Design Framework that directs optimisation toward unique design solutions. A particular interpretation of the Pareto design objective space enables required levels of performance margin and meaningful measures of confidence in design objectives to be achieved. The approach can be used to provide a systematic series of optimal hull geometries (geosims) that meet the most likely variations in design requirements, and so mitigate the impact of change early in the design cycle. This paper follows on from and earlier publication (IJME648) on multi-objective, multi-discipline, optimisation and uses the same case study.

KEYWORDS

Robust Design, Optimisation, Uncertainty Analysis, Non-Intrusive Polynomial Chaos, Inverse Design Methods

NOMENCLATURE

BM	Second moment of water-plane area/immersed volume
C_1, C_2	Constants relating Steel and Outfit Mass to Displacement
GM	Metacentric height (m)
Hs	Significant wave height (m)
KB	Height of centre of buoyancy above keel (m)
KG	Height of the centre of gravity above keel (m)
U_1, U_2	Genetic algorithm objective functions
M_{SOW}	Steel and Outfit Mass.
P	Payload/Topsides mass (Te)
P_0, $P_R(t)$	Initial and time varying production rate (BOPD)
RAO	Response Amplitude Operator
R_{NPV}	Net Present Value of field reserves (USD)
F	GA fitness measure
T_N	GA Target function
Zs	Significant heave (double amplitude, m)
α	Weightings for mean objective functions
γ	Weighting for standard deviations
β	Weighting between mean and standard deviations.
μ_1,μ_2	Mean values of objective functions U_1, U_2
σ_1,σ_2	Standard Deviations of U_1, U_2
Δ	Hull mass displacement (Te)

Hull geometry definitions are given separately in Appendix A1.

UNITS

MKS units are used throughout with the following additional accepted industry definitions:

bbl	International standard volume – Barrel.
BOPD	Production rate; Barrels of Oil Per Day
MM	Short form for millions, as in MMBBL
M	Short form for thousands, as in MBOPD
Te	Metric Tonnes

1. INTRODUCTION

1.1 STUDY BACKGROUND

This paper describes the application of robust design optimisation (RDO), and a novel inverse design method (IRDO), to early-stage concept development and selection of offshore floating systems.

The use of optimisation at the concept development stage might seem premature, not least because the level of design definition is often insufficient for the application of sophisticated engineering models, and levels of uncertainty in design requirements are high. However, decisions taken at the early stages of design and system definition are fundamental to success (Andrews, 2018), and so optimisation tools that incorporate and mitigate such uncertainties are of great value.

When choosing between different concepts, it is common to develop preliminary designs for each and carry out performance and cost/benefit comparisons. Decisions are often based on a single design point, and later changes that might lead in a different direction may either result in sub-optimal choices or significant re-work and delay.

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The solution proposed here is to model the potential for change and other uncertainties using robust design optimisation, but focus on generating "populations" of designs, rather than comparing "one-off" cases.

A successful concept select would therefore generate a systematic series of geometries (geosims) that show the greatest resilience to early-stage uncertainties in design definition and performance prediction. In design objective space, such a series is represented by a line segment (rather than a single design point), bounded at each end by statistical limits in design requirements, and with uncertainties in performance prediction built in as a margin.

To make this possible, it is assumed that the design objective space is smoothly varying as a function of design requirements and constraints, and that Pareto Fronts are contours representing unique solutions which are optimal with respect to different weightings of design objectives (performance, weight, cost etc.).

The range of design requirements, constraints and performance margin can be set deterministically, but it is more useful to use statistical models from which levels of confidence can be derived, and so elements from both Robust Design Optimisation and Inverse Design are adopted here.

The key elements of the resulting search algorithm are that it:

- First selects a mean (P50) optimal concept geometry with a defined performance margin against uncertainties in design modelling (intrinsic uncertainty).
- Then generates additional geometries meeting the same performance margin that allow calculation of its statistical properties subject to uncertainties in design requirements (extrinsic uncertainty).
- And finally, combines the resulting statistical properties of the design objectives arising from both sources of uncertainty to fully define the line segment along which robust solutions can be found.

By using this approach there is no need for complete Pareto Fronts to be generated nor does it require "manual" post-processing to further select design solutions from a Pareto set.

This paper describes the methodology in detail and provides a worked example to illustrate its application in practice.

1.2 TECHNICAL BACKGROUND

The use of formal mathematical methods in optimisation for the refinement of design performance is now well established across a wide range of engineering disciplines.

For ship design, applications to hull performance that take advantage of advances in parametric modelling and advanced analysis tools (Papanikolaou, 2010, Biliotti, *et al*. 2011, Guha, & Falzarano, 2015, Vasudev, *et al*., 2017, Maisonneuve, *et al*., 2018, Cheng, *et al*, 2019), demonstrate how specific design objectives can be optimised.

Other publications in the field either focus on application of these techniques during detailed design and analysis (Diez, M *et al*. 2015, Serani, A., *et al*. 2015), and/or, holistic principles that integrate the many different tools used throughout the design lifecycle (Papanikolaou, 2020).

For ships, use in concept development often includes factors that drive design requirements such as routing analysis, cargo and transport economics etc., and focus on defining principal particulars, dimensions, hull form coefficients and powering, that support a through-life perspective (Diez & Peri., 2010, Kim & Vlahopolous, 2012).

Sensitivity to uncertainty has been included within optimisation to both quantify effects (Wei *et al*. 2019), and more practically, apply the principles of Robust Design Optimisation to ship design more generally (Hannapel & Vlahopolous, 2010, Li, *et al*., 2016, Diez *et al*. 2013).

However, for other types of floating systems, for example FPUs, FPSOs, or in offshore wind, Floating Offshore Wind Turbines (FOWT), there is insufficient data from which to develop the types of correlation and design relationships needed for early-stage application of Robust Design Optimisation using typical parametric approaches.

This paper aims to provide the detail necessary to enable use of these methods for early-stage decision making in concept design and the selection of such offshore floating systems, but without needing to resort to an extensive toolbox of engineering analysis techniques to gain the benefits of the approach.

1.3 RESEARCH METHOD

The research approach adopted here is one of systematic numerical experimentation and testing.

The underlying robust optimisation algorithm has several key elements, described within the Appendices to this paper, that were tested individually and, where needed, tuned accordingly. This applied chiefly to the so-called Target (penalty) functions and associated power laws used to apply selection pressure within the genetic algorithm, and the order of numerical integration rules used for statistical modelling.

As is typical for most iterative numerical methods, measures of convergence are critical. Throughout this work, a maximum of 1.0% error in the sum of target functions was applied before examining whether results

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were Pareto. Runs were carried out repeatedly to ensure consistency and uniqueness in solutions.

A Case Study based on the single discipline of hull definition is used to illustrate the method. A previous publication (Gallagher, 2020) demonstrated multi-discipline optimisation, combining an Oil & Gas Process facility with hull definition for the same novel FPSO. Here however, the process facility definition is instead linked to oil reservoir characteristics, and so provides the source of uncertainty in design requirements, specifically topsides weight (payload) and storage capacity.

A summary description of the Case Study is given in Appendix 1 for completeness.

1.4 STRUCTURE OF THIS PAPER

The core elements of the work presented here are:

- Tools that allow the design objective space to be explored in a more directed approach than in conventional optimisation.
- An efficient statistical model for dealing with uncertainties in modelling (intrinsic uncertainties) and design requirements (extrinsic uncertainties), namely by Non-Intrusive Polynomial Chaos (NIPC).
- Robust design optimisation and the IRDO approach itself.

The first two of these core elements are key steps on the way to the IRDO method. They provide useful tools in themselves but are largely drawn from existing literature and so are confined here to a summary in section 2, and Appendices. The exception is a variation on the usual NDSA method that allows specific solutions on the Pareto Front to be identified, which is described in Appendix 3.

The main body of this paper therefore concentrates on robust design optimisation and the IRDO approach to concept design.

The next section summarises the overall modelling objectives and the methods used for uncertainty analysis. Section 3 describes robust design optimisation (RDO) and the inverse design method IRDO. Section 4 provides a worked example to illustrate the different treatment of intrinsic and extrinsic uncertainties. Section 5 provides discussion and conclusions.

2. OVERVIEW OF KEY PRINCIPLES

2.1 MODELLING OBJECTIVES

At the concept development stage, we need to generate design solutions that:

- Provide unique, consistent and smoothly varying results for geometry and other principal particulars.
- Are optimised for their design objectives.
- Meet specified performance criteria.
- Are robust with respect to modelling uncertainties.
- Are robust with respect to uncertainty and potential for change in design requirements.
- Provide bounding values for the effect of statistical uncertainty (even when minimised) on factors such as cost/weight or similar design objectives.

The first three objectives are key to good quality optimisation, independent of uncertainty modelling. Appendices 2, 3 and 4 describe the approach to optimisation used in this paper, which is based on the non-dominated search (NDSA) genetic algorithms (Deb, 2001., Konak *et al*., 2006, Seada & Deb, 2015), along with modifications that enable designs meeting specific performance targets to be found directly.

The remaining objectives are the main subject of this paper. They reflect the need to minimise the uncertainty in the location of the Pareto Front itself and the practical application of the IRDO method.

2.2 MODELLING UNCERTAINTY

Modelling the impact of uncertainties, either in design requirements or modelling inputs, is critical to rational concept design and selection. Such uncertainties are large in the early stages of design and change as projects become better defined. Concept designs that are flexible in the face of change and are robust with respect to uncertainty are highly desirable.

Non-Intrusive Polynomial Chaos (NIPC) provides an efficient numerical method for estimating such effects and has been used across many fields such as CFD and FEA in recent years (Lee *et al*. 2009). The NIPC approach used here is described in Appendix 5.

It is important to distinguish between optimisation incorporating uncertainties, and robust design optimisation (RDO). The former treats optimisation in the same "deterministic" way as for example, CFD or FEA, and so allows results to be stated using statistical measures of confidence.

In Robust Design Optimisation, the additional aim is to find Pareto optimal solutions that minimise the impact of uncertainty by incorporating statistical measures within the fitness ranking of competing design solutions.
Optimisation incorporating uncertainty in either design requirements or modelling is nevertheless one of the building blocks of RDO and IRDO, and so is described in more detail in Appendix 6.

The following section describes the integration of these concepts into an inverse robust design optimisation approach that can be applied to generating robust design solutions with defined statistical properties and performance margins.

3. ROBUST DESIGN

3.1 RDO FOR CONCEPT ENGINEERING

For early-stage concept engineering, Extrinsic uncertainties that lead to changing requirements during the Front-End and Detailed Design and Engineering are of particular concern. Similarly, as the level of engineering detail increases, modelling assumptions made earlier may come under challenge from more detailed computational analysis and/or physical testing.

Therefore, this study places particular emphasis on the need to focus on not only optimisation as a tool to reduce sensitivity to uncertainty, but also to develop:

- Forms of mitigation to changing requirements and increased levels of engineering detail,
- Rational design margins that account for uncertainties and at the same time support decision making.

To this end, two different forms of RDO algorithm are described here.

The first is a conventional RDO approach using a weighted minimisation of the mean and standard deviation of design objectives applied to find a single elite solution, but with additional mitigation measures built into the design.

The second is based on the use of a design performance margin to drive the selection of elite robust solutions. This is effectively an inverse approach and so the acronym IRDO is used.

3.2 THE GENERAL RDO METHOD

The key changes to the conventional NDSA are in the calculation of the fitness measure (F) based on the weighted design objectives U_1, U_2, U_M, and the selection of the parent designs for each new generation of populations within the genetic algorithm.

For a simple two objective optimisation problem, the usual (α) weighted fitness function:

$$F = \alpha.\ U_1 + (1-\alpha)\ U_2 \quad (1)$$

Becomes:

$$F = \beta.[\alpha.\mu_1 + (1-\alpha).\ \mu_2] + (1-\beta).[\gamma.\sigma_1 + (1-\gamma).\sigma_2] \quad (2)$$

Where:

μ_1, μ_2 are the mean values of design objectives (U_1,U_2)

σ_1, σ_2 are the standard deviations of the design objectives (U_1,U_2)

α is, as before, weighting between the mean values of the design objectives.

β is an additional weighting between mean and standard deviation of the design objectives

γ is an additional weighting between the standard deviations of design objectives.

This is a more generalised version of weighted fitness functions than can be found elsewhere (Murphy, Allen *et. al*. 2010), and gives greater flexibility, particularly when there is a need to weight the relative standard deviations of the design objectives.

For the optimisation scheme used here, the fitness of each design is further weighted by Target Functions as described in Appendix 2, which ensure that selection pressure favours members of the population which are closest to the stated design requirements (e.g. payload, storage capacity, GM etc), and indeed converge to satisfy them to within a suitable tolerance criterion.

The optimisation process therefore remains as described in Appendix 2, but with the additional need to calculate the mean and standard deviation of the design objectives for each member of the population.

When using NIPC as the statistical integration model, this is achieved by:

- Using the mean value of the inputs as the basis of a mean design (a P50 design basis) to be optimised.
- Generating (MxN – 1) additional design realisations for each of the quadrature points (N) for range (M) of uncertain input values.

These additional design realisations are evaluated within an inner loop at each generation within the genetic algorithm (See Figure A2.1 for a summary flow chart).

Values of β and γ are often set at 0.5 (or equivalent in other formulations), i.e. an equal ranking between mean and standard deviation, and the value of α varied between 0 and 1 to generate a Pareto Front for the mean design solution, with accompanying values for standard deviations in the design objectives.

For RDO methods, it is generally observed (Jin & Sendhoff, 2003) that the mean design optimum falls slightly short of the equivalent deterministic Pareto design, but with reduced sensitivity to uncertainty (minimised standard deviations in design objectives).

An example of the application of this general RDO method, which also describes a practical approach to mitigating changes in design requirements, is given in Appendix 7. A summary of weighting values used is given in Appendix 8.

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3.3 INVERSE RDO (IRDO)

The aim of Inverse Robust Design Optimisation is to generate only those design solutions which satisfy specific criteria with respect to the design objectives (Lim *et al*, 2005, Nellippallil *et al*., 2020). The method described here generates designs that meet given performance criteria by including an additional Target Function to favour members of the population that have statistical properties leading to a required margin or level of confidence. This can be achieved within a conventional RDO algorithm using the following additional steps:

1. For each generation, use the mean and standard deviation in the performance objective to calculate a suitable statistical measure, such as the 90^{th} percentile (0.9 probability level) or P90.
2. Compare this with a target P90 performance level to weight each member of the population and apply to its overall ranking, favouring those that are closest to the target.
3. Additionally, adjust the value of the weighting (α) dynamically (Appendix 3), to ensure convergence to the target level is met by a Pareto solution.

As described in Appendix 6.3, this approach requires the definition of a characteristic line (for example, a normal to the Pareto Front) along which to integrate, and so relies on knowledge of the mathematical form of the Pareto Front for the mean of the modelling inputs.

This dependency can be replaced by selecting a specific ratio of standard deviations (γ) within the Fitness relationship (equation 2), and modifying parent selection within the optimisation algorithm to include members of those populations of quadrature points that both minimise the Fitness function and align dynamically according to the value of γ. This approach leads to the following additional steps within the IRDO loop described above:

1. Perform each optimisation step for all quadrature point sets of modelling inputs simultaneously, adjusting the value of weighting α of each to drive alignment as needed (note: α for the mean is fixed, γ is fixed, within this inner loop).
2. For each generation of design solutions at each quadrature point, select the Top 10 using the local value of weighting α of the design objectives to rank fitness.
3. Generate a new "group population" (around 100 members) made up by collecting sets of randomly selected members of the Top 10 quadrature point design solutions.
4. Calculate the fitness F of each group using Equation 2, with mean and standard deviation values (μ, σ) calculated using the Gauss-Hermite quadrature rules described (Appendix 5)
5. Rank according to this new Group Fitness level and select new "Elite" population set.
6. Select new parents the next generation of design solutions at each quadrature point using the Elite population set and randomly selected others.

This process is consistent with the fundamentals of the NDSA, but with parents across all quadrature points selected not only by their individual ranking, but also by those of their population's statistical measures.

The characteristic line itself is evaluated dynamically using a least-squares best fit to the design objectives calculated for the Elite population set. Figure 1 below shows an example of the above population-based selection process for a simplified case involving uncertain inputs for the steel and outfit mass relationship (See Appendix 6), and with the critical damping ratio assumed in the calculation of the heave RAO (ζ); based on a mean of 0.125 with a standard deviation of 10% of the mean.

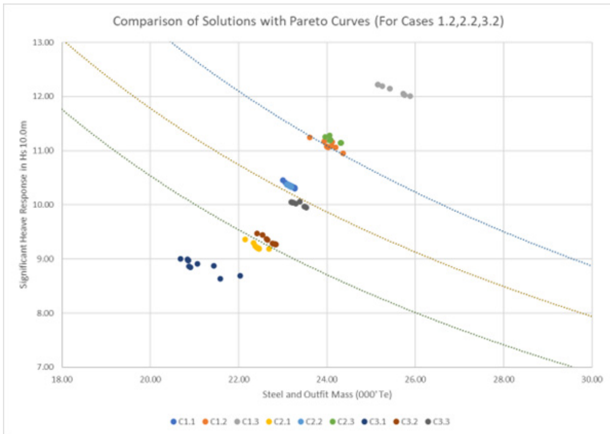

Figure 1. The top 10 design solutions for each set of Gauss-Hermite quadrature input values at convergence, γ = 0.25, α = 0.5 for mean input solution (C2.2)).

The Pareto fronts for the mean, upper and lower inputs for the damping ratio are shown for comparison (Cases 1.2, 2.2 and 3.2) showing that the converged solutions for these quadrature points are close to or coincident with their respective Pareto Fronts.

Figure 2 shows how the best fit characteristic line through all quadrature point design solutions can be reasonably defined for the purposes of Gauss-Hermite integration.

As applied to intrinsic uncertainty modelling, and for any fixed value of the weighting α, this "inner loop" solution provides the mean and standard deviation for the location of the Pareto Front. When applied within the IRDO structure, the weighting α for the mean of the modelling inputs is allowed to vary according to the additional selection pressure exerted by the performance requirement and using the search method described in Appendix 3.

In summary therefore, optimisation of the solution for the mean inputs is driven by the inverse design process, and the

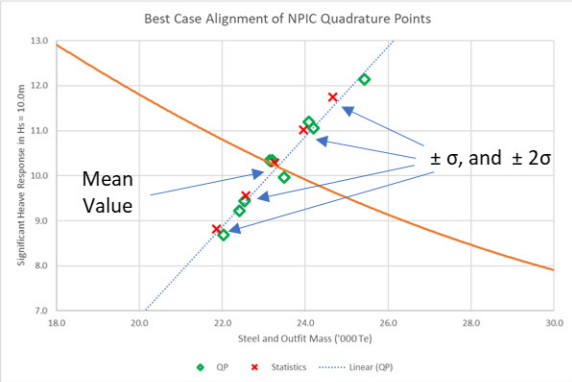

Figure 2. Example of best fit characteristic line as evolved during the RDO simulation.

dynamic alignment of the optimised solutions for all other Gauss-Hermite Quadrature Points to the characteristic line is a function of the selected ratio of the standard deviations (γ). For each value of weighting (γ) and chosen performance criterion, there is a unique design solution at the intersection of the characteristic line so formed with the Pareto Front for the mean of the modelling inputs.

3.4 EXTRINSIC UNCERTAINTY AND GENERATING A SYSTEMATIC SERIES.

The IRDO approach described above gives a Pareto solution for the mean of the uncertain modelling inputs with a built-in performance margin. It does not need any complete Pareto Fronts to be generated and, by definition, goes directly to a robust mean design solution for the defined modelling (intrinsic) uncertainties.

We now wish to generate a line segment in design objective space along which Pareto design solutions that satisfy the performance margin can be found, and for which bounding values of other design objectives can be defined by suitable statistical measures.

Figure 3 illustrates this mean or P50 design point and the margin ΔT_2 against a defined P90 performance Target level T_2.

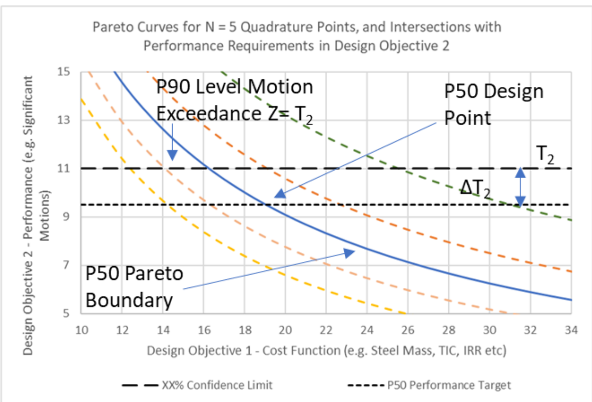

Figure 3. Illustration of where a P50 Robust design solution may sit relative to a P90 performance limit.

The horizontal line $Z = T_2 - \Delta T_2$ can be viewed as the line along which a systematic series of Pareto design solutions can be generated, all of which will satisfy the performance margin. It is also a line along which optimal solutions subject to extrinsic uncertainty (Appendix 6.2) may be calculated.

Indeed, a systematic series of robust solutions generated along this line and bounded by P10 to P90 confidence limits, represent a range of designs that are robust in terms of performance and bounded in terms of weight and cost.

Finally, the effect of extrinsic and intrinsic uncertainties can be combined as they are independent. This can be achieved by either simply summing the variances in the design objectives, or to be more rigorous, performing a full numerical Gauss-Hermite integration across all uncertain variables. These two approaches are compared in the following case study.

4. CASE STUDY

The following case study is aimed at showing how the RDO/IRDO algorithms are applied in practice following the three key stages described in 1.1.

The concept of interest is the so-called Deep Draught Production, Storage and Offloading (DDPSO) floating described previously (Gallagher, 2020), and summarised in Appendix 1. Here, we are interested only in optimising the hull geometry for performance and capital cost, with the process facility definition provided as input requirements and subject to (extrinsic) uncertainty.

A summary of the weighting values (α,β,γ) used for each stage is given in Appendix 8.

4.1 PROBLEM DEFINITION

Consider a marginal field development scenario defined as follows:

Table 1: Defined Field Development Data.

Inputs	Mean (μ)	Std Dev (σ)
Reserves	50.0 mmbbl	2.5mmbbl
Field Life	8 Years	1 year
Water Cut	50%	10%
Offloading Interval	14 days	1 day

These properties may be combined to give an initial production rate (P_0) based on the assumption of an exponential depletion of reserves, and that after 8 years of production, the total fluids handling capacity including water-cut is equal to P_0, and that the total production is equal to the estimated reserves (Gallagher, 2020).

This simple model may be re-formulated statistically, using either Monte-Carlo Simulation or Gauss-Hermite

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quadrature, and can be shown to give a P10 lifetime production of 69.6Mbbl, and a P90 lifetime production of 33.1Mbbl, along with matching upper and lower limits on production rate.

We use a simple model (Gallagher, 2020) to equate the total topsides mass/payload (P) to the production rate:

$$P = K_1 + K_2.P_0 \tag{3}$$

With uncertainties defined as follows:

Mean $K_{1\mu}$ = 2000Te,

Standard Deviation $K_{1\sigma}$ = 100Te

Mean $K_{2\mu}$ = 0.15,

Standard Deviation $K_{2\sigma}$ = 0.015

Assuming a normal distribution for all variables, the following statistical inputs for optimisation subject to Extrinsic uncertainties in Cargo volume and Topsides Mass are:

Table 2: Statistical Definition of Design Requirements.

Statistical Model - Mean (μ) and Std Deviation (σ)				
	μ - σ	Mean (μ)	μ + σ	
Prod Rate	20,625	24,195	27,766	BOPD
Cargo Vol	280,968	336,301	391,635	BBL
Topside Mass	**4980.2**	**5629.3**	**6278.4**	**Te**

Which are used to calculate the NIPC quadrature points for target design inputs to the optimisation model incorporating extrinsic uncertainties.

Intrinsic uncertainties in the modelling of hull steel and outfit mass (M_{SOW}) and heave damping ratio are as described earlier in section 3.3.

4.2 MEAN (P50) DESIGN USING IRDO

We use the IRDO formulation to calculate the P50 design geometry for the mean design inputs of payload (5,629Te) and cargo capacity (336.3MBBL), with the additional constraint that GM = 2.0m.

Figures 4 and 5 below show results for two sets of IRDO calculations for which P90 targets of 10.0m and 11.0m significant heave in a 1-year maximum sea-state are set. For illustration, the full Pareto Front for the mean design inputs is also shown.

The following tables summarise the resulting statistical properties of these robust solutions subject to intrinsic uncertainty.

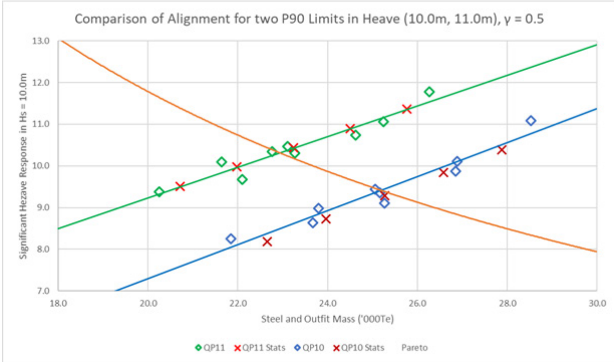

Figure 4. Alignment of Quadrature Points for γ = 0.5 for two P90 Significant Heave Targets of 10.0m (lower line) and 11.0m.

Figure 5. Alignment of Quadrature Points for γ = 0.125 for two P90 Significant Heave Targets of 10.0m (lower line) and 11.0m.

Table 3: Summaries of P90 Mean results for γ = 0.125 (above) and γ = 0.5 (below).

Target Zs	DO	Alpha	Mean	Std Dev	P10	P90
10.0	Zs	0.79	9.05	0.750	8.09	10.03
	M_{SOW}		26.27	0.352	25.81	26.72
11.0	Zs	0.59	9.99	0.797	8.97	11.01
	M_{SOW}		24.23	0.473	23.62	24.83

Target Zs	DO	Alpha	Mean	Std Dev	P10	P90
10.0	Zs	0.76	9.28	0.552	8.58	9.99
	M_{SOW}		25.27	1.308	23.59	26.94
11.0	Zs	0.49	10.43	0.462	9.84	11.02
	M_{SOW}		23.24	1.265	21.62	24.86

The principal observations from these results are that:

- The parameter γ, being the weighting between standard deviations in the fitness ranking, controls the gradient of the characteristic line intersecting with the Pareto Fronts for each set of input quadrature points.
- The different α values found for the mean P50 design reflect the adjustments needed to ensure that the P90 performance criteria are met and are consistent.
- Although the mean P50 designs differ, their P90 steel and outfit mass values are practically the same for the same target P90 Zs values.

That the P90 steel and outfit mass values should agree (as also found for other values of γ) suggests that there is both smoothness and consistency within the design objective space.

This also implies that (for this case study at least) robust design optimisation may give upper bound P90 estimates that are relatively insensitive to the choice of γ, even though different values for the P50 mean result from IRDO.

4.3 RDO FOR EXTRINSIC UNCERTAINTY

The next stage is to apply robust design optimisation subject to extrinsic uncertainties at the performance level calculated for the intrinsically robust mean P50 designs at selected γ. For example, the following cases might be of interest (β = 0.4 all cases).

Table 4: Definition of Mean Values for Performance Design Objective Zs in each Case of Interest.

Case	P90 Zs	γ	α	Zs (Mean)
1	10.0	0.125	0.79	9.05
2	10.0	0.5	0.76	9.28
3	11.0	0.125	0.59	9.99
4	11.0	0.5	0.49	10.43

As described in Appendices 3 and 4, the new mean Zs values represent horizontal lines in the design objective space.

To apply RDO we combine the approach described in Appendix 6.2, along with fitness Equation 2 to rank each generation, and the selection process described in 3.4, to establish the population of quadrature points along this line segment in design objective space that gives the required minimum standard deviation in steel and outfit mass.

In this case, values for weights (β, γ) are held fixed, and α varied as described previously to locate intersections with the relevant Pareto Fronts.

Figure 6 below illustrates the result of performing such a robust optimisation process to the cases identified above.

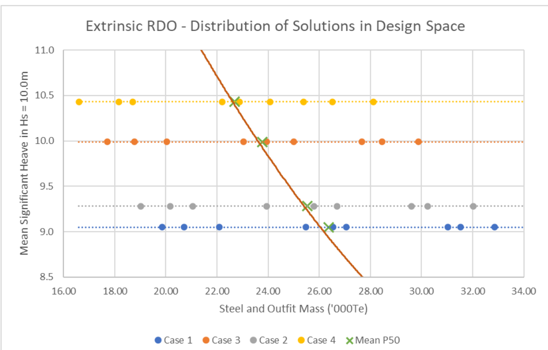

Figure 6. Location of RDO solutions at Quadrature Points for Extrinsic Uncertainties.

The table below summarises the resulting statistics for the steel and outfit mass (Te), calculated using the Gauss Hermite integration rules as before, applied to the above.

Table 5: Summary of RDO Solutions for Steel and Outfit Mass.

Case	P90 Zs	Mean (μ)	σ - M_{SOW}	P90 - M_{SOW}
1	10.0	26,370	3,107	30,352
2	10.0	25,522	3,034	29,410
3	11.0	23,766	2,844	27,411
4	11.0	22,682	2,362	25,709

4.4 COMBINING UNCERTAINTIES

To complete the picture, we need to combine the effect of intrinsic and extrinsic uncertainties. Assuming the PDF for each of the extrinsic uncertainty quadrature points arising from local intrinsic uncertainty is, to close approximation, the same as that calculated for the basis P50 design solution, the standard deviation of the combined intrinsic and extrinsic uncertainties (σ_c) can be calculated by simply summing variances for the mean P50 design point, i.e.

$$\sigma_c = (\sigma_I^2 + \sigma_E^2)^{½} \qquad (4)$$

However, a more rigorous approach is to calculate the effect of local intrinsic uncertainty at each of the Gauss-Hermite quadrature points for the extrinsic uncertainty, and then integrating the (now) 4D problem.

Figure 7 below shows the results of carrying out this analysis for Cases 1 and 2 using the same values of weights (β,γ) for each as originally applied for the P50 mean in 4.2 above. The weighting of α is varied as before, but with the target for Zs now set at the calculated means (9.05m and 9.28m respectively) rather than the P90 level used in the IRDO calculation.

A summary of the steel and outfit weight statistics for Cases 1 and 2 is given below, showing the resulting standard deviations for the combined intrinsic and extrinsic uncertainties as calculated using equation 4, and the full 4D integration.

Table 6: Comparison of 4D Gauss-Hermite Integration with Simplified sum of Variances.

	Case 1		Case 2	
	$\Sigma\sigma^2$	4D	$\Sigma\sigma^2$	4D
σ_c	3,127	3,238	3,304	3,242
P10	22,363	22,313	21,287	21,424
P50	26,370	26,463	25,522	25,579
P90	30,378	30,613	29,756	29,734

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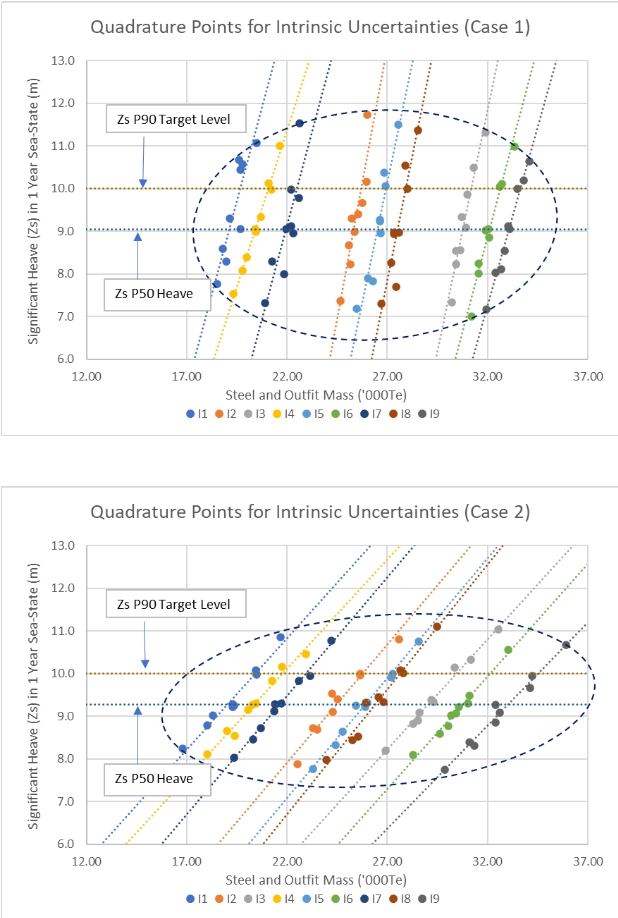

Figure 7. Plot Showing all Quadrature Points for Intrinsic and Extrinsic Uncertainty Models for Cases 1 and 2 (γ =0.125 above, 0.5 below).

The reasonable agreement in results for total standard deviation achieved in both cases appears to point toward the assumption underlying Equation 4 to provide a reasonable approximation in this case, given the consistent values of weights (β,γ).

The differences in P10 and P90 estimates between Cases 1 and 2 therefore attributed for by the differences in the mean values associated with choice of weight γ (0.125 and 0.5 respectively) and the different margins in significant heave that implies. It would therefore appear sufficient, at least during very early concept select, to use Equation 4 to make rapid assessment of the P90 cost estimate.

4.5 APPLICATION

The following table provides a complete picture of the results from this robust concept design optimisation process as might be reviewed in practice. Results for P90 targets in significant heave (P90-Zs) of 10m and 11m in a one-year sea-state (Hs = 10.0m) are given as indicated, with some selected key measures of hull geometry (Cases 1 and 3 from above).

Table 7: Summary of Design Properties for two Performance Criteria.

Requirements	**Mean**	**σ**	**P90**	**P10**
Reserves (MMbbl)	50.0	2.5	33.1	69.6
Field Life (Years)	8.0	1.0	6.7	9.3
Prod Rate (BOPD)	24,190	3,570	19,615	28,765
Topsides Mass (Te)	5,629	649	4,797	6,461
Cargo Vol (Kbbl)	336.3	55.3	265.4	407.2

Design bounds	**Mean**	**σ**	**P90**	**P10**
M_{SOW} – 10 (Te)	26,370	3,127	22,363	30,378
M_{SOW} – 11 (Te)	23,766	2,883	20,071	27,461

P90-Z_S =10m	**Mean**	**σ**	**P90**	**P10**
Displacement (Te)	99,387	11,960	84,060	114,714
Draught (m)	86.72	0.99	85.45	87.99
Max Beam (m)	42.95	2.628	39.58	46.32

P90-Zs =11m	**Mean**	**σ**	**P90**	**P10**
Displacement (Te)	96,246	11,369	81,676	110,816
Draught (m)	80.22	0.542	79.53	80.91
Max Beam (m)	43.78	2.618	40.42	47.14

It is important to note here the convention for the definition of requirements and design outputs. For the former, P10 is the most optimistic and P90 the most conservative. For the IRDO method, the P90 measure also gives a conservative view of the performance margin. Matching design solutions to the highest confidence in reserves and production rate (P90), leads to the smaller facility and visa-versa.

The "Line Segments" in design objective space of interest are given here by the horizonal lines Zs = 9.05m (Case 1) and 9.99m (Case 3) and bounded by the P10 and P90 values for Steel and Outfit Mass (M_{SOW}) as shown.

The primary initial use for these data is to provide estimates of capital cost (CAPEX). At early concept stage, this would typically be based on simple norms for key items; steel and outfit mass (Msow), fixed/solid ballast, and topsides mass (i.e. Process equipment and secondary structures) for construction (ex-works).

Clearly, these data might also be sought from industry statistical databases, and so a further level of modelling using Monte-Carlo or the simpler integration methods used here is possible but is not the subject of this paper.

As such projects progress through from concept development and selection to Pre-FEED, FEED and detailed design, the effect of change can be monitored. New optimal hull geometries can be generated quickly in response to updates in payload and storage requirements using the methods described in Appendix A3 to find the intersection of the Pareto front for each new set of requirements with the line segments in design objective space identified using this approach.

5. DISCUSSION

The case study described above illustrates the three main elements of the inverse and robust design method proposed here for early-stage concept development.

As applied to the influence of intrinsic uncertainty on optimisation, the IRDO method allows a set performance level, and performance margin, to be used to select design solutions that are Pareto for a set of given design requirements. The critical factor is the choice of weighting values used in calculation of fitness (Equation 2), and the value of γ, defining the relative importance of the standard deviations of the design objectives.

Nevertheless, the approaches described here avoid the need to work through many combinations of the weightings (α,β,γ) and the significantly greater computational effort that would entail.

The results presented in section 4.2 infer that, for this case study and choice of performance limits, predictions of the P90 steel and outfit mass subject to the intrinsic uncertainties as defined are the practically the same. This is because variations in standard deviation were compensated for by changes in the location of the mean design solution along the Pareto Front.

Clearly, this may not always be the case, and it is conceivable that a steeper Pareto Front and higher levels of intrinsic uncertainty would lead to a greater dependence in the calculated statistics of the design objectives to the choice of the weighting γ.

Section 4.3 shows that, not surprisingly, changes to design requirements provide the most significant uncertainties and largest influence with respect to overall weight and cost. The spread of mean design solutions that lay between the P10 and P90 estimates however provide good bounding values for geometries that are Pareto and meet required performance margins.

For the specific case study used here, combining intrinsic and extrinsic uncertainties can be achieved with reasonable accuracy by summing their individual variances. Consequently, in practical applications, it may be sufficient to carry out just one IRDO simulation for the required performance margin/confidence level against intrinsic uncertainty, and one RDO calculation at the subsequent mean performance level to calculate bounding values for cost/weight subject to extrinsic uncertainties.

Computationally, the work presented here was carried out using a combination of MS Excel and VBA, and so is easily repeatable in any design office environment. This does however give a practical limit with respect to the number of uncertain variables and order of numerical integration ($M<3$, $N<5$ as individual limits).

Clearly, if the algorithm were to be implemented using a compiled and parallel programming language, this would greatly improve run-times and give scope to increase both N and M. However, for the purpose of early-stage concept development, it is more likely that only leading order uncertainties (such as heave damping) and competing design objectives need to be addressed in practice, making such calculations more easily manageable.

6. CONCLUSIONS

An Inverse Robust Design Optimisation approach has been presented that takes advantage of various techniques for exploring design objective space and non-intrusive polynomial chaos based statistical modelling, to establish an efficient and directed approach to concept design.

Its principal benefit is that, through an inverse design approach, it directs solutions efficiently toward satisfying specific performance targets, is robust with respect to changing design requirements, and gives statistically bounded cost estimates that better reflect best practice in early project investment decision making.

The application demonstrated is based on a combination of empirical relationships and classical hydrodynamic theory, but the overall IRDO/RDO approach is independent of the models used. It is fully general and can be applied to the preliminary design of all forms of floating systems, particularly those that are novel and for which classical ship hull form definition coefficients are not appropriate, such as semi-submersibles, SPARs and TLPS as found in offshore engineering.

7. REFERENCES

1. ANDREWS, D., *The Sophistication of Early-Stage Design for Complex Vessels*. Trans RINA, Special Edition, IJME 472, 2018.
2. CAMPANA, E.F., FASANO, G., PERI, D., 2012, *Penalty Function Approaches for Ship Multi-disciplinary Design Optimisation (MDO),* European Journal Industrial Engineering, Vol 6, No. 6.
3. DEB. K., 2001, *Multi-Objective Optimization using Evolutionary Algorithms. Chichester*: Wiley & Sons.
4. DIEZ, M., CHEN, X., CAMPANA. E.F., STERN, F., *Reliability-Based Robust Design Optimisation for Ships in Real Ocean Environment*. 12th International Conference on Fast Sea Transport, FAST 2013. Netherlands.
5. DIEZ, M., PERI, D., FASANO, G., CAMPANA, E.F., *Multidisciplinary Robust Optimisation for Ship*

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Design, 28th Symposium on Naval Hydrodynamics, Pasadena, California, September, 2010.
6. DIEZ, M., PERI, D., *Two-Stage Stochastic Programming Formulation for Ship Design Optimisation under Uncertainty.* Ship Technology Research, August 2010.
7. DIEZ M., VOLPI S., SERANI A., STERN F., CAMPANA E.F., *Simulation-based Design Optimization by Sequential Multi-criterion Adaptive Sampling and Dynamic Radial Basis Functions.*" Proc. EUROGEN 2015 - International Conference on Evolutionary and Deterministic Methods for Design, Optimization and Control with Applications to Industrial and Societal Problems, University of Strathclyde, Glasgow, UK, (14-16 Sept. 2015)
8. GALLAGHER, P., Multi-Objective Optimisation and its Application to Concept Development in Floating Offshore Systems. Trans RINA, IJME 648, 2020.
9. GUHA, A. AND FALZARANO, J. 2015, *Application of multi objective genetic algorithm in ship hull optimization*, Ocean Systems Engineering, Vol. 5, No. 2, 91-107.
10. HANNAPEL, S.E., VLAHOPOLOUS, N. *Robust and Reliable Multidiscipline Ship Design.* 2010, 13th AIAA/ISSMO Multidisciplinary Analysis Optimisation Conference, AIAA 2010-9394.
11. HASLUM. H.A., 2000, *Simplified Methods Applied to Non-linear Motions of SPAR Platforms.* PhD Thesis, NTNU Trondheim.
12. JIN, Y., SENDHOFF, B., *Trade-off between Performance and Robustness: An Evolutionary Multi-Objective Approach.* Evolutionary Multi-Criterion Optimisation, LNCS 2632, pp 237-252, 2003.
13. KIM, H.Y., VLAHOPOULOS, N., *A Multi-Level Optimization Algorithm, and a Ship Design Application.* 12th AIAA Aviation Technology, Integration and Operations Conference, 2010. AIAA 2012-555
14. KONAK. A, COIT, D.W., SMITH A.E., 2006, *Multi-Objective Optimisation using Genetic Algorithms: A Tutorial*. Reliability Engineering and System Safety 91, 992-1007.
15. LEE, S-H., CHEN, W., KWAK, B., *Robust Design with Arbitrary Distributions using Gauss-Type Quadrature.* Structural and Multidisciplinary Optimisation, September 2009.
16. LI, D-Q., JIANG, Z., ZHAO, X., *Ship Multidisciplinary Robust Design Optimization under Multidimensional Stochastic Uncertainties.* 26th International and Polar Engineering Conference, June 2016.
17. LIM, D., ONG, Y-S., LEE, B-S., *Inverse Multi-Objective Robust Evolutionary Design Optimisation in the Presence of Uncertainty*. GECCO '05, June 25-29, 2005 Washington.
18. MAISONNEUVE, JJ., HARRIES, S., MARZI, J., RAVEN, H.C., VIVIANI, U., PIIPPO, H., 2018, *Towards Optimal Design of Ship Hull Shapes,* Project FANTASTIC, ResearchGate Publications.
19. MURPHY, T.E., TSUI, K-L., ALLEN, J.K., *A Review of Robust Design Methods for Multiple Responses.* Research in Engineering and Design, 2005, Vol. 16: pp 118-132.
20. NELLIPPALLIL, A.B., MOHAN, P., ALLEN, J.K., MISTREE, F., *An Inverse, Decision Based Design Method for Robust Concept Exploration.* ASME Journal of Mechanical Design, Vol 142, August 2020.
21. NEWMAN. J.N., 2017, *Marine Hydrodynamics*, MIT Press, 40th Anniversary Edition.
22. PAPANIKOLAOU, A., 2010, *Holistic Ship Design Optimisation*, Journal Computer-Aided Design, Elsevier, Vol. 42, Issue 11, pp. 1028-1044.
23. PAPANIKOLAOU, A., *et al*, 2020, *Tools and Applications for the Holistic Ship Design,* Proceedings of 8th Transport Research Arena TRA 2020, ResearchGate Publications.
24. SARPKAYA. T., ISAACSON. M., 1981, *Mechanics of Wave Forces on Offshore Structures.* Van Nostrand Reinhold co.
25. SEADA, H., DEB, K., *U-NSDA-III: A Unified Evolutionary Algorithm for Single, Multiple and Many-Objective Optimisation*. COIN Report Number 2014022. Springer, pp.34-49, 2015.
26. SERANI A., DIEZ M., CAMPANA E.F., *Single- and multi-objective design optimization study for DTMB 5415, based on low-fidelity solvers*. INSEAN Tech, rep. 2015-TR-002 (2015a)
27. TAO, L., MOLIN, B., SCOLAN, Y.-M., & THIAGARAJAN, K., 2007, *Spacing Effects on Hydrodynamics of Heave Plates on Offshore Structures.* Journal of Fluids and Structures, 23, pp.1119-1136.
28. WEI, X., CHANG, H., FENF. B., LUI. Z., *Sensitivity Analysis Based on Polynomial Chaos Expansions and Its Application in Ship Uncertainty-Based Design Optimization.* Hindawi Press, Mathematical Problems in Engineering, 2019, Article ID 7498526.

A1. CASE STUDY GEOMETRY

The geometry of the DDPSO concept used in the case studies was described in detail in IJME648 (Gallagher, 2020) and is illustrated in Figure A1.1 below.

The DDPSO was originally conceived as a marginal field development solution for the UKCS, with the focus being on a combination of low CAPEX (less than $500m TIC) and good motion response characteristics for harsh environments for a relatively small floating system. It is aimed at the "niche" that fits around a storage capacity of around 300Kbbl, with production rates around 25,000 BOPD or less.

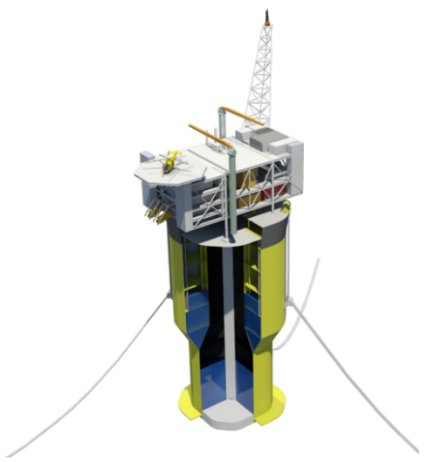

Figure A1.1. General view of the DDPSO concept used in these case studies.

It has the following main features:

- An upper hull section to provide the buoyancy required.
- A central oil-over-water storage tank, extending from the main deck down almost to the keel, within which lower density crude oil floats on seawater, such that the tank is permanently pressed full.
- An upper deck and hull paces for various hull utilities, mooring system equipment, offloading facilities etc.
- A central, internal seawater caisson to provide pressure balance.
- A fixed solid ballast compartment at the keel.
- One or more heave plates, or for this case study, two heave plates combined also to form a keel box, open to the sea.
- Minimal water ballast needed to meet damage stability regulations and balance differing specific gravities of crude oil and seawater.
- Additional water ballast to allow mitigation of changes in requirements during concept optimisation.

For the purposes of optimisation, the hull form is described using selected key dimensions as shown in Figures A1.2 and A1.3, along with certain parametric relationships that maintain the fundamental DDPSO concept. Starting conditions for the optimisation algorithm are based around a prototype, with the above dimensions varied randomly within a set of bounds to establish an initial population of (n ϵ N) designs.

$$X_{i,n} = X_{i,0} \pm R.\Delta x_i \qquad \text{A.1.1}$$

Where:

$X_{i,n}$ represents any of the chosen dimensions
Δx_i is a small variation in $X_{i,n}$ (or so called search radius)
R is a random number between 0 and 1.

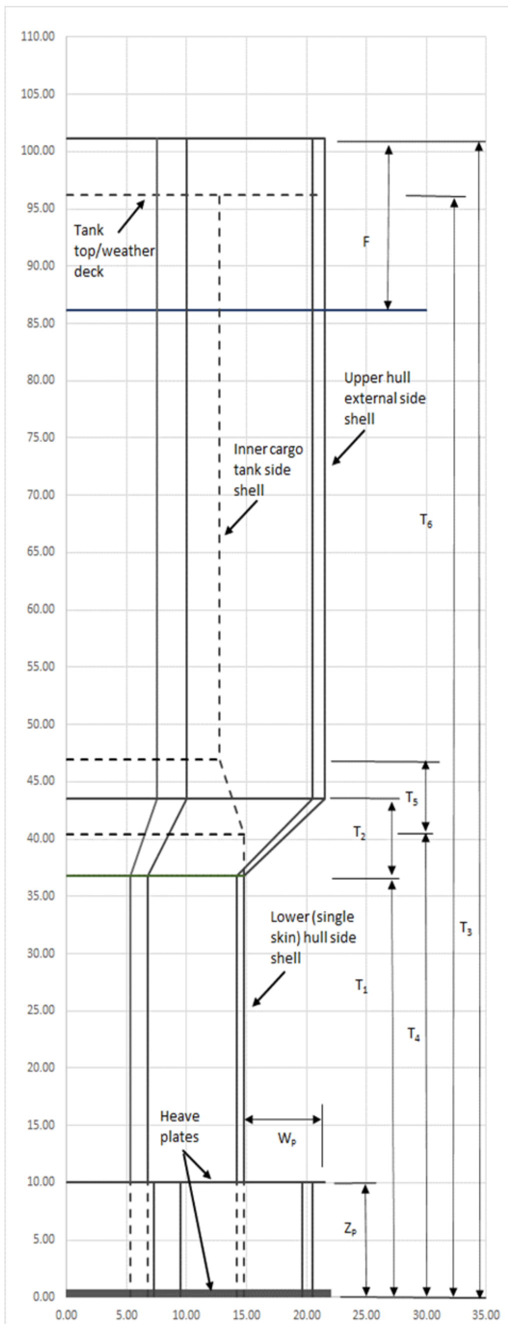

Figure A1.2. Elevation, Principal Dimensions of DDPSO.

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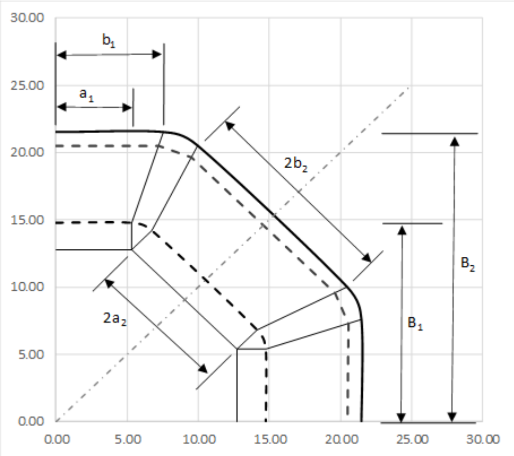

Figure A1.3. Plan View, Principal Dimensions of DDPSO.

The optimisation of the hull form is based on two key (and conflicting) design objectives.

- Hull Steel and Outfit Mass (M_{SOW}) as a proxi for capital cost
- Significant heave motions (Z_s) in a given sea-state with a one-year return period.

The critical weight items for the system as modelled here were the payload (topsides mass), hull steel and outfit mass, fixed (solid) ballast, water ballast and cargo mass.

For the purpose of the current work, the total mass of the topsides and the position of its centre of gravity, are treated as an inputs, albeit subject to (extrinsic) uncertainty (see Equation 3, Section 4.1).

The hull steel mass is assumed to be linearly proportional to a combination of the overall displacement and the draught (See equation A6.1 later). This relationship was based on a number of separate studies into typical scantlings established as weight estimates and using appropriate Codes Of Practice, for different sizes of hull, with the likely levels of uncertainty based on typically accepted margins.

The treatment of the other weights is as described in specific example cases reported here.

Significant heave motions were calculated using first principles linear hydrodynamic theory to generate a heave response amplitude operator (heave RAO). The hull was assumed sufficiently slender and deep draught such that a non-diffracting approximation to wave loading (i.e., Froude-Krylov, inertia and linearised drag loading), could be applied.

Froude-Krylov and inertial wave loads were integrated analytically for the relevant wetted surfaces (Newman, 2017, Sarpkaya & Isaacson, 1981).

The additional effect of heave plates on inertia and damping were used published data (Tao, Molin, *et al.* 2007) to provide the basis of a mean damping ratio, along with its standard deviation, as part of the intrinsic uncertainty analysis.

Additional design constraints were applied to avoid second order effects (Haslum, 2000). These cover both designs for which the pitch period is an integer multiple of the heave period, and those within the envelope of the natural pitch period and wave spectrum peak period. Such design configurations are penalised within the optimisation algorithm such that they cannot be carried through the selection process.

The significant heave response for a one-year maximum significant wave height (Hs) of 10m, modal period 15.85s, (SMB spectrum) was used throughout.

A2. SUMMARY OF NDSA OPTIMISATION ALGORITHM

A Non-Dominated Search Algorithm (Deb, 2001, Konak, 2006) is used throughout the work described here, and is illustrated in the flow chart shown in Figure A2.1 below.

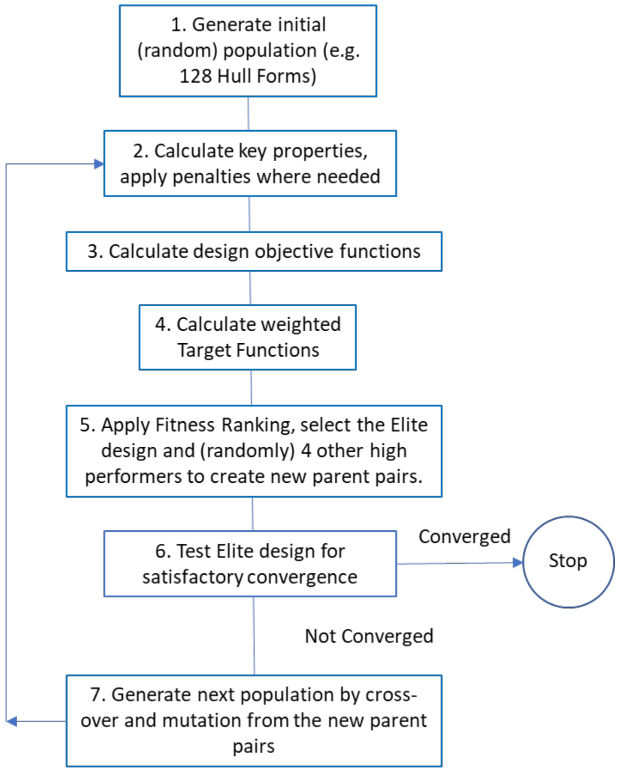

Figure A2.1. Flow Chart for Underlying Non-Dominated Search Algorithm

The optimisation process starts with a prototype geometry, from which a much larger population is generated (in this study, 128 members) by making small random variations in selected dimensions. Its main properties, volumes, mass distribution, stability, natural periods, etc. are then calculated according to the design model/ correlations being applied. Performance and cost/weight design objectives are calculated for each member of the population, along with any penalties (e.g. the need to avoid particular combinations of natural periods of motion).

As in previous work (Gallagher, 2020) target functions that provide selection pressure towards specific requirements (e.g., payload, storage capacity, GM etc), are calculated.

$$W_{Ti} = 1 + (| X – T_i |/ T_i)^N \qquad (A2.1)$$

Where:

- T_i: is the required target value (e.g. payload, GM)
- X: is the calculated value for each member of the population
- N: is a power law based on the current "error"

For the work reported here, these are supplemented by additional target functions as follows:

- Following the NDSA III method, additional selection pressure toward a so-called "Utopia Point" (Seada & Deb, 2015) are used (See Appendix A3).
- For the IRDO method described here, additional selection pressure toward solutions meeting specified performance criteria for one or more design objectives is applied.

Fitness is the product of the of weighted design objectives (Equations 1 or 2 previously), and the target functions, giving overall ranking of each member of the population.

For the general NDSA method a single elite design is selected, along with 4 other randomly selected members of the population from the top 20, to be paired together for the next stage; cross-over and mutation, which generates the next, 128-member, generation.

For the RDO and IRDO methods specific to this paper, an alternative selection strategy is used as described in Section 3 of this paper.

The overall process its iterative and can be shown to converge to form the well-known Pareto Front of optimal design geometries in the design objective space as a function of the Fitness weightings used.

A3. FINDING SPECIFIC PARETO SOLUTIONS

Figure A3.1, taken from (Gallagher, 2020) illustrates a typical set of Pareto Fronts for 3 different sets of design requirements (crude oil storage capacity and payload in this case).

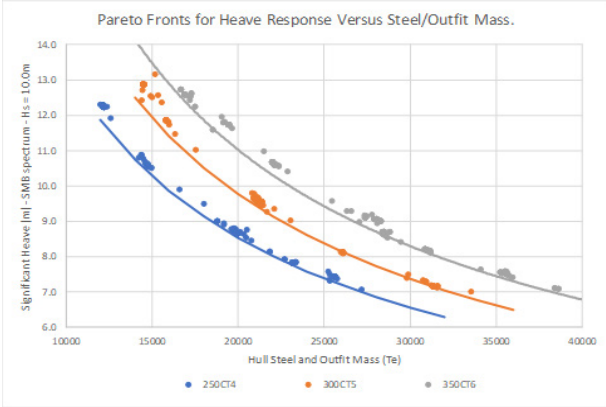

Figure A3.1. Typical Pareto Fronts for a range of design requirements from 250Kbbl storage with a 4000Te payload to 350Kbbl and 6000Te Payload.

It illustrates the general form of such Pareto Fronts, and leads to some key assumptions for this study that:

- That the Pareto Fronts for the different design requirements have similar characteristics and form a series of locally near parallel contours in the design objective space.
- Similarly, the effect of changes to constants/power laws within the various design models used lead to similar and consistent shifts in the location of the Pareto Fronts.
- Consequently, it can be assumed that within the Pareto Space, families of geometries forming systematic series can be generated that meet varying design requirements and variations in underlying design model coefficients or correlations.

Figure A3.2, illustrates this principle.

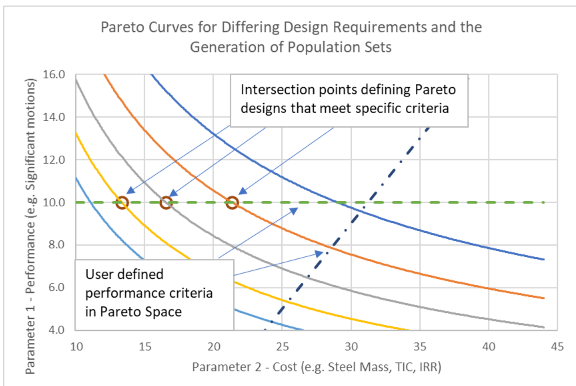

Figure A3.2. The concept of Population Sets that are Pareto for Specified Design Performance Requirements

This may be applied in two ways:

- To generate a systematic series of optimum designs to meet varying design requirements, all of which satisfy a given performance criterion.

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- To model uncertainties using Non-Intrusive Polynomial Chaos (NIPC), and subsequent application of Robust Design to optimise designs (RDO)

The first of the above applications is met by applying two modifications to the NDSA-II/III model. They are:

- to target the performance requirement (e.g. the horizontal line Zs = 10.0m in Figure A3.2), with a fitness weighting proportional to the difference between the design objective and the requirement,
- to include the weighting α used in the ranking (R_α) of each population within the overall search algorithm.

For the Pareto Fronts representing the different design requirements, the intersection points with the line (Zs = 10m) are achieved each with different values of the weighting α. A regular updating of α is therefore applied based on:

If: $U_1 > Zs + \varepsilon$: $\alpha = \alpha + \Delta\alpha$

$$U_1 < Zs - \varepsilon: \alpha = \alpha - \Delta\alpha \quad \text{(A3.1)}$$

Where:

U_1: Is the current value of the design objective 1 (the performance objective).
ε: Is a tolerance value for Zs
Δα: Is a small fraction of α

The combined effect of weighting the fitness of each member of the design population according to its proximity to the target performance, iteratively adjusting α as shown, and the optimisation process itself, leads to the evolution of solutions at the required intersection points. Figure A3.3 below illustrates convergence to an intersection point using this iterative approach.

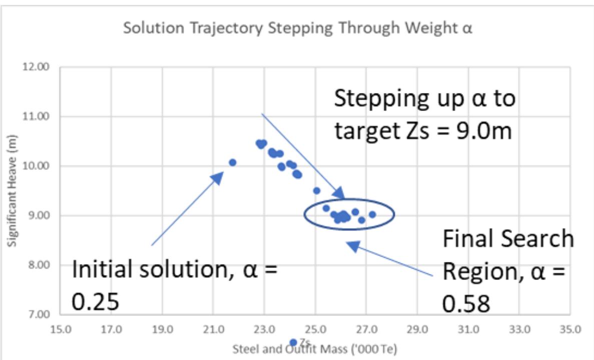


Figure A3.3. Example of how by stepping through the weighting α, the search region can be focused on a specific target requirement.

It should be noted that the NDSA-III algorithm, which includes a so-called "Utopia Point" in the design space to accelerate the evolution process, is helpful here. The Utopia Point can be located on the characteristic line (Zs = 9.0 in Figure A3.3) and used to weight the ranking of solutions closest to it more positively.

Appendix A4 describes with an example how this technique can be applied to generating a systematic series of design solutions for varying design requirements that meet a specific performance criterion.

A4. GENERATING A SYSTEMATIC SERIES

The performance targeting approach can be used to generate a systematic series of optimal designs in response to varying design requirements as illustrated by the following example case.

Figure A4.1 illustrates a typical variation in topsides mass with storage capacity using relationships linking them to oil production rate and offloading intervals (Gallagher, 2020).

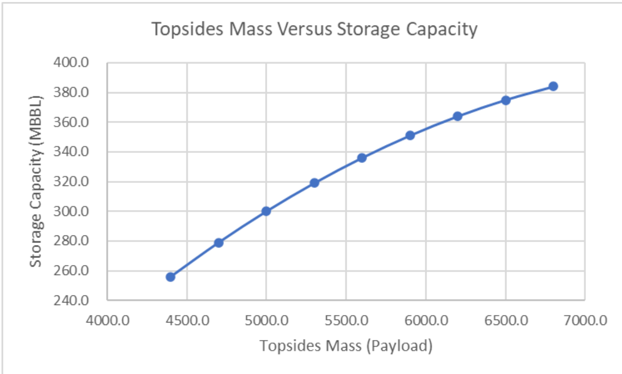
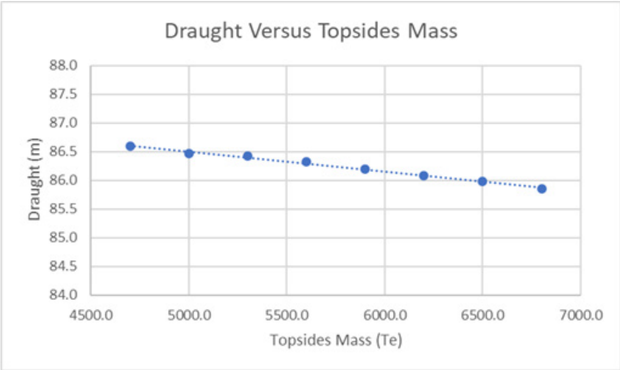


Figure A4.1. Design Inputs for Storage Capacity and Topsides Mass.

A significant heave response of 9.0m in a one/year sea state (Hs = 10.0m, Tz = 15.8s) was used as the performance requirement and represented in the design objective space by a characteristic horizontal line; Zs = 9.0m.

Using the approach described in Appendix 3, there is no need to calculate complete Pareto Fronts for each of the design requirements. Instead, convergence to the target performance level in significant heave and to a consistent value of the weighting α, are used as a stopping condition. Figure A4.2 shows how certain principal dimensions vary, quite smoothly, as a function of topsides mass for Pareto optimal solutions at the required performance objective, Zs = 9.0m. Figure A4.3 shows the subsequent variation in hull steel and outfit mass for the same cases.

In conclusion, the above example demonstrates that systematic series of optimal solutions can be targeted without the need for full Pareto front definition. Such results are useful for decision makers needing to quickly understand the effect of changing requirements on capital cost.



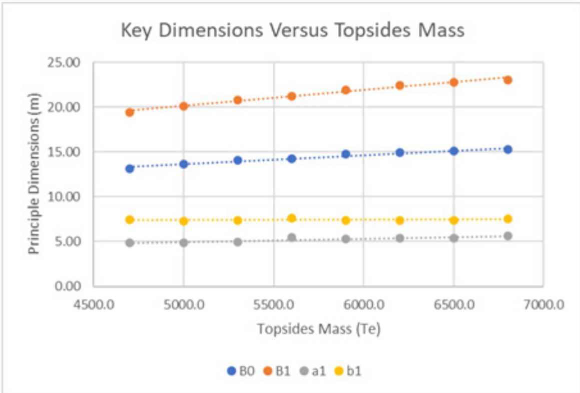


Figure A4.2. Variation in Draught and other key Dimensions for Pareto Optimal Solutions with Varying Inputs.

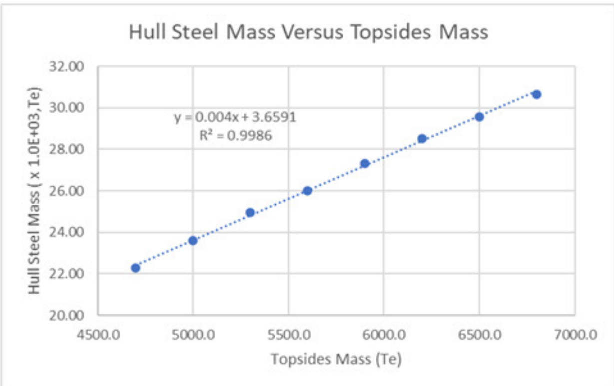


Figure A4.3. Variation in Hull Steel and Outfit Mass for the Example Case.

A5. NON-INTRUSIVE POLYNOMIAL CHAOS

Non-Intrusive Polynomial Chaos (Expansion) refer to a class of statistical modelling methods for which it is possible to apply deterministic analysis tools (e.g. FEM, CFD etc.) subject to uncertain inputs and boundary conditions and generate statistically meaningful levels of confidence in outputs.

We assume that uncertain modelling inputs can be represented by typical probability density functions (PDFs) as shown below:

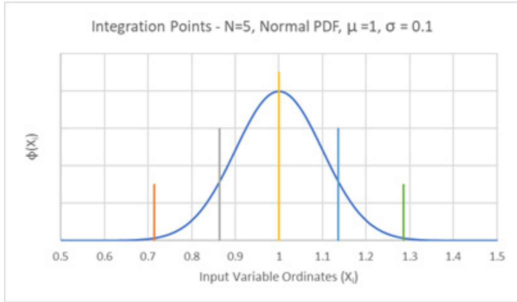


Figure A5.1. Illustration of Typical, Normal, PDF

And that we can apply Gauss-Hermite integration rules with quadrature points given at:

$$X_i = \mu \pm C_i.\sigma.\sqrt{2}$$

Where:

μ,σ: are the mean and standard deviation of the inputs
C_i: are a series of coefficients according to the order of integration (number of quadrature points)

The deterministic analysis is carried out for inputs at each of these quadrature points, and the resulting solutions post-processed to generate outputs expressed statistically (i.e. mean and standard deviations etc.)

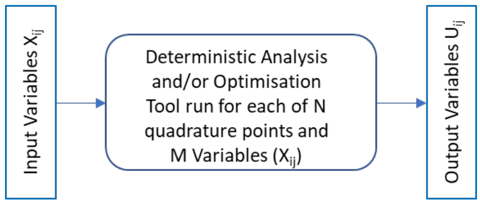


Figure A5.2. Schematic View of Non-Intrusive Modelling Approach.

The output statistics are generated using a weighted sum of output values (U_{ij}), with weights derived again, from the chosen quadrature rules, i.e. for mean and standard deviation:

$$\bar{U} = \Sigma\,\Sigma\, w_i.w_j.U_{ij} \,/\, \Sigma\,\Sigma\, w_i.w_j \qquad \text{(A5.1)}$$

$$\sigma_U = (\,\Sigma\,\Sigma\, w_i.w_j.U^2_{ij}/\Sigma\,\Sigma\, w_i.w_j - \bar{U}^2\,)^{1/2} \qquad \text{(A5.2)}$$

The coefficients C_i and weights w_i can be found in standard texts and on-line resources (e.g. Hermite-Gauss Quadrature -- from Wolfram MathWorld).

To be clear, this is a convenient and highly efficient method for introducing the effects of uncertainties in all forms of deterministic modelling.

For the purposes of RDO as used here in early-stage concept design, it relies on simplifying assumptions with respect to the PDFs used, i.e. those for which numerical

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integration rules can be applied. Thus, inputs or modelling coefficients expressed as Normal or Raleigh PDFs are the most straightforward. Where input data is sparce or non-linear, there are several more complex data analytic tools that can be applied. However, at the early stages of concept development and selection, this is likely to over-complicate matters

Similarly, despite significant advantages over techniques such as Monte-Carlo Simulation, there are practical limitations regarding the number of variables and order of integration that would be appropriate, particularly at concept stage, as discussed in the main body of the paper.

A6. OPTIMISATION UNDER UNCERTAINTY

A6.1 MODELLING UNCERTAINTY

The Non-Intrusive Polynomial Chaos approach described in Appendix A5 is used here to model the effect of statistical uncertainty on optimisation.

At concept stage, it is sufficient to assume particular forms of PDF, e.g., Normal/Gaussian, or Raleigh, for inputs of interest, and apply the Gauss-Hermite integration rules described earlier. Input quantities are simply defined by their mean and standard deviation, from which quadrature points are calculated according to the specific rule and order of integration applied as shown earlier.

Optimisation is carried out for each of these sets of inputs and then outputs post-processed to derive values for mean and standard deviation of the design objectives.

Clearly, this form of statistical modelling also leads to multiple Pareto Fronts (for each of the input quadrature points). If using an NDSA type optimisation model, we are again faced with the problem of which weighting of design objectives (α) to apply, and if being used for Robust Design Optimisation (RDO), how to minimise the standard deviation, or variance, of the outputs.

In these studies, NIPC integration is applied along characteristic lines in the Pareto design space, with quadrature points being at their intersection with the relevant Pareto Fronts and found as described in A3 above. The definition of the characteristic lines depends on the class of uncertainty of interest, and are of two types:

- For uncertainty in design requirements (i.e. payload, storage capacity, minimum GM etc. – so called "extrinsic uncertainty").
- For uncertainty in modelling (i.e in relationships used to model critical weight items, hydrodynamic performance, shear force/bending moments etc.), so called "intrinsic uncertainty".

Each of these is now discussed in turn.

A6.2 EXTRINSIC UNCERTAINTY

The main principle behind design for extrinsic uncertainties is that we wish to understand their impact on weight, cost, or similar decision critical factors, for a defined level of performance.

This is represented in design objective space by a horizontal characteristic line. The objective is to find the intersection points made by this line and the Pareto Fronts of designs evolved using statistically modelled inputs. The approach described in Appendix A3 is applied in order to align solutions with the specified target level, adjusting the weighting (α) for each.

Here however, the approach is applied simultaneously to all extrinsic quadrature points in order that calculations of mean and standard deviation can be made and also used in the evaluation of fitness for RDO.

Figure A6.1 below shows how the intersection of Pareto Fronts for each of the input combinations of payload and storage capacity used in the Case Study, and an arbitrary performance criterion (Zs = 9.8m in this case), can be found. These represent specific design solutions whose properties can then be post-processed to provide resulting statistics.

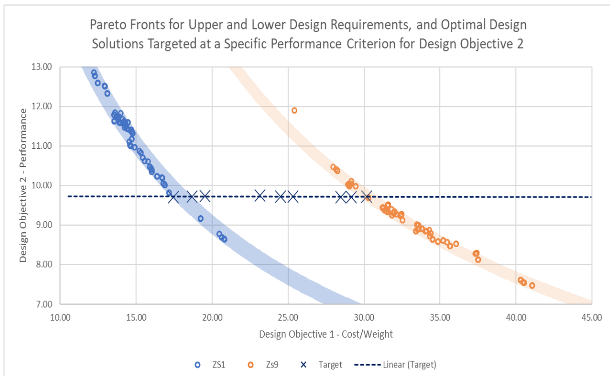

Figure A6.1. Example plot showing the intersections of Pareto Solutions and a Defined Performance Criterion.

To be clear, if the above method is applied without including the calculated standard deviation within the fitness measure, it simply finds the intersection of each of the Pareto Fronts with the stated performance criterion. In such a case, the calculated mean and standard deviation are measures of statistical variation properties of the Pareto solutions along this line, which can also be of value.

A6.3 INTRINSIC UNCERTAINTY

The principle used here is that uncertainties in modelling lead to uncertainty in the location of the Pareto front itself according to the statistical properties of the design objectives.

Consider how the position of the Pareto Front might vary as the result of uncertainty in, for example, the design model used for the steel and outfit mass of the DDPSO Case Study. The model used (Gallagher, 2020) is:

$$M_{SOW}/\Delta = C_1.(1 + C_2.(T-T_R)/T_R) \qquad (A6.1)$$

Where:

C_1: is the ratio of steel and outfit mass to total mass displacement (Δ) at reference draught (T_R).
C_2: is an additional factor to account for variations in steel mass with draught (T).

The deterministic values $C_1 = 0.225$, $C_2 = 1.25$, used in (IJME648) are replaced by means of 0.225 and 1.25, and standard deviations of 0.0225 and 0.125 respectively.

Uncertainty in performance modelling is also included, in this case in terms of the level of linearised heave damping applied, being in this case, a mean of 12.5% of critical damping with a standard deviation of 10% of this.

Figure A6.2 illustrates this concept, showing how for example, Pareto fronts for an order N=5 Gauss-Hermite quadrature for a normal distribution of the coefficient C_1 might look.

Any characteristic line through the design point as shown can be used to define intersection points with the other Pareto fronts and therefore estimate the statistics of the design objectives for that point.

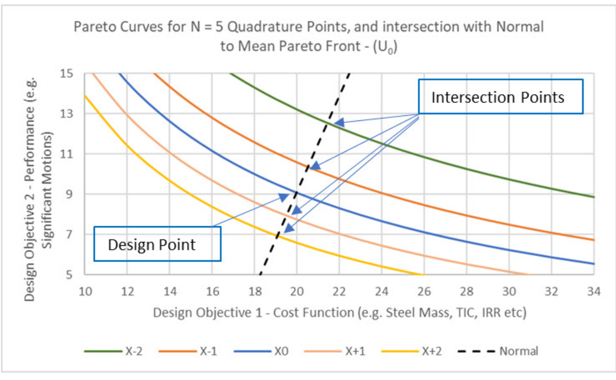

Figure A6.2. Geometric model in design objective space for estimating the effect of intrinsic uncertainty.

The key question here is how best to define this line, forming as it does the abscissa for the numerical integration. Its slope governs the relative proportions of standard deviation in the design objectives, and so becomes part of the decision making, as it reflects the properties of all design solutions lying along it.

This is illustrated in Figure A6.3, which shows the result of applying the methodology described in A3 to align each of the Pareto optimal solutions at intersections between the chosen characteristic line (the normal to the mean Pareto Front in this case), and so making up the quadrature points for the Gauss-Hermite integration (the small diamond points on the plot).

Clearly, a different choice of gradient would lead to a change in proportion of standard deviations of the design objectives because there are inherently different populations of designs aligned along each such characteristic line.

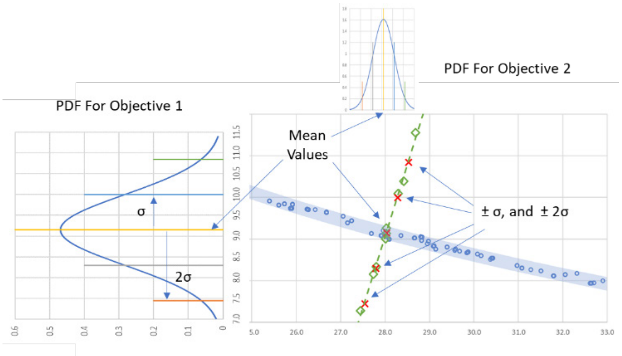

Figure A6.3. Illustration of the effect of choice of characteristic line on the relative statistical properties of the design objectives.

As with the Extrinsic modelling described in A6.2, this model of design uncertainty does not yet constitute a robust design method, rather it is a simple application of NIPC to an otherwise deterministic optimisation model, much as might be carried out with other forms of engineering analysis.

It does therefore have some value to decision makers who might prefer at concept stage to just to understand the impact of uncertainty and use that understanding to build confidence and establish performance margins.

Nevertheless, the following section describes how a useful Robust Design Optimisation methodology suitable for concept development can be built on the above principles.

A7. EXAMPLE APPLICATION OF RDO.

This following is an example of the application of the conventional robust design optimisation approach to the case study used here and previously (Gallagher, 2020).

Generally, RDO allows the selection of designs for which the sensitivity of their design objectives to uncertainty is minimised. But a key consequence of introducing statistical variation in input quantities, is that other dependent variables also now exhibit statistical variation.

For example, in the case study used here, the intrinsic uncertainty in estimating steel and outfit mass will, for a given hull displacement, impact both payload capacity and the amount of fixed solid ballast required.

Consequently, fulfilling the design requirement for a particular payload capacity becomes subject to additional

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uncertainty. This is clearly undesirable, but inevitable. A practical measure commonly used by naval architects to mitigate the effect of change is to include some flexibility in variable ballasting arrangements. In this way, any changes in weight items may be absorbed without the need for re-work.

The following example demonstrates how RDO can be used to help define how much variable ballast might be required when incorporating intrinsic uncertainty in steel and outfit mass as before.

The model used for steel and outfit mass (Gallagher, 2020) is as given in statistical form as in section A6.3, Equation A6.1. The same treatment of heave damping as given in A6.3 is also applied.

Figure A7.1 illustrates a typical Pareto Boundary based on a mean payload of 5600Te and storage capacity of 336 KBBL generated using a wide range of weightings (α), for the DDPSO case study, using this relationship for Steel and Outfit Mass.

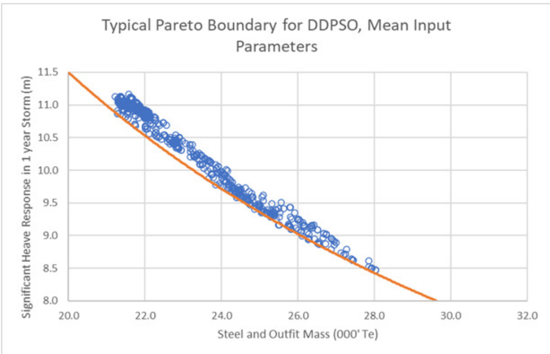

Figure A7.1. Typical Pareto Boundary for the DDPSO Case Study

The measure of Fitness used in the optimisation algorithm (Equation 2 in the main text of this paper) now includes the standard deviation in the design objectives, but with fixed weighting values (α,β,γ). The figure below shows a few discrete solutions for this RDO problem for γ=0.5, β=0.5, and weightings of α = 0.4, 0.5, 0.6 and 0.7.

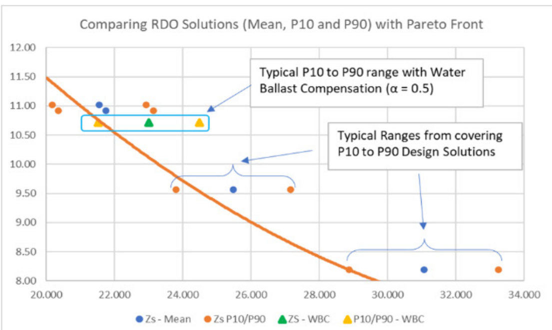

Figure A7.2. Comparison of RDO and Conventional Pareto Solutions (circles), and solution with water-ballast margin (solid triangles).

The first general observation is that the mean design solution for each of the four α values, now optimised to also minimise the standard deviation of the design objectives, falls short of the original Pareto front. This feature is common in RDO, i.e. that the penalty for robustness is loss of optimality (Jin & Sendhoff, 2003).

Key design requirements such as payload capacity, now also show statistical variability because, given constant displacement, they must be adjusted for equilibrium and stability.

To counter this compensating water ballast can be included. This is modelled by carrying out the RDO calculations for a range of compensating water ballast mass values, with only the difference between this and the changes in steel and outfit mass now appearing as variations in payload capacity.

Figure A7.3 shows how, by including this "designed in" margin, uncertainties in the coefficients C_1/C_2 can be managed, minimising their impact on topsides payload capacity by reducing its standard deviation.

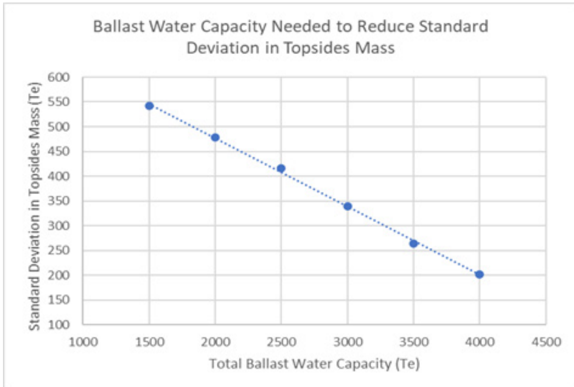

Figure A7.3. Variation in standard deviation of topsides payload capacity with increasing compensating water-ballast.

In summary, applying a conventional RDO approach does help minimise the variance in design objectives due to uncertain design and modelling inputs, but as observed elsewhere, at the cost of reduced optimality, for example, increasing steel weight and cost in the example shown here.

It also reflects the very real issue that statistical variability in design modelling introduces uncertainty in achieving key design requirements but also shows how this problem can be mitigated.

A8. SUMMARY OF FITNESS WEIGHTINGS

The following table summarises values and provides comments on the weightings (α,β,γ) used in the fitness ranking of the design objectives (Equation 2) before application of the target/penalty functions used to exert selection pressure within the NDSA genetic algorithm.

Table A8.1: Summary of Weighting Values used for each type of Robust Optimisation Approach.

Analysis Type		**Fitness Weightings**				
		β	**α**	**Comments**	**γ**	**Comments**
Deterministic		1.0	0.1 – 0.9	Set range to generate Pareto Front, variable to meet performance targets.	N/A	Not used in deterministic calculations
Intrinsic	RDO	0.5	0.25 – 0.75	User defined to give position on Pareto Front of interest	0.5	Generally equal balance between σ values
	IRDO	0.4	Variable	Included in search to match required performance level and alignment	Variable	User defined to give required balance in σ values
Extrinsic	RDO	0.6 – 0.75	Variable	To ensure alignment with horizontal line segment defining performance.	0.25	To give σ weighting toward design objective 2

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