

# MULTI-OBJECTIVE OPTIMISATION AND ITS APPLICATION TO CONCEPT DEVELOPMENT IN FLOATING OFFSHORE SYSTEMS

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## SUMMARY

This paper addresses the need for a rapid, multi-disciplined and rational approach to floating system concept development and selection during the very earliest stages of project definition. It describes the implementation of a modified multi-objective Genetic Algorithm for this purpose. A formulation of the NSGA-II algorithm is combined with additional Target Functions to reduce otherwise large multi-disciplined problems to more tractable solution using tools commonly available in the design office. It also provides a rational basis for the comparison of different design solutions each of which are Pareto Optimal with respect to the technical and economic performance of each underlying concept. A specific example of marginal field development using a novel FPSO concept is presented. Starting with just the oil field location and reserves estimate, the algorithm provides the means to define preliminary hull form and production facility capacities, match performance to payload, and give preliminary indicators of likely investment performance. The method may also be applied more generally in preliminary ship design, particularly where it is possible to model economic performance alongside efficiency, safety and key technical factors in hydrodynamics and structures.

## NOMENCLATURE

BM	Second moment of water-plane area/immersed volume
$C_1, C_2$	Constants relating Steel and Outfit Mass to Displacement
DDPSO	Deep Draft Production, Storage and Offloading
$F(t)$	Total reservoir fluids handling capacity (BOPD)
GM	Metacentric height (m)
$H_s$	Significant wave height (m)
IRR	Internal Rate of Return
$K_1, K_2$	Constants relating topsides mass to initial production rate $P_0$
KB	Height of the centre of buoyancy above keel (m)
KG	Height of the centre of gravity above keel (m)
$M_1, M_2$	Genetic algorithm objective functions
$M_{SOW}$	Steel and Outfit Mass.
P	Payload/Topsides mass (Te)
$P_0, P_R(t)$	Initial and time varying production rate (BOPD)
$P(S)$	Probability of crossover swap
$P(M)$	Probability of a mutation
R	Oil reservoir depletion rate (% vol/year)
RAO	Response Amplitude Operator
$R_{NPV}$	Net Present Value of field reserves (USD)
ROI	Return on Investment
$S, S_0$	GA fitness measures
TIC	Total Installed Cost
$T_N$	GA Target function
WACC	Weighted Average Cost of Capital
$Z_s$	Significant heave (double amplitude, m)
$\alpha$	Weighting for objective functions
$\Delta$	Hull mass displacement (Te)

Hull geometry definitions are given separately in Appendix A1.

## UNITS

MKS units are used throughout with the following additional accepted industry definitions:

bbl	International standard volume – Barrel.
BOPD	Production rate; Barrels of Oil Per Day
MM	Short form for millions, as in MMBBL
M	Short form for thousands, as in MBOPD
Te	Metric Tonnes

## 1. INTRODUCTION

### 1.1 STUDY BACKGROUND

Many naval and offshore engineering projects involve early development studies to select the most economically advantageous and technically feasible engineering solutions. Such concept studies are critical to success since good decisions made early save considerable time and effort later in the project life cycle.

For projects involving marginal economics, new technologies or safety challenges, it is essential that competing solutions are compared rationally, and on the basis that each alternative is, of its kind, “optimal” in terms of its potential performance, risks and returns on investment.

A recent industry survey for the Energy sector (Jamieson, 2020) points to a halving of the timescale involved in early stage engineering studies since 2015. This paper describes the development of a straightforward approach to multi-disciplined optimisation that meets the challenge of such reduced timescales and provides an improved basis to compare alternatives at the earliest stages of

concept selection before the commitment of resources toward major design effort.

## 1.2 TECHNICAL BACKGROUND

The use of formal mathematical methods in optimisation for the refinement of design performance is now well established across a wide range of engineering disciplines. Examples include drag reduction or structural weight saving, typically using CFD and FE analysis (Tahara, et al., 2006, Trapani, et al., 2012, Nasser, et al., 2014).

In ship design, there has been extensive work in multi-objective optimisation for hull performance, combining resistance, seakeeping, stability etc., taking advantage of advances in parametric modelling and CFD tools in hydrodynamics (Papanikolaou, 2010, Biliotti, et al. 2011, Guha, & Falzarano, 2015, Vasudev, et al., 2017, Maisonneuve, et al., 2018, Cheng, et al, 2019).

However, application to offshore engineering problems is relatively rare despite extensive early work in the field (Claus & Birk, 1996, Klanac & Jelovoka, 2008), although there are some recent signs of increasing interest (De Oliveira, et al, 2017, Nasser, et al, 2014, Pillai, et al, 2017).

Commercial analysis software now often features optimisation as part of parametric design, and stand-alone software platforms such as ModeFrontier™ provide a broad range of options in data analytics, integrated analysis and design optimisation. However, this can be an expensive exercise and requires high levels of design definition and significant computing resources.

Unlike the above advanced design applications, early stage concept definition starts with relatively little data and limited scope for detailed performance analysis. Requirements are often both multi-disciplined, and multi-objective, with significant potential for conflict in the absence of design detail.

Here this problem is addressed using a multi-objective genetic algorithm based on the Non-dominated Sorting Genetic Algorithm, NSGA-II, (Deb, 2001), but modified to include novel features aimed at managing complexities common to multi-disciplined early stage design problems.

Key objectives are: that the method used is simple and cost effective; that it can be implemented without the need for expensive engineering software and hardware; and that it can be performed efficiently and repeatedly to shorten the time-scale of a typical concept development or feasibility study (i.e. days rather than weeks or months).

The example chosen to test the approach is that of concept development for a marginal, stranded offshore oil field.

The economic viability of such oil and gas fields is often dependent on the choices made during the earliest stages

of concept selection and engineering. This issue becomes acute at the lower end of field size, for example for reserves between 25 and 75 million barrels of oil equivalent, and in a climate of low oil prices.

The suitability of small ship shaped FPSOs to operate in harsh environments, particularly in the Northern North Sea or West of Shetland, is often debated. The problem is therefore to find concepts that balance the conflicting pressures of low capital and operating costs, safety and hydrodynamic performance, and also meet specific design targets and constraints.

A working example is presented based on the Atkins Deep Draught Production, Storage and Offloading (DDPSO) floating system. This provides a clear demonstration of how genetic algorithms can be used to drive the preliminary definition of a novel hull form subject to multiple constraints and objectives, and so enable rational comparison with traditional alternatives.

## 2. THE DDPSO CONCEPT

Figure 1 illustrates the DDPSO concept used in this study. The design intent is to provide a small, low CAPEX FPSO suitable for marginal field developments in relatively harsh environments.

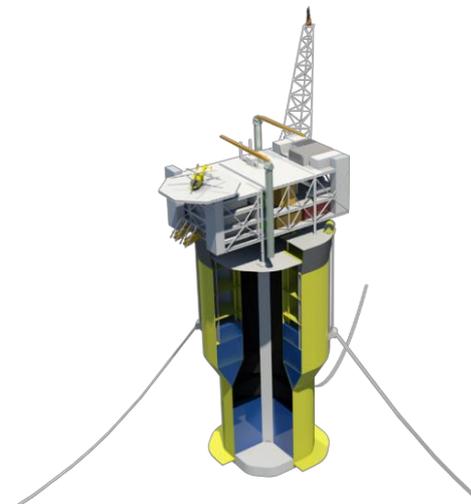


Figure 1: Atkins DDPSO Concept (GB Patent 2507370, US Patent 98228072).

The hull form is “SPAR-like” in appearance, and certainly the aim is to gain the benefit of deep draught with respect to motion response. However, unlike SPARs, intact stability is derived largely from the second moment of the water-plane (i.e. BM), with a hull form and distribution of mass such that the centres of gravity (KG) and buoyancy (KB) are practically co-incident. The use of a so-called “oil-over-water” crude oil storage tank as shown in Figure 1, ensures that there are only small variations in KG and KB, and displacement throughout operation. This removes the need

for an extensive water ballast system, giving savings in steel mass and outfit costs.

Consequently, this hull form is quite specialised, with a specific balance of dimensions needed to give its desired characteristics (i.e. a minimum practical GM and a high heave natural period (~ 20s)). It is therefore a good candidate for demonstration of the methods described in this paper.

### 3. THE OPTIMISATION ALGORITHM

#### 3.1 OVERVIEW AND CHOICE OF GA

The optimisation technique used here is based on a genetic algorithm (GA), wherein design solutions are found through the repeated generation and testing of discrete populations by simulation. Only the fittest survive each testing phase, to be used as the “parents” of each new generation. Design solutions evolve to a point where each new population cannot improve on the performance of the previous one.

Such so called “Evolutionary Algorithms” were originally developed by Holland (Holland, 1992) and others (Deb, 2001, Konak, et al, 2006), and have seen wide application in engineering and design as noted earlier.

The NSGA-II algorithm used here was chosen because:

- It adheres closely to the simple but fundamental principles of evolutionary algorithms, and so may easily be implemented or programmed.
- It works with primary variables, rather than constructing binary code representations of gene sets.
- It utilises simple scoring and ranking, and the concept of an Elite within each population, which aligns with the practical engineering need to “converge” on a clear solution.
- Nevertheless, it can be shown to enhance the fitness of populations more generally, ensuring that the parents of each generation also approach Pareto status.
- It appears able to work with relatively small populations and has rates of convergence that are reasonable and reliable.

Alternatives to NSGA-II, applications and comparative performance may be found in Konak, et al, 2006, and more recently, Sobey, et al. 2019. An overview of the key elements of the method as applied here is given below, with further added detail where relevant in later sections.

#### 3.2 FITNESS

Two primary performance objective functions are used here, namely:

- $M_1$ : A measure of first order motion response in a one-year maximum sea-state ( $H_S$ ).
- $M_2$ : A suitable measure of weight, cost, or return on investment.

The DDPSO hull form is symmetric in the X-Y plane and so critical modes of motion are surge, heave and pitch. For SPAR type platforms, surge (when moored) and pitch modes have high natural periods, well above any wave spectrum energy content. This is not the case for heave, and it is a fundamental requirement of the optimisation process to de-tune this natural period, and overall response, from the peak of the wave spectrum. Therefore, the ratio of significant heave to significant wave height ( $Z_S/H_S$ ) is chosen as the measure  $M_1$  to be minimised.

$M_1$  is calculated for each design variant using a first principles calculation of its heave response amplitude operator (RAO) derived from geometry and weight models, and a suitable wave spectrum for a given significant wave height ( $H_S$ ).

$M_2$  needs to be a performance measure that is at conflict with  $M_1$ , and typically, steel weight, capital cost or ROI are all suitable candidates.

These two objective functions are combined into a single measure of fitness given by:

$$S_0 = \alpha.M_1 + (1 - \alpha).M_2 \quad (1)$$

Where the weighting  $\alpha$  is varied from 0 to 1 through a sequence of simulations in order to map out the Pareto Front. The value of  $\alpha$  may be allowed to vary about a defined mean following a normal distribution with a small standard deviation. This helps increase variation in the population and, as the solution converges, supports the generation of parents that are close to the Pareto Front.

To provide consistent scaling both  $M_1$  and  $M_2$  are normalised by their minimum values in each population. Thus,  $M_1$  and  $M_2$  become measures of the relative performance of each design and scaled such that  $S_0$  will always tend to unity for the best solution within a population.

Equation 1 can be expanded to take on multiple objective functions,  $M_N$  ( $N = 2, 3 \dots$ ) involving  $N-1$  weights ( $\alpha_{N-1}$ ). However, this increases computational effort significantly and introduces unwanted levels of complexity for this early stage of design and feasibility assessment.

This problem can in part be addressed through the introduction of targets that meet specific design requirements. The measure of fitness ( $S_0$ ) is weighted with additional Target functions ( $T_N$ ) such that the overall fitness measure of each design is expressed as the product:

$$S = (T_1.T_2 \dots T_N).S_0 \quad (2)$$

The general form of Target function ( $T_N$ ) as applied to a typical target parameter ( $X_N$ ) used here is:

$$T_N = (1 + |X_N - T| / T)^{(1+m)} \quad (3)$$

As  $X_N$  tends toward the target value  $T$ ,  $T_N$  tends to unity, regardless of whether  $X_N$  is greater or less than  $T$  initially, and thus  $S$  tends to  $S_0$ . Here, the power  $m$  has also been introduced as a factor to modify the relative influence of the Target constraint.

Typical target values for the DDPSO are measures such as topsides payload, cargo capacity and BM. However, targets can also be set that account for the multi-discipline nature of the problem, such as the total crude oil reserves that can be extracted, offloading frequency (to match storage capacity with production rate), etc. examples of which are given in later sections.

### 3.3 HULL PARAMETRIC MODEL

A simplified parametric model for the hull and process facility is used to create each generation of designs. This consists of:

1. A preliminary set of dimensions as described in Appendix A1, which form the so-called “Genes” of each design.
2. A set of remaining calculated dimensions that complete the definition of the hull form.
3. A mass and centre of gravity model for hull steel and outfit.
4. The payload model – representing the topsides process facility and its capacities, production rate and relationship to reservoir characteristics.

Each population of designs is initially generated from random variations in selected key dimensions from a baseline geometry. These key dimensions are tabulated in Appendix A1. The range of variation used was typically  $\pm 15\%$  of the baseline, applied both in the generation of the initial population, and in subsequent mutation operations.

All other dimensions, weights, payload and capacities are derived using a mix of geometrical relationships and design correlations.

For the DDPSO parametric model there are 4 critical mass groups, namely:

- a) The hull steel and outfit mass and centre of gravity.
- b) The cargo mass and centre of gravity.
- c) The topsides process facility (payload) and its centre of gravity above the deck.
- d) The solid fixed ballast added at the keel.

The payload capacity and solid ballast mass for each of the generated hull forms are calculated for required values (or range) of GM. In other words, each hull form that is randomly generated has a unique combination of GM, payload and fixed ballast mass.

The payload capacity of each hull form and the payload requirement (derived from a simplified process facility model) are used as one of the target functions (Equation 3). The hull steel and outfit mass ( $M_{SOW}$ ) is calculated using a design correlation as described in Section 4.1 below.

The models used to represent process facilities, reservoir and field economics that make up fitness parameter  $M_2$  are case dependent and also described in section 4.1.

### 3.4 HYDRODYNAMIC MODEL

The heave response amplitude operator (RAO) is calculated from first principles and subsequently used to generate a heave response spectrum for the specified wave climate. This is then integrated numerically to calculate significant heave ( $Z_s$ ) in the usual way.

Direct use of a 3D boundary element or panel diffraction model to calculate the heave RAO would lead to an excessive computing requirement (of order 50,000 geometries may be needed) inappropriate for early stage concept design.

For optimisation studies, alternatives do exist including various forms of surrogate or so-called Metamodels using diffraction analysis results, CFD or even experimental data on a much reduced but still representative sets of geometries (Viana, Simpson, et al., 2014, Harries, Papanikolaou, et al. 2017). Such methods would be less time consuming than 3D diffraction calculations but would still represent significant effort at the earliest stages of concept development.

The approach taken here therefore, was to apply well established empirical models and assumptions for heave added mass and damping coefficients, with Froude-Krylov and inertial wave loads integrated analytically for the relevant wetted surfaces (Newman, 2017, Sarpkaya & Isaacson, 1981). Data for the additional effect of heave plates on inertia and damping were also used (Tao, Molin, et al. 2007, Thiagarajan, et al., 2002). However, to reduce complexity and ensure consistency through the optimisation process, a simple linearised damping ratio of 0.1, consistent with the above for the relatively high sea-states and large Keulegan-Carpenter numbers chosen as a basis for optimisation, was applied.

Additional hydrodynamic constraints have been applied to avoid Mathieu instabilities. These cover both, the pitch period being an integer multiple of the heave period, and the envelope between the heave natural period and the wave peak period giving a second order excitation in pitch (Haslum, 2000).

The significant heave response for a one-year maximum significant wave height ( $H_s$ ) of 10m, modal period 15.85s, (SMB spectrum) was used here as the basis of

measure of fitness  $M_1$ , but this could be extended to cover more detailed descriptions of environmental statistics.

### 3.5 THE ALGORITHM

The overall algorithm consists of the following stages:

1. Generate the starting population of design geometries by random variation (see Appendix A1).
2. Calculate their fitness parameters,  $M_1$  and  $M_2$ , and Target functions ( $T_N$ ), to give the  $\alpha$  weighted fitness measure  $S$  (Equations 1 and 2).
3. Select a group of so-called “Elite” solutions based on the minimum value of  $S$  from the population of possible geometries.
4. Generate the next population using a mix of genetic algorithm operators (described below) from this elite group.
5. Calculate the fitness parameters of this new population and repeat this process from step 3 until convergence is achieved.

This process is repeated for a range of  $\alpha$  values to create the Pareto Front describing the boundary of design solutions with the best possible combinations of measures  $M_1$  and  $M_2$ .

The initial Elite group (step 1) is made up of 8 design solutions selected as the best (i.e. lowest value of  $S$ ) from 8 populations of 128 randomly generated geometries each. This variation on the standard NSGA-II approach was introduced to help pre-condition each initial parent group within the design space, minimise the risk of false minima, and improve starting population quality.

These 8 Elite designs are then paired off as parents and used to generate 4 new sub-populations of 32 new candidate solutions, i.e. a single new population of 128.

Objective functions  $M_1$ ,  $M_2$  and fitness  $S$  are re-calculated for this new population and ranked by minimum  $S$  values.

The design with the lowest  $S$  value is the new single Elite for this population and is retained. A further 4 design geometries are selected randomly from the next best 20 performing members of the population. The new Elite design is then paired with each of these 4 geometries to generate the next 4 sub-populations of 32, i.e. the next full generation of 128 designs. This process is repeated until convergence is achieved.

The generation of each new population from the parent pairs is achieved using the two primary GA operators, Crossover and Mutation. The principle dimensions of the DDPSO hull form (Appendix A1) are used to form the so-called “Gene set” to which the GA operators are applied.

The Crossover operator involves the random swapping of elements of the Gene set between each parent pair. This is the so called “uniform crossover” approach, chosen for simplicity and practicality. The probability of a swap  $P(S)$

is fixed, (typically 0.3 to 0.5) and crossover controlled by generating a random number  $R$  between 0 and 1, such that if  $R < P(S)$ , then a swap is made.

For each parent pair, this is repeated 7 times, to create 16 new sets of design dimensions, including the original parent pair.

The Mutation operator applies random variation to the above new set of 16. The probability  $P(M)$  of whether a mutation occurs is handled in the same way as for Cross-over. The range of possible variation in each dimension is a set fraction of its current value, and the actual variation is calculated as a random proportion of that fraction.

The 50/50 split between Crossover and Mutation was arrived at through numerical experiment but is typical of similar work (Pillai, et al., 2017). It appears that Crossover provides the initial drive to the overall improvement in performance of the population, but then Mutation takes over to create variations that test whether the Elite solution is optimal.

The scoring and subsequent ranking of each design was based on both the fitness measure  $S$  (equations 2 and 3), and some specific additional weights used to eliminate undesirable solutions. These include for example, those that violate the Mathieu instability criteria, or hull geometries that have unfeasible payload capacities. Such cases are simply assigned an additional weighting factor in the calculation of their overall performance score which has the effect of ranking them at the bottom of the population. This is a similar philosophy to the Penalty Function approach to constraints (Klanac & Jelovoka, 2008, Campana, et al., 2012), but much simplified.

Convergence is assumed to have occurred when:

- The same design geometry is repeatedly retained over 10 generations and has evolved such that both its  $M_1$  and  $M_2$  measures are ranked highest.
- Certain tolerances are met with respect to the Target values used. A value of 1.0% of the Target was used in all calculations presented here.

## 4. CASE STUDIES

The following case studies are aimed at showing how the algorithm described here selects principle hull dimensions and process system characteristics to produce a consistent Pareto boundary for a range of different working scenarios.

### 4.1 GENERAL RELATIONSHIPS

For concept development simple but consistent relationships between design variables provide a reliable basis for preliminary hull and systems definition.

For this study, hull and outfit steel mass for past designs based on scantlings derived from Codes of Practice were used as reference benchmarks for a simple design model:

$$M_{SOW}/\Delta = C_1.(1 + C_2. (T-T_R)/T_R) \quad (4)$$

Where:

$C_1$ : is the ratio of steel and outfit mass to total mass displacement ( $\Delta$ ) at reference draught ( $T_R$ ).

$C_2$ : is an additional factor to account for variations in steel mass with draught ( $T$ ).

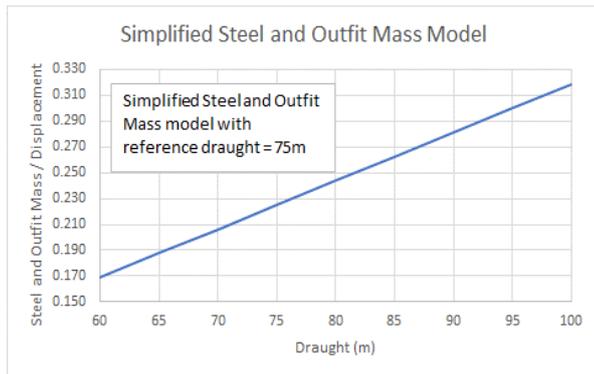


Figure 2: Simplified model for steel and outfit mass.

Reference values of  $C_1 = 0.225$ ,  $C_2 = 1.25$  and  $T_R = 75m$  were selected. The factor  $C_2$  accounts for an increasing shell thickness with draught to maintain a consistent compression stress due to self-weight across the designs. It has an important role in counteracting trivial solutions that otherwise exhibit ever increasing draught to minimise heave response.

Conventionally, a combination of reservoir modelling, exploration well testing and process design drives specification of topsides systems, with hull selection based on resulting weights and oil storage requirements. Here, a simple empirical model is used to match reservoir and process design with hull definition, relating initial daily production rate ( $P_0$ , in Mbbbl per day) to payload ( $P$  - Tonnes), and crude oil storage requirements:

$$P = K_1 + K_2.P_0 \quad (5)$$

Where  $K_1$  is a constant representing a minimum fixed mass of topsides steel (in Tonnes), and  $K_2$  linearly relates process facilities mass to production rate (Tonnes per bbl). Values of  $K_1 = 2000Te$  and  $K_2 = 0.15Te/bbl/day$  were used throughout these studies. Crude oil volume storage requirements were subsequently calculated using an assumed year 1 offloading frequency of once every 15 days throughout.

We also seek to optimise the system for a specific reservoir size and life-of-field. Therefore, a simple exponential decay in oil production with time with reservoir depletion rate  $R$  is assumed:

$$P_R(t) = P_0.e^{-Rt} \quad (6)$$

For a given value of depletion rate  $R$ ,  $P_0$  can be found by equating (6), integrated with respect to time, to a target field size. In practice, depletion rates need their own detailed reservoir modelling and optimisation (e.g. Barnes *et al.*, 2002, Hoffmann *et al.* 2019), but this simple approach was considered sufficient for design comparison purposes.

The net present value (NPV) of the crude oil extracted over the life of field is found using equation 6 factored by the weighted average cost of capital (WACC) and again integrated analytically.

The NPV of the field increases with increasing initial production rate (oil extracted and sold now is more valuable than oil extracted in the future). However, increasing  $P_0$  leads to an increased payload and storage requirement, driving up the CAPEX of both hull and topsides. This gives one of the critical competing factors within the optimisation algorithm.

An additional, practical factor for process system definition is the increasing proportion of water that makes up the composition of well fluids, from which oil and gas are separated, through the life-of-field. This so-called “water-cut” is a key driver in the design of the topsides separation systems, and later life revenue generation.

The simple model for  $P_0$  used here is therefore based on:

$$P_0 = \text{MAX} [ P_{oil} (t=0), F_{(oil+water)} (t = T_{max}) ] \quad (7)$$

There is an optimal combination of depletion rate  $R$  and water-cut for which  $P_{oil}$  at  $t=0$  and  $F_{(oil+water)}$  at  $t=T_{max}$  are equal, i.e. the initial production capacity is matched to the through-life fluids handling capacity  $F(t)$ , and so represents a minimum payload requirement.

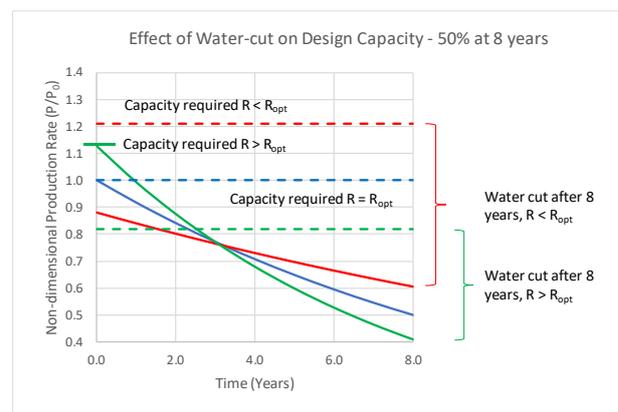


Figure 3: Simplified illustration of the effect of including water-cut on sizing process system capacity (Water-cut of 50% equivalent to a 50/50 composition of oil and water process fluids after 8 years).

Whether this represents the best return on investment however depends on whether the rate of change of

CAPEX and OPEX with  $P_0$ , payload and storage capacity, outstrips the rate of change in NPV of the reserves recovered.

The following Case Studies feature increasing levels of complexity as follows:

**Case 1:** Matching hull and topsides according to set target values for storage volume and payload, with the objectives being to minimise response in waves and steel mass (hull + outfit).

**Case 2:** Matching of hull and topsides for a given target level of recoverable reserves and fixed life-of-field, to minimise response in waves and maximize the return on investment on the Total Installed Cost (TIC).

**Case 3:** Matching of hull and topsides for a given level of recoverable reserves and life-of-field, to minimise response in waves and maximise return on total through-life cost expressed as an Internal Rate of Return (IRR), and including the effect of water-cut.

For all cases, the fitness measure  $M_1$  is the non-dimensional significant heave ( $Z_s/H_s$ ). The definition of  $M_2$  varies according to the Case definition as will be described in each section. For Cases 2 and 3, a fixed oil field reserves target of 50.0 MMbbl recovered over a period of 8 years, at an oil price of \$50.0 per barrel, and WACC of 8% was used.

#### 4.2 CASE 1

Case 1 considers only hull form and sizing for target values of payload and storage capacity. The target values considered are:

Table 1: Summary of Case 1 Target Parameters.

Case	Label	Payload (Te)	Storage (Mbbl)
1.1	250CT4	4000.0	250.0
1.2	300CT5	5000.0	300.0
1.3	350CT6	6000.0	350.0

The calculations follow the methodology described in Section 3. The Pareto front describing the design the best possible combinations of heave responses versus steel and outfit mass, is generated by successive runs across values of weights  $\alpha$  from 0.1 to 0.9.

Figure 4 shows a “snapshot” of the heave RAO for a typical population of designs from the GA simulation for Case 1.3.

Figure 5 shows the results from each of the cases 1.1, 1.2 and 1.3. Each data point represents a specific design combination of hull dimensions for the final parent populations for each set of  $\alpha$  values. A lower bound line has been fitted to illustrate the expected Pareto front in each case.

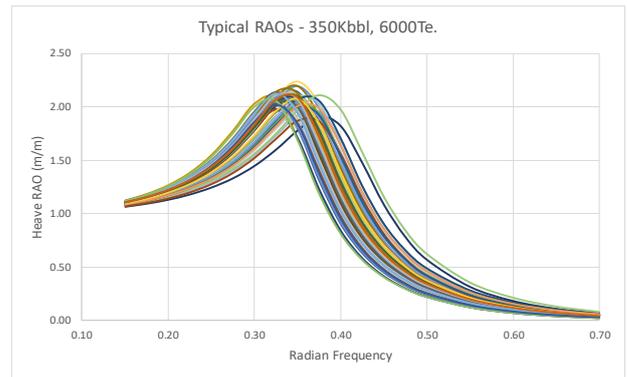


Figure 4: Typical heave response amplitude operators for a single population

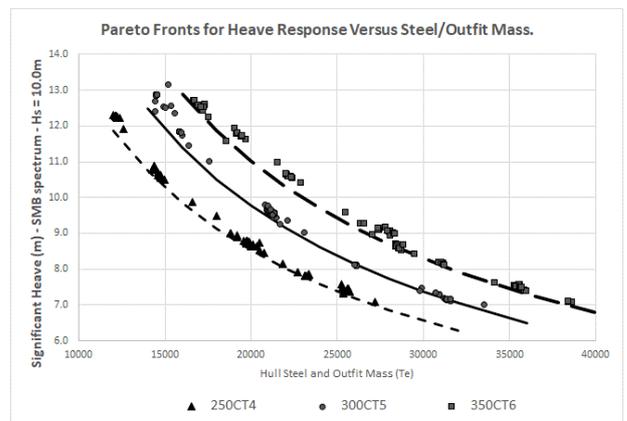


Figure 5: Pareto front solutions for Case 1.

The general shape of the Pareto front follows the expected convex form. There is some variation which is attributed to the 1% Target Function convergence criterion used being not quite sufficient to eliminate all scatter.

The emergence of two different water-plane configurations is also observed, i.e. populations that are (see definitions in Appendix A1):

- exactly octagonal, i.e. dimensions  $a_1 = a_2$  and  $b_1 = b_2$  - Type A.
- those for which  $a_1 > a_2$ , and  $b_1 > b_2$ , tending toward larger water-plane areas and second moments, and shallower draughts – Type B.

Type A populations are most prominent for high  $\alpha$  and deeper draughts, hence lower heave response, and low  $\alpha$  Type B configurations have lower steel and outfit mass but higher response.

#### 4.3 CASE 2

In Case 2, we seek the Pareto front that represents the optimal combinations of hull sizing to give minimum significant heave versus Total Installed Cost (TIC).

The explicit steel and outfit mass objective function is replaced by a measure of the TIC based on the sum of the

hull and topsides CAPEX, mooring system and installation costs. The former is calculated using a simple cost per tonne norm based on steel and topsides fabrication costs (\$7,500 and \$45,000 per Tonne respectively), and the latter is combination of fixed and a variable cost proportional to displacement.

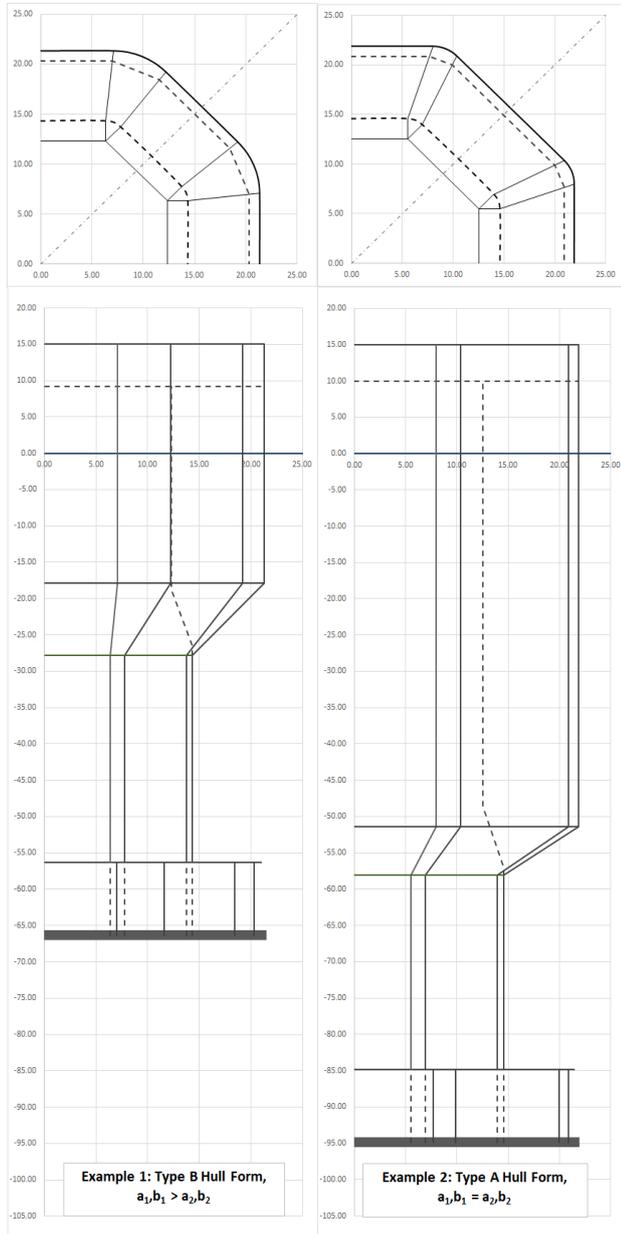


Figure 6: Examples of hull form variation with  $\alpha = 0.2$  (left) and  $\alpha = 0.7$  (right).

Equation 5 is used to equate year 1 oil production rate to payload, and an offloading frequency of 15 days selected to give the crude oil storage capacity. The depletion rate (R) is randomly generated for each design and, along with the year 1 production rate, is used in Equation 6 integrated to give a life-time production volume and its target factor ( $T_N$ ) from Equation 3.

The fitness measure  $M_2$  is calculated as the Total Installed Cost divided by the NPV of the crude oil revenue ( $R_{NPV}$ ) earned at the production capacity and depletion rate (R) of each design, and so drives the solution towards the lowest TIC for the largest possible crude oil revenue.

GA simulations follow the process as described before, with the Pareto front generated for a range of values of  $\alpha$ . Figure 6 shows the typical variation from low  $\alpha$  (left) to high  $\alpha$  (right), type B and A hull forms respectively, as identified in Case 1.

Table 2 summarises properties of each example design configuration for comparison.

Table 2: Example properties of design configurations generated at low and high  $\alpha$ .

Property	Example 1	Example 2
Displacement	68,290 Te	118,890 Te
Steel/Outfit	13,720 Te	36,730 Te
Payload	4,930 Te	5,755 Te
Draught	66.3 m	94.9 m
Cargo Capacity	289,950 bbl	378,640 bbl
Heave Response	12.5 m	6.6 m
Production Rate	19,950 BOPD	25,033 BOPD
Depletion (R)	3.6%	9.6%
Oil Revenue	\$1878 Bn	\$1980 Bn
TIC	\$374 MM	\$ 608 MM

Figure 7 shows the trend across the range of  $\alpha$  values for depletion rate (R) and initial production rate ( $P_0$ ) for converged elite solutions in each case.

Converged (Elite) design solutions and their approximate Pareto Fronts are shown in Figure 8 for TIC and steel and outfit mass respectively. Each black square represents an Elite solution at the end of the simulation, accompanied by previous iterations (grey triangles).

Solutions for low  $\alpha$ , that are weighted toward generating designs with lowest total installed cost, tend toward lower year 1 production rates and reservoir depletion rates. Although they generate lower NPV from the 50MMbbl of reserves, it remains sufficiently high to result in a lower ratio of total installed cost to oil field revenue (TIC/ $R_{NPV}$ ).

Therefore, according to this analysis, the optimum solution that gives the best overall return on initial capital cost should be that which spreads production across the full life-of-field, rather than that which seeks a high year 1 production rate, higher NPV, and fastest time to break-even. However, the penalty can be seen clearly in the significantly larger heave motions that the smaller platform suffers in the chosen environmental conditions.

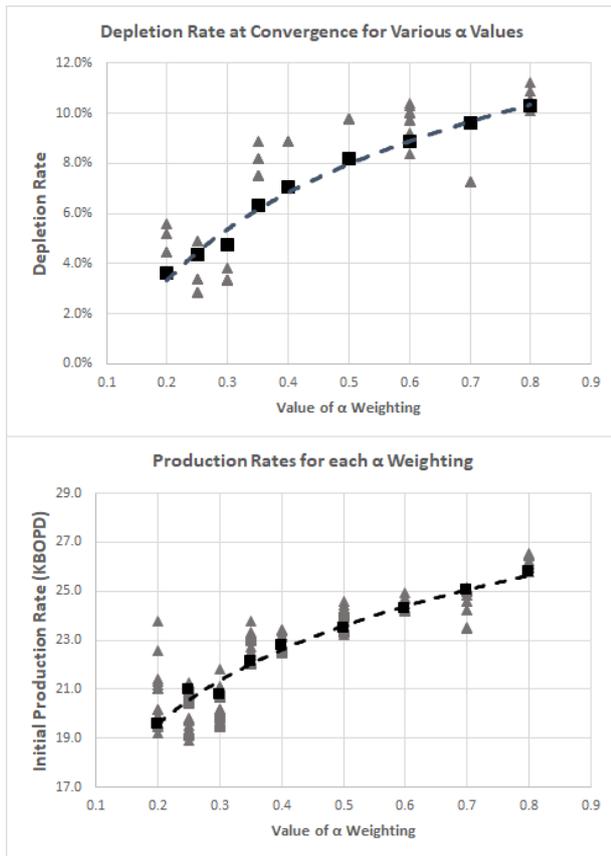


Figure 7: Trends in Depletion Rate (R) and Production Rate with Weighting  $\alpha$  (Converged Values as black squares)

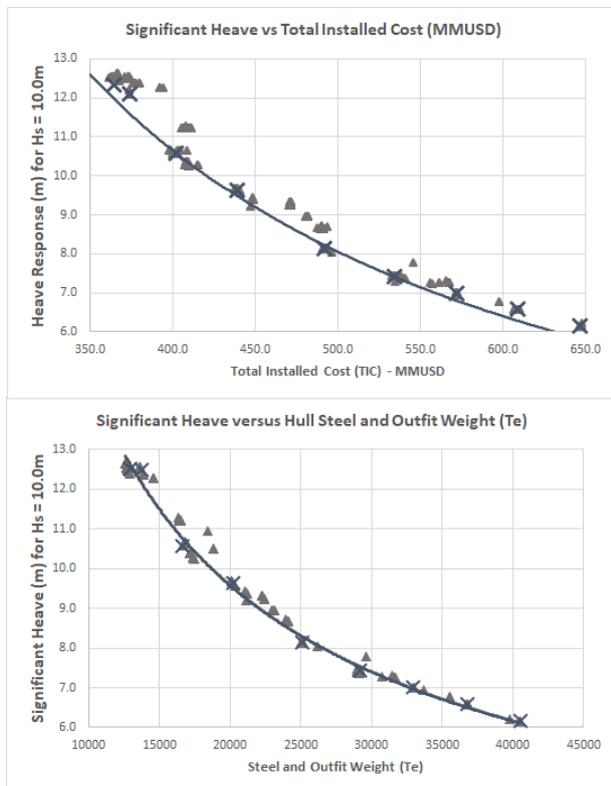


Figure 8: Trend and approximate Pareto Front for Heave Response versus Total Installed Cost and Steel and Outfit mass with Elite solutions highlighted (crosses).

#### 4.4 CASE 3

In Case 3 we seek the Pareto front that represents the optimal combinations of hull sizing to give minimum significant heave versus a through-life return on investment expressed as an Internal Rate of Return (IRR).

There are two additional factors in this case. The first is the introduction of the operating cost (OPEX) into the field economics. Here, we define the operating cost ( $O_{NPV}$ ) as the sum of a fixed percentage of the capital cost and a variable lifting cost per barrel at year 1 production rates, adjusted for Net Present Value. The fitness measure  $M_2$  is therefore now calculated as:

$$M_2 = TIC / (R_{NPV} - O_{NPV}) \quad (8)$$

The second is the inclusion of water-cut. Equation 7 is used alongside an assumed water-cut of 50% at year 8. This implies an optimum topside mass (Payload) using equation 5 equivalent to the total through life liquid processing capacity and depletion rate R.

The question of interest is whether the design solutions generated will converge around a process facility that has this local optimum in payload and processing capacity, for all  $\alpha$  values along the Pareto front, or whether it will behave as Case 2. Two sets of calculations have therefore been performed:

- with R set at 8.66% per year, the optimum payload condition,  $P_0 = 23,750\text{BOPD}$ .
- With R allowed vary randomly (as in Case 2).

Figure 9 shows the results of these combined calculations.

Results are presented grouped into three distinct populations as they emerged from the simulations. The first (Non-Optimal Depletion rate - NODR), are those that converged to a solution that had a depletion rate significantly at variance with the optimal value of 8.66% per year. The next two populations are those that evolved either from the benchmark simulations or converged to within 1% of the optimal value of depletion rate R.

These two populations were further categorised into so-called Type A ( $a_1, b_1 = a_2, b_2$ ) and Type B ( $a_1, b_1 > a_2, b_2$ ) geometries that were observed in the Case 1 studies.

Through repeated calculations, it was observed that the Pareto Front for heave performance versus IRR is comprised of populations that align with both the optimal depletion rate  $R_{OPT}$  of 8.66% and are of the Type A geometry.

Plots of significant heave versus steel and outfit mass, and TIC, show some similar trends, but with more scatter around the best fit trend-lines through the data points representing the Type A Pareto population. Unlike the plot for IRR, (where all non-Pareto solutions fall clearly above the line), there is more scatter of the various converged designs around the best fit curve to the Type A

designs. In other words, designs that are Pareto for heave response versus IRR may be neither least steel and outfit mass nor least Total Installed Cost.

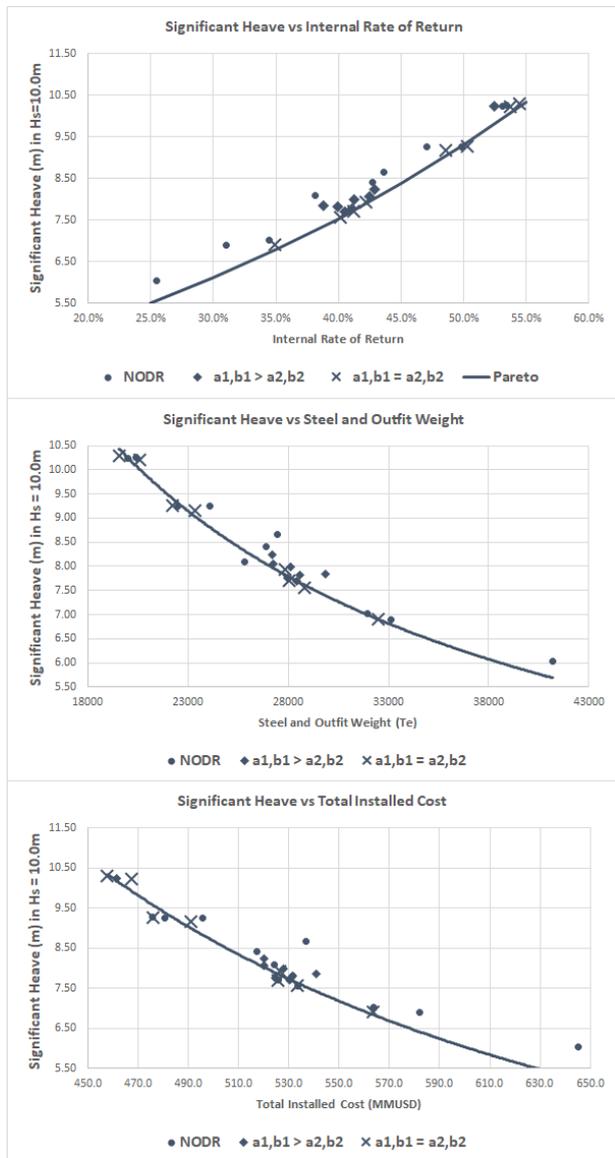


Figure 9: Pareto fit to Heave vs IRR and best fit curves to Steel & Outfit Mass and TIC.

Table 3: Properties of Pareto solutions across all case studies at a nominal significant heave of 8.0m

Design Parameter	Case 1: 250CT4	Case 1: 300CT5	Case 1: 350CT6	Case 2	Case 3
Significant Heave (m)	<b>7.92</b>	<b>8.11</b>	<b>8.14</b>	<b>8.13</b>	<b>7.93</b>
Displacement (Te)	81,465	92,079	108,106	98,105	101,935
Cargo Capacity (bbl)	251,204	300,699	349,983	334,882	361,383
Steel/Outfit Mass (Te)	<b>22,699</b>	<b>26,091</b>	<b>31,155</b>	25,092	27,844
Payload (Te)	4,022	4,992	5,993	5,321	5,593
Production Rate (BOPD)	<i>13,480</i>	<i>19,950</i>	<i>26,620</i>	22,140	23,955
Total Installed Cost (\$MM)	<i>406.9</i>	<i>493.1</i>	<i>572.3</i>	491.7	526.5
Reservoir depletion rate (R)	-	3.83%	11.30%	6.70%	8.66%
Reservoir NPV (\$MM)	<i>(&lt;50MMBBL)</i>	<i>1920.0</i>	<i>2002.0</i>	1970.0	1975.0
Ratio: RNPV/TIC	<i>N/A</i>	3.83	3.50	<b>3.91</b>	3.75
IRR (%)	<i>N/A</i>	40.7%	40.3%	44.1%	<b>42.2%</b>

## 5. DISCUSSION

### 5.1 RESULTS

The three case studies presented in this paper illustrate a range of ways in which genetic algorithms may be used to establish optimal combinations of hull geometry and process facilities definition in concept design.

For Case 1, using fixed targets for the DDPSO storage capacity and topsides mass (and so notionally unrelated to each other) the GA generates clearly defined Pareto fronts for heave response versus steel and outfit mass.

For Case 2, allowing topsides mass and storage capacity to vary according to random variation in the reservoir depletion rate, the lowest TIC is associated with lowest reservoir depletion rates and initial production rates (and therefore topsides mass/cost, and oil storage requirement). The low-cost penalty is increased heave response.

For Case 3, with additional factors included for the effect of water-cut and operational costs, the Pareto Front is formed by designs for which the optimal depletion rate defines the minimum topsides mass. Pareto designs also favour a regular octagonal waterplane, Type A hull forms ( $a_1, b_1 = a_2, b_2$ ), which feature deeper draughts, and improved heave performance.

Design solutions that give a response of around 8.0m significant heave have been selected for comparison as might typically be considered during a concept design review and are summarised in Table 3. Figures given in italics are post-processed values to aid comparison, those in bold are the specific parameters for which each case was optimised (i.e.  $M_2$  objective).

Exact comparison is not strictly possible as there are some subtle differences in the formulation of each case. For example, simulation 250CT4 yields a production rate that would not generate 50MMbbl over 8 years as indicated.

Nevertheless, there are some interesting observations that can be made across these different design solutions.

Firstly, minimising hull steel and outfit mass alone (Case 1 studies) is less effective than optimisation with respect to the ratio of reservoir NPV to Total Installed Cost (Case 2) in maximising IRR. This confirms the benefits of using a multi-disciplined approach over otherwise fixed combinations of payload and storage capacity.

Next, as noted earlier, the effect of modelling total fluid processing capacity in Case 3, as opposed to simply oil production rate (Case 2), creates a local optimum in payload and production rate for the genetic algorithm to search for.

At first sight however, Case 2 appears to generate a better Internal Rate of Return for a lower production rate, topsides mass and reservoir depletion rate than Case 3.

However, this direct comparison is misleading. If the fluids handling capacity for a 50% water-cut in year 8 were to be specified for Case 2, its payload capacity would need to increase by around 500Te, with increasing fixed ballast and hull displacement accordingly, pushing up capital cost and reducing IRR.

This demonstrates that the proposed Target Function approach correctly models the effect of including water-cut, reducing IRR in comparison with designs for which TIC is targeted, but capturing the optimum depletion rate for minimum topsides mass. It therefore offers a good alternative to increasing the number of objective functions and complexity in multi-disciplined optimisation problems.

Finally, solutions achieved with depletion rates that are higher than the local optimum (e.g. Case 1, 350CT6), always lead to higher TIC and lower IRR, regardless of their capacity to generate higher net present value from the available reserves.

## 5.2 COMMENTS ON APPLICATION

The application of optimisation tools as described in this paper uses some key design correlations and simplified relationships to both drive solutions to convergence and link together the different disciplines.

The form of these models (e.g. Equations 4, 5 and 7) may be derived either from historical design data or separate benchmark studies.

For example, the process system mass model, relating production rate to payload, may be derived from preliminary topsides facilities studies for which well-established commercial tools are available. Similarly, estimates of steel-mass to minimum scantlings, according to offshore Codes of Practice, with suitable margins form

a good starting point for the hull where historic “as-built” data might be unavailable or sparse.

In practice, all such relationships are subject to uncertainties. These can arise from the statistics of underlying databases, or changes to design definition as the project evolves, i.e. so called “growth”, as each discipline progresses in level of detail.

Sensitivity studies are therefore essential to decision making. Similarly, the benefit of maintaining extensive design and “as-built” databases from which to extract design correlations is clear. Modern AI tools and the growing development of Metamodels (Li, *et al.*, 2008, Viana, *et al.*, 2014) should enable better exploitation of such data in optimisation.

The application of formal methods of robust design that include such modelling uncertainties (e.g. Diez, *et al.*, 2010), is worth further investigation if an efficient approach can be developed that is consistent with the key objective here of simplicity in application.

The methods described here are equally applicable across all ship and floating system preliminary design so long as suitable sets of design correlations that link the key disciplines can be developed. Alternative performance measures such as efficiency and risk may easily be included as either primary Objective Functions ( $M_1$ ,  $M_2$ ) or Target Functions ( $T_N$ ).

## 5.3 THE ALGORITHM

The genetic algorithm used here includes key features that should prove useful in other applications, including:

- The NSGA-II algorithm structure provides an effective approach to optimisation in early stage concept development that is straightforward to implement as part of preliminary design definition.
- The use of Target functions as an alternative approach to full multi-disciplined optimisation, reduces computational effort, and generates additional selection pressure in the evolution of design solutions.
- The initial definition of the parent group, by selecting the first 8 elite from 8 randomly generated populations, reduces the number of generations needed for convergence.
- The selection of a single Elite design solution from each population as a common parent for each new generation is well suited to concept design applications where there is a need for rapid convergence and testing of ideas.

Typically, solutions that might be close to Pareto would emerge within 10 to 20 generations from the initial parent group, albeit that up to 40 generations were used in practice to ensure consistency.

There was an occasional tendency for simulations to get “stuck”, the prime cause of which appeared to be conflict between Target Functions. The power law  $m$  in Equation 3 proved an effective way to modify the ranking of solutions and generate local variation in the population to solve this problem. Similarly, the use of random, bounded values for weights  $\alpha$ , was successful in creating variation in the population close to the Pareto front and again helped avoid such issues.

Finally, it should be noted that this algorithm has been implemented in nothing more complex than an Excel workbook. This demonstrates that the method is well suited to the earliest stages of concept development offers significant benefits in cost and ease of application.

## 6. CONCLUSION

The application of a simplified genetic algorithm has been shown to be well suited to concept development or feasibility studies in multi-disciplined floating systems projects. The underlying approach of combining of both fitness and target measures from different disciplines into a single ranking as described here, simplifies what might otherwise be a significantly more complex multi-objective problem.

This study therefore demonstrates that the benefits of formal optimisation techniques are not just limited to advanced engineering design, but rather can be exploited at the very earliest stages of project definition.

Crucially, this approach reduces the timescale needed in concept development or selection and improves the quality of engineering decision making. It also provides a rational basis upon which to make comparisons between competing solutions, i.e. the Pareto front.

Furthermore, it offers the opportunity at the very earliest stages of project definition for technical alignment of all disciplines and for the development of the design to proceed in parallel with consistent objectives and understanding of their inter-relationships.

The method may be generalised to suit the early stage definition of any form of floating systems, and particularly multi-disciplined design where the matching of overall functional requirements is critical. It is also valuable where other target criteria in areas such as efficiency, operability or safety, are key objectives, as is often the case in ship design.

Further work to refine this approach is recommended the areas of:

- Faster and more efficient methods in hydrodynamic response prediction are needed to widen application where performance assessment to higher order is required. Traditional, purely analytical methods, or combinations of numerical and empirical approaches

with meta-modelling, might offer improved prediction tools in this context.

- Non-Intrusive approaches to Robust Design, to include the influence of uncertainties in the underlying design model correlations, functional requirements or other variables, would further inform good decision making. Development of an efficient method for their use with the simplified genetic algorithm described here would also be welcome.

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## APPENDIX 1 – CASE STUDY GEOMETRY

The geometry of the DDPSO concept used as the case study is illustrated in Figure A1.1. It has the following key design features:

- A central oil-over-water storage tank, extending from the main deck down almost to the keel, in which lower density crude oil floats on seawater, such that the tank is permanently pressed full.
- An upper hull section to provide the buoyancy required.
- An upper deck and hull paces for various hull utilities, mooring system equipment, offloading facilities etc.
- A central, internal seawater caisson to provide pressure balance.
- A fixed solid ballast compartment at the keel.
- One or more heave plates, or for this case study, two heave plates combined also to form a keel box, open to the sea.
- Minimal water ballast needed only to meet damage stability regulations and balance differing specific gravities of crude oil and seawater.

The Figures A1.2 and A1.3 show the basic geometry definition and key dimensions, in plan and elevation, used in this paper.

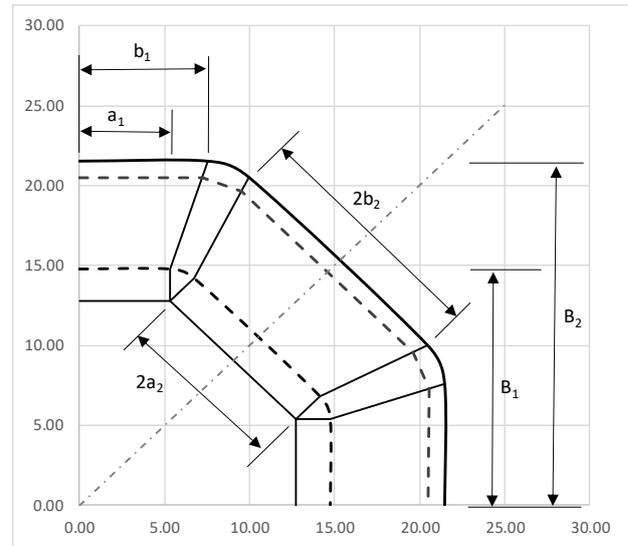


Figure A1.2 – Plan view showing upper side shell (width  $B_2$ ), lower tank shell (width  $B_1$ ), tank top and heave plate/box shell.

### General Arrangement Sketch of the Deep Draft Production, Storage and Offloading Floating System (GB Patent 2507370, US Patent 98228072).

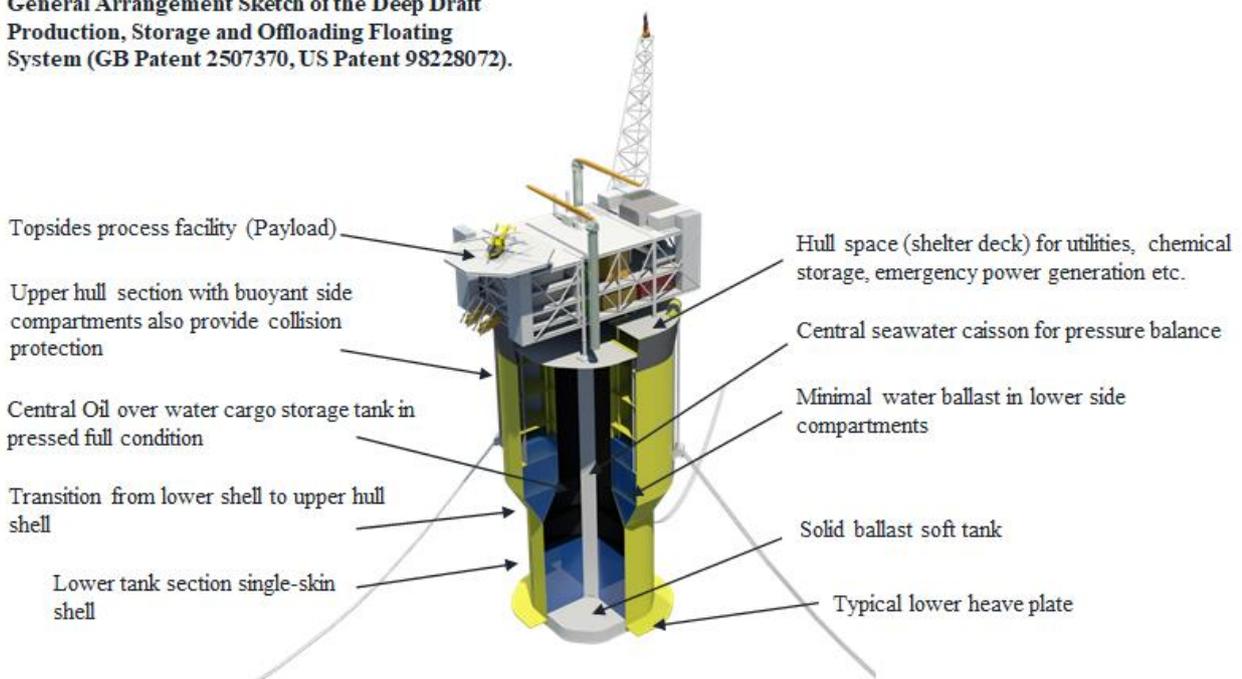


Figure A1.1 – General Arrangement Sketch of DDPSO Concept

Compared with Figure A1.1, the geometry model is both generalised and simplified as follows:

- The vertical height of transition from the lower section of the crude oil storage tank to its upper section (dimension T<sub>4</sub>) is above (not co-incident with) the external shell transition to the upper hull (T<sub>1</sub>).
- Vertical transition extents T<sub>2</sub> and T<sub>5</sub> are independent variables.
- The lower tank corner radius is fixed.
- The upper tank top plan view is octagonal, of sides 2a<sub>1</sub> and 2a<sub>2</sub>, with no corner radius or flat.
- Freeboard is fixed and tank top height above the external waterplane may not exceed 10.0m (to control internal pressure load).

The principal dimensions and how they are treated within the genetic algorithm are summarised in Table A1.1 below.

Table A1.1 – Summary of Principal Dimensions

Plan View – Figure A1.2		
Item	Description	Treatment
B <sub>1</sub>	Half beam -lower tank	GA1
B <sub>2</sub>	Half beam - hull/water-plane	GA1
b <sub>1</sub>	Half width - upper side shell 1	GA2
b <sub>2</sub>	Half width - upper side shell 2	GA2
a <sub>1</sub>	Half width - lower tank side 1	GA1
a <sub>2</sub>	Half width - lower tank side 2	GA2
R	Lower tank corner radius	Fixed
Elevation – Figure A1.3		
Item	Description	Treatment
T1	Height to hull Flare abv keel	GA1
T2	Vertical extent of hull flare	GA1
T3	Total height to sheer-line	GA1
T4	Lower tank height	GA1
T5	Inner tank reverse flare	GA1
T6	Tank Top height abv keel	Calculated
ZP	Depth of heave plate “box”	Fixed
WP	Width of heave plate	= B <sub>1</sub> -B <sub>2</sub>
F	Freeboard	Fixed

Here, GA1 and GA2 refer to two different treatments for the random generation of dimensions used in the algorithm.

GA1 refers to the random variation formula:

$$X_{N+1} = X_N + \beta \cdot \delta X \dots\dots\dots A2.1$$

Used to both create the initial population from the base-line geometry, or parent in later mutation steps, where:

- β a random number between -1.0 and +1.0
- X<sub>N</sub> the parent variable value at generation N
- X<sub>N+1</sub> the variable value at the next generation
- δX maximum variation in the variable

GA2 refers to cases where geometric constraints are needed, with random variation limited to specific elements of the

geometrical definition. For example, upper hull dimensions b<sub>1</sub> and b<sub>2</sub> are subject to the constraints that the radius of the side shell section between them must be greater than that of the lower tank, and lower than the radius that would make b<sub>1</sub> < a<sub>1</sub> and/or b<sub>2</sub> < a<sub>2</sub>.

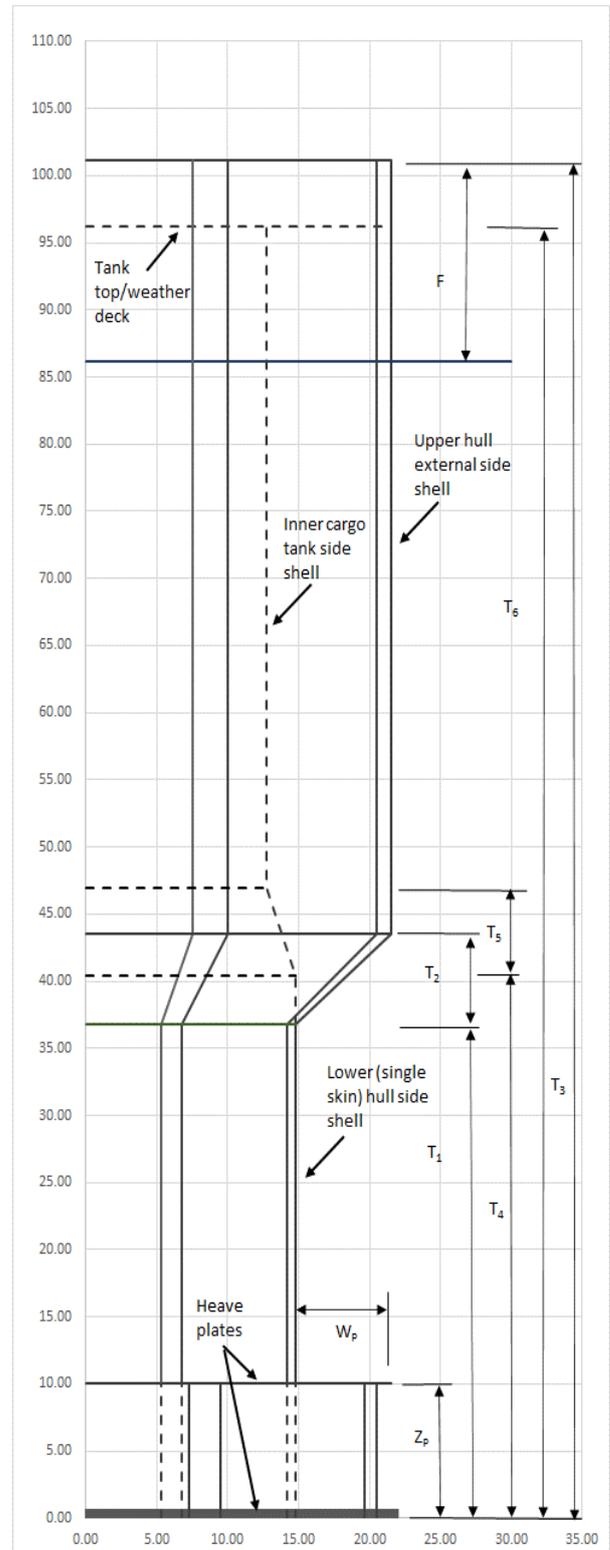


Figure A1.3 – Elevation showing details of the single skin lower tank, extending internally (dashed line), and principal dimensions.