FREE VIBRATION OF SKEW LAMINATES – A BRIEF REVIEW AND SOME BENCHMARK RESULTS

(DOI No: 10.3940/rina.ijme.2019.a4.540)

S Haldar and **S Pal,** Indian Institute of Engineering Science and Technology, India, **K Kalita**, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, India

SUMMARY

This study investigates and reviews prior research works on skew composite laminates. The equivalent single layer theories are explored and discussed. An exhaustive review on static and dynamic analysis of composite skew laminates is also presented. Subsequently, a nine node isoparametric plate bending element is used for free vibration analysis of laminated composite skew plate with central skew cut out. The effect of shear deformation is incorporated in the formulation considering first order shear deformation theory. Two types of mass lumping schemes are analysed to study the effect of rotary inertia. Certain numerical examples of plates having different skew angles, skew cut out sizes, boundary conditions, thickness ratios (h/a), aspect ratios (a/b), fiber orientations and number of layers are solved which will be useful for benchmarking of future studies.

NOMENCLATURE

[B]	Strain-displacement matrix
[D]	Rigidity matrix
[K]	Global stiffness matrix
[N]	Shape function
$[N_0]$	Null matrix
$[M]$	Consistent mass matrix
$\|J\|$	Jacobian matrix
$[N_r]$	Interpolation function of the r^{th} point
$[K_0]$	Overall stiffness matrix
$[M_0]$	Overall Mass matrix
u, v	In-plane displacement
w	Transverse displacement
E	Modulus of elasticity
G	Modulus of rigidity
ν	Poisson's ratio
h	Thickness of plate
a, b	Plate dimensions
D	Flexural rigidity
ω	Natural frequency
ϕ_x ϕ_y	Average shear rotation
θ_x θ_y	Total rotation in bending
$\{\sigma\}$	Stress vector
$\{\varepsilon\}$	Strain vector
M_x, M_y	Bending moments in x and y direction
M_{xy}	Twisting moment
Q_x Q_y	Transverse shear forces
ξ, η	Natural coordinates
ρ	Density
CLPT	Classical laminate plate theory
DSCM	Discrete singular convolution method
DSC-EM	Discrete singular convolution-element method
DQM	Differential quadrature method
EFGM	Element-free Galerkin method
ESLT	Equivalent single layer theory
FDM	Finite difference method
FEM	Finite element method
FSDT	First-order shear deformation theory
FSM	Finite strip method
HSDT	Higher order shear deformation theory
Iso	Isogeometric method
MFVM	Meshless finite volume method
MLS-RM	Moving least square Ritz method
MM	Meshfree method
MTEKM	Multi-Term extended Kantorovich method
QEM	Quadrature element method
RBF	Radial basis function
R-DQM	Ritz-differential quadrature methodology
RM	Ritz method
RRM	Rayleigh-Ritz method

1. INTRODUCTION

Free vibration analysis of laminated composite plates is very important in the field of structural engineering. Many structures such as ships and containers require the complete enclosure of plates. With the advancement in fiber-reinforced laminated composite materials, the use of composite plates and shells has increased greatly due to their high strength to weight ratio. Fiber reinforced laminated composite plates are generally used in architectural structures, bridges, hydraulic structures, pavements, containers, airplanes, missiles, ships, instrument and automobile structures. Skew plates are often used in such modern structures. Swept wing of airplanes, for example, can be idealized by introducing substructures in the form of oblique plates. Similarly, complex alignment problems in bridge designs are often designed by using skew plates. Plates with cut-outs are also commonly encountered in engineering practice. Cut-outs are introduced to provide access, reduce weight and alter the dynamic response of structures.

In the present work, a brief literature review on equivalent single layer theories is presented. This is followed by an exhaustive review of the literature on skew plates. Both static and dynamic analysis involving skew plates are surveyed. A first-order shear deformation based finite element method is introduced and some benchmark results on skew plates are reported for certain test cases which are sparse in literature.

2. LITERATURE REVIEW ON EQUIVALENT SINGLE LAYER THEORIES

The static and dynamic behavior of composite plates and shells can be simulated using either equivalent single layer theories or three-dimensional elasticity theories. Using suitable assumptions, equivalent single layer theories are derived from three-dimensional elasticity theories (Reddy, 2004). In general, the equivalent single layer theories account for shear deformation using certain assumptions. Equivalent Single Layer theories (ESL) can be further classified Classical Laminate Plate Theory (CLPT), First-Order Shear Deformation Theory (FSDT) and Higher Order Shear Deformation Theories (HSDT). In the context of his paper, three-dimensional theories are not discussed. Readers may look at the excellent works of Jin (Jin et al., 2015a, 2015b, 2015c), (Jin et al., 2014a, 2014b) and Su (Su, et al., 2015a, 2015b) (Su, et al., 2014a, 2014b, 2014c, 2014d) (Su, et al., 2016a, 2016b, 2016c) and (Ye et al.,2014), (Ye et al.,2015), (Ye and Jin, 2016), (Ye et al.,2016a, 2016b), (Ye et al.,2017) on three-dimensional vibrational analysis.

2.1 CLASSICAL LAMINATE PLATE THEORY (CLPT)

Classical laminate plate theory (CLPT) is the simplest of the equivalent single layer theories. CLPT which is based on Kirchhoff–Love hypothesis assumes that the straight lines remain straight and perpendicular to the midplane after deformation. Due to this shear and normal strains vanishes which in turn leads to neglecting the transverse shear and normal deformation effects (Kirchoff, 1850). Thus, the applicability of CLPT is limited to thin plates/shells which leads to erroneous solutions for thick and moderately thick plates and shells where the shear and normal deformation effects are considerable. Further, CLPT violates stress-free boundary conditions at top, bottom surfaces. It underpredicts the deflections in plates and shells and overpredicts Eigenfrequencies and buckling loads (Cosentino & Weaver, 2010). However, CLPT gives relatively good results for symmetric and balanced laminates under the effect of pure bending or pure tension (Khandan, et al., 2012). The displacement fields of CLPT may be expressed as,

$$
\begin{aligned}
u &= u_o(x,y) - z\frac{\partial w}{\partial x} \\
v &= v_o(x,y) - z\frac{\partial w}{\partial y} \\
w &= w(x,y)
\end{aligned} \qquad (1)
$$

Where u, v, w are displacements in x, y, z directions respectively.u_o, v_o, w are unknown functions of position(x, y).

CLPT despite its shortcoming has been popular among researchers due to its simple form and computational inexpensive nature. Since 3D plate or shells are idealized as 2D plate or shells there is a significant reduction in the total number of variables which in turn saves a lot of computational costs. CLPT was initially propounded by Kirchhoff (Kirchhoff, 1850) and was later extended by Love (Love, 2013), Timoshenko and Goodier (Timoshenko & Goodier, 1971) and Volokh (Volokh, 1994). Volokh (Volokh, 1994) tried to enhance the classical form of CLPT by assuming the shear forces as statically equivalent to “rotated” bending and twisting moments instead of defining it as an integral over the plate thickness of the transversal shear stresses. Timoshenko and Krieger (Timoshenko & Woinowsky-Krieger, 1959), Timoshenko and Gere (Timoshenko & Gere, 1961), Dym and Shames (Dym, et al., 1973), Szilard (Szilard & Nash, 1974), Ugural (Ugural & Ugural, 1999), Ashton and Whitney (Ashton & Whitney, 1970), Ambartsumyan (Ambartsumian, 1970), Lekhnitskii (Lekhnitskii, 1968), Arkhipov (Arkhipov, 1968) and Tamurov and Grud’eva (Tamurov & Grud'eva, 1974) also made significant contributions that helped in making the theory more popular.

Reissner and Stavsky (Reissner & Stavsky, 1961) were the first researchers to apply the CLPT to heterogeneous aeolotropic elastic plates. Stavsky (Sky, 1961) use CLPT to study multilayer aeolotropic plate subject to in-plane forces and transverse loading. Dong et al. (Dong, 1962) formulated the CLPT for analysing electrostatic extension and flexure of laminated plates and shells having small thickness.

By using CLPT and including Von Karman nonlinear terms Whitney and Leissa (Whitney & Leissa, 1969) formulated the governing equations of laminated plates. They also included the inertia effect and thermal stresses. Whitney (Whitney, 1969a) further used the CLPT to study bending of simply supported rectangular plates. He also successfully modelled the effect of transverse shear deformation to predict flexural vibration frequencies and buckling loads. He then extended the theory to study anti-symmetric cross-ply and angle-ply laminates under transverse loading (Whitney, 1969b). Whitney (Whitney, 1969c) also showed the effect of bending-extensional coupling in cylindrical bending of laminated plates.

Konieczny and Wozniak (Konieczny & Wozniak, 1994) used CLPT to study composites plates of arbitrary inhomogeneous linear-elastic material. Wang et al. (Wang, et al., 1997) used CLPT to strip element method is presented to determine bending solutions of orthotropic plates. CLPT has been extensively reviewed by Vasil’ev (Vasil'Ev, 1992) for isotropic plates and by Vinson and Chou (Vinson & Chou, 1975) for anisotropic plates. The limitations of CLPT have been shown by a few researchers, notably Pagano (Pagano, 1969) (Pagano, 1970a, 1970b).

By comparing the CLPT results with the theory of elasticity solutions. Pagano (Pagano, 1969) highlighted that at low span-to-depth ratios CLPT leads to poor approximation but convergences towards an exact solution as the span-to-depth ratio increases. He also showed the limitations of the theory for sandwich plate (Pagano, 1970a) and unidirectional and angle-ply composites (Pagano, 1970b).

2.2 FIRST-ORDER SHEAR DEFORMATION THEORY (FSDT)

Due to the inherent flaws of CLPT, the first-order shear deformation theory (FSDT) was propounded by Mindlin (Mindlin, 1951). By considering a linear variation of in-plane displacements through the thickness, FSDT accounts for the shear deformation effect. The displacement fields of FSDT may be expressed as,

$$\begin{aligned} u &= u_o(x,y) + z\theta_x(x,y) \\ v &= v_o(x,y) + z\theta_y(x,y) \\ w &= w(x,y) \end{aligned} \quad (2)$$

Where u, v, w are displacements in x, y, z directions respectively; u_o, v_o, w are unknown functions of position(x, y); θ_x and θ_y are the rotations of a transverse normal about the y-axis and x-axis, respectively.

However, FSDT requires a shear correction factor. Thus, the predictions of FSDT are largely dependent on the considered shear correction factor which accounts for the strain energy of shear deformation. The shear correction factor depends on geometry, loading and boundary conditions and thus may be difficult to determine. In fact, the accurate estimation of the shear correction factor for FSDT has been a research concern by itself.

Bolle (Bolle, 1947), Hencky (Hencky, 1947), Uflyand (Uflyand, 1948), Yang et al. (Yang, et al., 1966), Whitney and Pagano (Whitney & Pagano, 1970), Qi and Knight (Qi & Knight Jr, 1996), Knight and Qi (Knight & Qi, 1997a, 1997b), Wang and Chou (Wang & Chou, 1972), Sun and Whitney (Sun & Whitney, 1973), Chow (Chow, 1971) (Chow, 1975) initiated further investigations on FSDTs.

Using energy principles Whitney (Whitney, 1973), Chatterjee and Kulkarni (Chatterjee & Kulkarni, 1979), Vlachoutsis (Vlachoutsis, 1992) presented a study on shear correction factors. They also established that multi-layered composite plates and homogeneous plates require separate values of shear correction factors. Gruttmann and Wagner (Gruttmann & Wagner, 2017) also detailed shear correction factors for layered plates and shells. FSDT is suitable for thin and moderately thick plates/shells. For thick plates, it deviates slightly from the exact solution.

It is worth mentioning here that Reissner (Reissner, 1947) (Reissner, 1945) also developed a theory that considers the shear deformation effect. However, Thai and Kim (Thai & Kim, 2015) have pointed out in a recent review that the Reissner theory is not similar to the Mindlin one. Wang et al. (Wang, et al., 2001) derived the bending relations between Mindlin and Reissner quantities to establish the differences between the two theories. The displacement variation across the thickness may or may not be linear in case of Reissner theory since it considers a linear bending stress distribution and a parabolic shear stress distribution (Wang, et al., 2001). Thai and Kim (Thai & Kim, 2015) argue that it is erroneous to refer to the Reissner theory as the FSDT since FSDT essentially implies a linear variation of the displacements through the thickness. Moreover, the normal stress is not included in the Mindlin theory (Panc, 1975).

Bhaskar and Varadan (Bhaskar & Varadan, 1993) used the combination of Navier's approach and a Laplace transform technique to solve the equations of equilibrium. Onsy et al. (Roufaeil & Tran-Cong, 2002) presented a finite strip solution for laminated plates. Pryor and Barker (Barker & Pryor Jr, 1971) developed a finite element formulation based upon the FSDT for cross-ply symmetric and unsymmetric laminated plates. Ha (Ha, 1990) developed the finite element model for sandwich plates based on FSDT. Byun and Kapania (Byun & Kapania, 1992) used FSDT to predict interlaminar stresses in laminated plates. Dobyns (Dobyns, 1981) employed FSDT for analysis of orthotropic plates. Turvey (Turvey, 1977) presented the analyses for laminated rectangular plates using FSDT. Kabir (Kabir, 1996) presented an analytical solution to shear flexible rectangular plates with arbitrary laminations based on FSDT. Some recent applications of FSDT may be found at (Alavi & Eipakchi, 2018) (Civalek, 2017) (Pandit, et al., 2007) (Yu, et al., 2015) (Yu, et al., 2016) (Zhang, et al., 2015a) (Kalita & Haldar, 2015, 2016, 2017, 2018) (Kalita, et al., 2018a, 2018b) (Kalita, et al., 2016a, 2016b, 2016c) (Kalita, et al., 2015).

2.3 HIGHER ORDER SHEAR DEFORMATION THEORIES (HSDT)

Since accurate estimation of the shear correction factor is essential for correct prediction by FSDT, higher-order shear deformation theories (HSDT) were introduced. In HSDT, the displacement components are expanded in a power series of the thickness coordinate. In general, by including more and more terms in the expansion series, the desired accuracy may be achieved. Higher-order variations of the in-plane displacements or both in-plane and transverse displacements through the thickness are considered in higher-order shear deformation theories. Thus, in HSDT the effects of shear deformation or both shear and normal deformations are accounted for. HSDT is realized by either considering polynomial shape functions or non-polynomial shape functions.

2.3 (a) Third-order shear deformation theory (TSDT)

It was Vlasov (Vlasov, 1957), who initially developed a third-order displacement field that could satisfy the stress-free boundary conditions at the top and bottom surfaces of a plate. Jemielita (Jemielita, 1975), Krishna Murty (Krishna Murty, 1987) and Schmidt (Schmidt, 1977) were some of the first researchers to propose TSDT. However, the TSDT developed by Reddy (Reddy, 1984) is the most commonly used one. The transverse shear deformation effect is considered in TSDT. It also satisfies the zero-traction boundary conditions on the top and bottom surfaces of a plate. Thus, a shear correction factor is not needed. Though the equations of motion for Reddy's TSDT and Levinson's theory (Levinson, 1980) are different both these theories use the same displacement field. The displacement field of Reddy's third-order shear deformation theory may be expressed as,

$$\begin{aligned} u &= u_o(x,y) + z\left[\theta_x(x,y) - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\theta_x(x,y) + \frac{\partial w}{\partial x}\right)\right] \\ v &= v_o(x,y) + z\left[\theta_y(x,y) - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\theta_y(x,y) + \frac{\partial w}{\partial y}\right)\right] \\ w &= w(x,y) \end{aligned} \quad (3)$$

Where u, v, w are displacements in x, y, z directions respectively; u_o, v_o, w are unknown functions of position(x, y); θ_x and θ_y are the rotations of a transverse normal about the y-axis and x-axis, respectively.

Murthy (Murthy, 1981) developed a higher-order shear deformation theory and formulated it for unsymmetric laminates, symmetric laminates and classical orthotropy. In such similar attempts, a few TSDTs were formulated by Ambartsumian (Ambartsumian, 1960) (Ambartsumian, 1969), Librescu (Librescu, 1967), Shirakawa (Shirakawa, 1983) and Bhimaraddi & Stevens (Bhimaraddi & Stevens, 1984) among others. In 1984, Reddy (Reddy, 1984) reviewed all TSDTs proposed until then and established an equivalence among them. Phan & Reddy (Phan & Reddy, 1985) proposed a higher-order shear deformation theory that accounted for parabolic distribution of the transverse shear stresses. Pandya & Kant (Pandya & Kant, 1988) incorporated a linear variation of transverse normal strains and parabolic variation of transverse shear strains through plate thickness. A nine-node Lagrangian parabolic isoparametric plate bending element was used by them for the finite element analysis. Murty & Vellaichamy (Murty & Vellaichamy, 1988) developed a higher-order shear deformation theory with provision for cubic variation of in-plane displacements and parabolic variation of the normal displacement. Using the principle of virtual displacements Ren-Huai & Ling-Hui (Ren-Huai & Ling-Hui, 1991) developed a TSDT that accounted for parabolic variation of transverse shear strains through the thickness. Singh & Rao (Singh & Rao, 1996) developed a four node rectangular element with fourteen degrees of freedom and it used it in conjunction with a TSDT to study the effect of various parameters such as lay-up, side to thickness ratio, aspect ratio, type of loadings, boundary conditions on stability characteristics of laminated plates. Vuksanovic (Vuksanovic, 2000) developed a TSDT that could take a parabolic distribution of shear strains across the plate thickness and cubic variation for in-plane displacements.

Idlbi et al. (Idlbi, et al., 1997) in 1997 made a comparative study of CLPT, FSDT, TSDT and TSDPT (sine type), through which they concluded TSDPT to better than the others especially when interlayer continuity requirements are included. Much later Carrera (Carrera, 2007) compared three different TSDT models – one having five displacement variables, and the other two having three displacement variables. While the second TSDT was reduced from five displacement variables to three by enforcing homogeneous transverse stress conditions, the third was done so by considering non-homogeneous transverse stress conditions. He concluded that the use of non-homogeneous transverse stress conditions led to superiority of the third model over the second one. However, in general, the original model (first one) still had better estimations the other two.

2.3 (b) Trigonometric shear deformation theory (TgSDT)

As the name suggests, trigonometric functions are used to describe the shear deformation plate theories called trigonometric shear deformation theory (TgSDT). TgSDT is richer than polynomial functions, simple, more accurate and the stress-free surface boundary conditions can be guaranteed a priori (Mantari, et al., 2012). TgSDT was realized by Levy (Levy, 1877) using sinusoidal functions in the displacement field. The displacement field of the Levy's TgSDT are as follows,

$$\begin{aligned} u &= \sum_{n=0}^{N} z^{2n+1}\, u_n(x,y) + \sum_{n=0}^{N} sin\frac{(2n+1)\pi z}{h}\,\theta_x(x,y) \\ v &= \sum_{n=0}^{N} z^{2n+1}\, v_n(x,y) + \sum_{n=0}^{N} sin\frac{(2n+1)\pi z}{h}\,\theta_y(x,y) \\ w &= \sum_{n=0}^{N} z^{2n}\, w_n(x,y) \end{aligned} \quad (4)$$

where u, v, w are displacements in x, y, z directions respectively; u_n, v_n, w_n are unknown functions of position(x, y) ;θ_x and θ_y are the rotations of a transverse normal about the y-axis and x-axis, respectively.

Other such TgSDTs have been developed for plate and shells using sine, hyperbolic sine and cosine functions. The trigonometric functions describe the warping through the thickness of the plate during rotation due to transverse shear. Kil'chevskiy (Kil'chevskiy, 1965) in his book discussed TgSDTs in detail. He solved several static and dynamic problems on shells which were used as benchmark problems till much later. However, he neglected the dissipative forces in the analysis. Stein and Jegley (Stein & Jegley, 1987) used a TgSDT and described the displacement fields using algebraic and trigonometric terms. To find the displacements and stresses they used both potential and complementary energy methods. Jegley (Jegley, 1988) in a technical report for NASA, USA studied the effects of transverse shear deformation and anisotropy on natural vibration frequencies of laminated cylinders by using a TgSDT. He also reported that the

TgSDT predicted buckling loads to be about 65% of those predicted by FSDT for certain thick-walled cylinders. Stein and Bains (Stein, et al., 1990) studied buckling of plates due to compressive load using sinusoidal terms for displacement fields. Touratier (Touratier, 1991) (Touratier, 1992) presented TgSDTs that accounted for cosine shear stress distribution and free boundary conditions for shear stress upon the top and bottom surfaces of the plate. His theory was based on the kinematical approach, where the shear was represented by a sinusoidal function. He further extended it for shells (Touratier & Faye, 1995). Bhimaraddi and Stevens (Bhimaraddi & Stevens, 1986), Stein (Stein, 1986), Becker (Becker, 1994) (Becker, 1993) and Lu and Liu (Lu & Liu, 1992) have also made some valuable contributions towards development of TgSDTs.

Using Hamilton's principle and Lagrange multipliers, Soldatos (Soldatos, 1992) developed a TgSDT for homogenous monoclinic plates. Beakou and Touratier (Beakou & Touratier, 1993) developed a 32 degree of freedom finite element that was used in conjunction with a TgSDT in which the transverse shear deformation was represented by cosine functions. They carried out static, buckling and dynamic analysis of composite shells. Muller and Touratier (Muller & Touratier, 1995) made a comparative study on the theory of Kirchhoff-love, Schmidt-Levinson theory, Reissner-Mindlin theory, Reddy theory and Touratier theory. Shimpi and Ghugal (P. Shimpi, 2000) used a sinusoidal function to represent the shear deformation. However, it contained only three variables, even less than FSDT. Kassapoglou and Lagace (Kassapoglou & Lagace, 1986) used a TgSDT to calculate the interlaminate stress field at straight free edges in symmetric composite plates under uniaxial load. They also extended the theory for angle-ply and cross-ply plates (Kassapoglou & Lagace, 1987). Later the method was further extended by Kassapoglou (Kassapoglou, 1990) to study the effect of combined loads on the free edges interlaminate stress. Similarly, Webber and Morton (Webber & Morton, 1993) used TgSDT to study free edge stress fields in laminated plates due to thermal effects.

3. LITERATURE REVIEW ON ANALYSIS OF SKEW PLATES

In this section, a brief literature survey on static and dynamic analysis of composite skew plates is presented. Some papers on isotropic plates/shells are also reviewed to maintain continuity. However, works on functionally graded structures are excluded. The structural behavior of isotropic skew plates has been studied previously by many investigators like Kennedy and Huggins (Kennedy, 1964), Kennedy and Tamberg (Kennedy & Tamberg, 1969), Mizusawa et al. (Mizusawa, et al., 1979) among others.

York and Williams (York & Williams, 1995) relied on CLPT to study buckling of skew plates. Reddy and Palaninathan (Reddy & Palaninathan, 1995) used the finite element method for buckling analysis of laminated skew plates. They used a high precision triangular plate bending element with three nodes located at vertices having 12 degrees of freedom per node. Auricchio and Taylor (Auricchio & Taylor, 1995) developed a new formulation for a triangular element. Using FSDT they calculated the cylindrical bending of simply supported skew plates. Ganapathi and Prakash (Ganapathi & Prakash, 2006) too used FSDT to estimate buckling of skew panels.

Bardell (Bardell, 1992) determined the natural frequencies for isotropic plates. McGee and Butalia (McGee & Butalia, 1994) used FSDT and HSDT in conjunction with a nine node Lagrangian isoparametric quadrilateral element based finite element analysis for estimating the natural frequencies of a cantilever skew plate. Using the same high precision triangular plate bending element that they developed in 1995, Reddy and Palaninathan (Reddy & Palaninathan, 1999) conducted an FE analysis to accurately predict the Eigenfrequencies of a skew plate. Singha and Ganapathi (Singha & Ganapathi, 2004) estimated the large amplitude free flexural vibrations using HSDT. Sundararajan et al. (Sundararajan, et al., 2005) conducted a finite element analysis using the 8-node quadrilateral element to calculate the nonlinear free flexural vibrations. Dey and Singha (Dey & Singha, 2006) carried out a dynamic stability analysis of composite skew plates subjected to periodic in-plane load. Singha and Daripa (Singha & Daripa, 2007) used a 4-node shear flexible quadrilateral high precision plate bending element to study nonlinear vibrations in a symmetric laminated skew panel. Nguyen-Van et al. (Nguyen-Van, et al., 2008) too relied on FSDT to estimate the Eigenfrequencies of skew plates. Park et al. (Park, et al., 2009) modelled delamination in composite skew plates using finite element method and studied their effect on natural frequencies. They considered HSDT for considering the shear deformation across the thickness of the plate.

Eftekhari and Jafari (Eftekhari & Jafari, 2012) developed a higher order FEM formulation to accurately model skew plates. Chalak et al. (Chalak, et al., 2014) carried out both static and dynamic analysis of skew rectangular laminated sandwich plates considering a higher order zigzag theory (HOZT). Experimental and numerical simulation in a commercial FE package was carried out by Srinivasa et al. (Srinivasa, et al., 2014) to study the natural frequencies of skew laminates. Yadav et al. (Yadav, et al., 2015) comprehensively studied the effect of skewness in stiffened plates using a commercial finite element package. Garcia-Macaisa et al. (García-Macías, et al., 2016) used a four-node skew element while considering FSDT to account for shear deformation. Zhang et al. (Zhang, et al., 2015b) also used a FSDT and moving least square-Ritz method. Zghal et al. (Zghal, et al., 2018) used a HSDT to carry out free vibration analysis of nanocomposite shells reinforced with carbon nanotubes. Lee (Lee, 2018) also used a HSDT and finite element analysis to assess the dynamic stability of multiscale composites. Delamination was also considered in his analysis. Using finite strip method Ashour (Ashour, 2009) studied vibration of skew plate. Rango et al. (Rango, et al., 2015) relied on trigonometric shear deformation theory to calculate the natural frequencies. Upadhyay and Shukla (Upadhyay & Shukla, 2013) and Shojaee et al. (Shojaee, et al., 2017) made use of Hamilton's principle to do so.

However, HSDT was used in (Upadhyay & Shukla, 2013) whereas (Shojaee, et al., 2017) used FSDT.

Ardestani et al. (Ardestani, et al., 2017) and Zhang et al. (Zhang, et al., 2017) used HSDT and TSDT respectively while modelling thick skew laminates. In both the works isogeometric method was adopted. Similarly, meshless methods were adopted by Fallah and Delzendeh (Fallah & Delzendeh, 2018) and Liew et al. (Liew, et al., 2004). The moving least square Ritz (MLS-Ritz) method was used by Zhou and Zheng (Zhou & Zheng, 2008) and Zhang (Zhang, 2017). Fallah et al. (Fallah, et al., 2011) considered the use of multi-term extended Kantorovich method most appropriate for skew plate analysis. The Rayleigh-Ritz approach was used by quite a few researchers Like Mizusawa et al. (Mizusawa, et al., 1979) (Mizusawa, et al., 1980), Liew and Lam (Liew & Lam, 1990), Liew et al. (Liew, et al., 1993), Singh and Chakraverty (Singh & Chakraverty, 1994), Zeng and Bert (Zeng & Bert, 2001), Kumar et al. (Kumar, et al., 2015) (Kumar, et al., 2017), He et al. (He, et al., 2017). Wang (Wang, 1997) used B-spline Rayleigh-Ritz method based first order shear deformation theory for free vibration analysis of laminated composite skew plates. For analysis of free vibration of laminated composite skew plates, Anlas and Gooker (Anlas & Göker, 2001) used orthogonal polynomials with Ritz method. Makhecha et al. (Makhecha, et al., 2001) investigated dynamic responses of thick skew sandwich plates using C0QUAD-8 finite element based on a realistic higher-order theory. Effect of skew angle and thickness ratio on the dynamic characteristics of sandwich laminates subjected to thermal and mechanical loads have been studied. A high precision thick plate element has been developed by Sheikh and Haldar (Sheikh, et al., 2004) for free vibration analysis of composite plates in different situations. Numerical examples of plates having different shapes, boundary conditions, thickness ratio and fiber orientations have been analysed. Examples of plates having an internal cut-out and concentrated mass have also been studied. A simple C0 isoparametric finite element model based on a higher order shear deformation theory has been presented by Garg et al. (Garg, et al., 2006) for free vibration of isotropic, orthotropic and layered composite and sandwich skew laminates. Numerical results have been presented for natural frequencies of cross-ply and angle-ply with different lamination parameters, skew angles and boundary conditions. A nine node isoparametric plate bending element formulation has been developed by Pandit et al. (Pandit, et al., 2007) for free vibration analysis of isotropic and laminated composite plates. Numerical examples of isotropic and composite plates having different fiber orientations, aspect ratios, and thickness ratios have been solved and compared. Examples of plates having an internal cut-out and uniformly distributed mass on the plate have also been studied. Bending response of functionally graded skew sandwich plates has been analysed by Taj et al. (Taj, et al., 2014). A comprehensive list of works on skew isotropic and composite plates is presented in Table 1. From this review, the followings insights into the analysis of skew composite laminates are gained:

- FSDT is by far the most popularly used theory for analysis of skew laminates.
- FEM, DQM and Rayleigh-Ritz are the most commonly applied numerical methods for this problem.
- Works involving static and dynamic analysis of skew shells are very limited.
- Works involving static and dynamic analysis of skew shells with cutouts are negligible.

Table 1. Research works on skew plates.

Source	**Theory**	**Method**	**Structure**	**Problem Type**
Eftekhari and Jafari (Eftekhari & Jafari, 2013)	FSDT	R-DQM	Plate	Vibration
Wang (Wang, 1997a)	FSDT	RRM	Plate	Vibration
Wang (Wang, 1997b)	FSDT	RRM	Plate	Buckling
Kiani et al. (Kiani, et al., 2018)	FSDT	RM	Shell	Vibration
Malekzadeh (Malekzadeh, 2008)	FSDT	DQM	Plate	Vibration
Bert and Malik (Bert & Malik, 1996)	FSDT	DQM	Plate	Vibration
Malekzadeh and Karami (Malekzadeh & Karami, 2005)	FSDT	DQM	Plate	Vibration
Malekzadeh (Malekzadeh, 2007)	FSDT	DQM	Plate	Vibration
Malekzadeh and Zarei (Malekzadeh & Zarei, 2014)	FSDT	DQM	Plate	Vibration
Wang and Wu (Wang & Wu, 2013)		DQM	Plate	Vibration
Wang and Yuan (Wang & Yuan, 2018)		DQM	Plate	Buckling
Wang et al. (Wang, et al., 2014)		DQM	Plate	Vibration
Zamani et al. (Zamani, et al., 2012)	FSDT	DQM	Plate	Vibration
Malekzadeh and Fiouz (Malekzadeh & Fiouz, 2007)	FSDT	DQM	Plate	Bending
Adineh and Kadkhodayan (Adineh & Kadkhodayan, 2017)	3D elasticity	DQM	Plate	Vibration
Malekzadeh and Karami (Malekzadeh & Karami, 2006)	FSDT	DQM	Plate	Vibration

Krisiinan and Deshpande (Krishnan & Deshpande, 1992)	CLPT	FEM	Plate	Vibration
Gurses et al. (Gürses, et al., 2009)	FSDT	DSCM	Plate	Vibration
Lai et al. (Lai, et al., 2011)	FSDT	DSC-EM	Plate	Vibration
Jaberzadeh et al. (Jaberzadeh, et al., 2013)		EFGM	Plate	Buckling
Watts et al. (Watts, et al., 2018)	FSDT	EFGM	Plate	Vibration
Naghsh and Azhari (Naghsh & Azhari, 2015)	CLPT	EFGM	Plate	Vibration
Zhao et al. (Zhao, et al., 2009)	FSDT	RM	Plate	Vibration
Kim and Hwang (Kim & Hwang, 2012)	FSDT	FDM	Plate	Vibration
Sundararajan et al. (Sundararajan, et al., 2005)	Lagrange's equations	FEM	Plate	Vibration
Reddy and Palaninathan (Reddy & Palaninathan, 1999)	FSDT	FEM	Plate	Vibration
Chalak et al. (Chalak, et al., 2014)	HOZT	FEM	Plate	Vibration/ Bending
Lee (Lee, 2018)	HSDT	FEM	Plate	Buckling
Park et al. (Park, et al., 2009)	HSDT	FEM	Plate	Vibration
Singha and Ganapathi (Singha & Ganapathi, 2004)	HSDT	FEM	Plate	Vibration
Singha and Daripa (Singha & Daripa, 2007)	FSDT	FEM	Plate	Vibration
Zghal et al. (Zghal, et al., 2018)	HSDT	FEM	Plate	Vibration
Auricchio and Taylor (Auricchio & Taylor, 1995)	FSDT	FEM	Plate	Bending
Reddy and Palaninathan (Reddy & Palaninathan, 1995)		FEM	Plate	Buckling
Ganapathi and Prakash (Ganapathi & Prakash, 2006)	FSDT	FEM	Plate	Buckling
Dey and Singha (Dey & Singha, 2006)	FSDT	FEM	Plate	Buckling
Vimal et al. (Vimal, et al., 2014)	FSDT	FEM	Plate	Vibration
Yadav et al. (Yadav, et al., 2015)	FSDT	FEM	Plate	Vibration
McGee and Butalia (McGee & Butalia, 1994)	FSDT, HSDT	FEM	Plate	Vibration
Nguyen-Van et al. (Nguyen-Van, et al., 2008)	FSDT	FEM	Plate	Vibration
Robinson (Robinson, 1985)	CLPT	FEM	Plate	Bending
Srinivasa et al. (Srinivasa, et al., 2014)	FSDT	FEM	Plate	Vibration
Ashour (Ashour, 2009)	FSDT	FSM	Plate	Vibration
Upadhyay and Shukla (Upadhyay & Shukla, 2013)	HSDT	RM	Plate	Vibration/ Bending
Shojaee et al. (Shojaee, et al., 2017)	FSDT	DQM	Plate	Vibration
Bardell (Bardell, 1992)	FSDT	FEM	Plate	Vibration
Rango et al. (Rango, et al., 2015)	TgSDT	FEM	Plate	Vibration
Eftekhari and Jafari (Eftekhari & Jafari, 2012)	FSDT	FEM	Plate	Vibration
Garcia-Macaisa et al. (García-Macías, et al., 2016)	FSDT	MLS-RM	Plate	Vibration
Zhang et al. (Zhang, et al., 2015)	FSDT	MLS-RM	Plate	Vibration
Ardestani et al. (Ardestani, et al., 2017)	HSDT	Iso	Plate	Vibration
Zhang et al. (Zhang, et al., 2017)	TSDT	Iso	Plate	Buckling
York and Williams (York & Williams, 1995)	CLPT	RRM	Plate	Buckling
Fallah and Delzendeh (Fallah & Delzendeh, 2018)	FSDT	MFVM	Plate	Vibration
Liew et al. (Liew, et al., 2004)	FSDT	MM	Plate	Vibration/ Buckling
Zhou and Zheng (Zhou & Zheng, 2008)	CLPT	MLS-RM	Plate	Vibration
Zhang (Zhang, 2017)	FSDT	MLS-RM	Plate	Vibration
Fallah et al. (Fallah, et al., 2011)	FSDT	MTEKM	Plate	Vibration
KitipornchaI et al. (Kitipornchai, et al., 1993)	FSDT	pb-2 RRM	Plate	Buckling
Xue et al. (Xue, et al., 2018)	RPT	Iso	Plate	Vibration
Wang et al. (Wang, et al., 2000)	FSDT	p-RM	Plate	Vibration
Woo et al. (Woo, et al., 2003)	FSDT	p-FEM	Plate	Vibration
Jin and Wang (Jin & Wang, 2015)		QEM	Plate	Vibration
Ferreira et al. (Ferreira, et al., 2005)	FSDT	RBF	Plate	Vibration

Asemi et al. (Asemi, et al., 2014)	3D elasticity	RRM	Plate	Vibration/ Bending
Zeng and Bert (Zeng & Bert, 2001)	FSDT	RRM	Plate	Vibration
Mizusawa et al. (Mizusawa, et al., 1979)	CPT	RRM	Plate	Vibration
Mizusawa et al. (Mizusawa, et al., 1980)	CPT	RRM	Plate	Vibration/ Bending/ Buckling
Kumar et al. (Kumar, et al., 2017)	TSDT	RRM	Plate	Buckling
Kumar et al. (Kumar, et al., 2015)	HSDT	RRM	Plate	Vibration
Singh and Chakraverty (Singh & Chakraverty, 1994)	FSDT	RRM	Plate	Vibration
He et al. (He, et al., 2017)	FSDT	RRM	Plate	Bending
Liew and Lam (Liew & Lam, 1990)	FSDT	RRM	Plate	Vibration
Liew et al. (Liew, et al., 1993)	FSDT	RRM	Plate	Vibration
Kiani (Kiani, 2016)	FSDT	RM	Plate	Vibration
Zhou et al. (Zhou, et al., 2006)		RM	Plate	Vibration
Anlas and Goker (Anlas & Göker, 2001)		RM	Plate	Vibration
Mizusawa and Kajita (Mizusawa & Kajita, 1987)	FSDT	RRM	Plate	Vibration
Cheung et al. (Cheung, et al., 1988)	FSDT	FSM	Plate	Vibration/ Bending
Liew et al. (Liew, et al., 1995)	3D elasticity	pb-2 RM	Plate	Vibration
Malekzadeh et al. (Malekzadeh, et al., 2014)	Layerwise theory	DQM	Plate	Vibration
Kiani et al. (Kiani & Mirzaei, 2018)	FSDT	RM	Shell	Vibration

(Table 1. continued)

4. FINITE ELEMENT FORMULATION

In the current work, first-order shear deformation theory (FSDT) is used. Independent field variables u, v and w are defined as per equation (2). The shear deformation effect is included by taking the bending rotations as independent variables in the field (Pandit, et al., 2007), which are as follows

$$\begin{Bmatrix}\theta_x\\ \theta_y\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial w}{\partial x} + \phi_x\\ \dfrac{\partial w}{\partial y} + \phi_y\end{Bmatrix} \quad (5)$$

here $\phi_x$and $\phi_y$are the average shear rotation over the entire plate thickness and θ_x and θ_y are the total rotations in bending.

A nine-node isoparametric plate bending element is used in the current finite element formulation. One of the main advantages of the element is that any form of plate can be well managed with a simple mapping technique that can be defined as

$$x = \sum_{r=1}^{9} N_r x_r \text{ and } y = \sum_{r=1}^{9} N_r y_r \quad (6)$$

where (x, y) are the coordinates of any point within the element are, (x_r, y_r) are the coordinates of rth nodal point and N_r is the corresponding interpolation function or shape function of the element. In this element, Lagrangian interpolation function is used for N_r.

The nodal displacements at any node 'r' of the plate element can be expressed as

$$\{\delta_r\} = \begin{Bmatrix}u_r\\ v_r\\ w_r\\ \theta_{xr}\\ \theta_{yr}\end{Bmatrix} \quad (7)$$

Where

$$u = \sum_{r=1}^{9} N_r u_r\,;\, v = \sum_{r=1}^{9} N_r v_r\,;\, w = \sum_{r=1}^{9} N_r w_r\,; \quad \theta_x = \sum_{r=1}^{9} N_r \theta_{x_r}\,;\, \theta_y = \sum_{r=1}^{9} N_r \theta_{y_r} \quad (8)$$

For a laminate, the generalized stress-strain relationship with respect to its reference plane may be expressed as

$$\{\sigma\} = [D]\{\varepsilon\} \quad (9)$$

where {σ} is the vector of stress resultants which can be expressed as

$$\{\sigma\}^T = [N_x \quad N_y \quad N_{xy} \quad M_x \quad M_y \quad M_{xy} \quad Q_x \quad Q_y] \quad (10)$$

where, N_x, N_y, N_{xy} are in-plane force resultants; M_x, M_y are the bending moments in x and y directions; M_{xy} is the twisting moment resultant; and Q_x, Q_y are the transverse shear force resultants.

The generalized strain in terms of displacement is written as

$$\{\varepsilon\}^T = \left[\frac{\partial u}{\partial x} \quad \frac{dv}{dy} \quad \left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \quad \frac{-\partial\theta_x}{\partial x} \quad \frac{-\partial\theta_y}{\partial y} \quad \left(\frac{-\partial\theta_x}{\partial y}-\frac{\partial\theta_y}{\partial x}\right) \quad \left(\frac{\partial w}{\partial x} - \right.\right. \tag{11}$$

and $[D]$ is the rigidity matrix of the laminate which is written as

$$[D] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_c.A_{55} & k_c.A_{54} \\ 0 & 0 & 0 & 0 & 0 & 0 & k_c.A_{45} & k_c.A_{44} \end{bmatrix} \tag{12}$$

where,

$$\begin{aligned} A_{ij} &= \textstyle\sum_{k=1}^{n} \left(Q_{ij}\right)_k (Z_{k+1} - Z_k) \\ B_{ij} &= \tfrac{1}{2}\textstyle\sum_{k=1}^{n} \left(Q_{ij}\right)_k (Z_{k+1}^2 - Z_k^2) \\ D_{ij} &= \tfrac{1}{3}\textstyle\sum_{k=1}^{n} \left(Q_{ij}\right)_k (Z_{k+1}^3 - Z_k^3) \end{aligned} \tag{13}$$

A_{ij}, B_{ij}, D_{ij} are the extensional, extensional-bending and bending stiffness coefficients, which are defined in terms of the lamina stiffness coefficients. Here n denotes the number of the laminas.

$\left(Q_{ij}\right)_k$ are the material coefficients. For any orthotropic material, they are known in terms of the engineering constants of the kth layer and given as (Jones, 1975),

$$Q_{11}^k = \frac{E_1}{1-\mu_{12}\mu_{21}}; Q_{22}^k = \frac{E_2}{1-\mu_{12}\mu_{21}}; Q_{12}^k = Q_{21}^k = \frac{\mu_{12}E_2}{1-\mu_{12}\mu_{21}} Q_{44}^k = G_{23}; Q_{55}^k = G_{13}; Q_{66}^k = G_{12} \tag{14}$$

where E_1 is the longitudinal modulus and E_2 is the transverse modulus, μ_{12} is the major Poisson's ratios, G_{12}, G_{13}, G_{23} are the shear moduli. μ_{21} is determined by using the relation $\mu_{21}E_1 = \mu_{12}E_2$.

In FSDT, a shear correction factor (k_c) is required to adjust the transverse shear stiffness for studying the static or dynamic problems of plates. The accuracy of solutions of the FSDT is strongly dependent on predicting better estimates for the shear correction factor. In this case the shear correction factor is assumed to be 5/6 (Kalita, et al., 2016b).

With the help of equation (8) and equation (11), the strain vector may be written as

$$\{\varepsilon\} = \sum_{r=1}^{9} \begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_r}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_r}{\partial y} & \frac{\partial N_r}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{-\partial N_r}{\partial y} \\ 0 & 0 & 0 & \frac{-\partial N_r}{\partial y} & \frac{-\partial N_r}{\partial x} \\ 0 & 0 & \frac{\partial N_r}{\partial x} & -N_r & 0 \\ 0 & 0 & \frac{\partial N_r}{\partial y} & 0 & -N_r \end{bmatrix} \begin{Bmatrix} u_r \\ v_r \\ w_r \\ \theta_{x_r} \\ \theta_{y_r} \end{Bmatrix} \tag{15}$$

or, $\{\varepsilon\} = \sum_{r=1}^{9} [B]_r \{\delta_r\}_e$

or, $\{\varepsilon\} = [B]\{\delta\}$

Where [B] is the strain matrix containing interpolation functions and their derivatives and $\{\delta\}$ is the nodal displacement vector having order 45×1

Once the matrices [B] and [D] are obtained, the stiffness matrix of the plate element $[K]_e$ can be easily derived by the virtual work method and it may be expressed as

$$[K]_e = \int_{-1}^{+1}\int_{-1}^{+1} [B]^T[D][B]|J| d\xi d\eta \tag{16}$$

In equation (16), the Jacobean $|J|$ is derived from equation (6) by taking the derivatives of the co-ordinates in equation (15). The integration is carried out numerically following Gauss quadrature technique.

Applying the concept of consistent mass matrix, a lumped mass matrix has been derived and it may be expressed as

$$[M] = \rho h \int_{-1}^{+1}\int_{-1}^{+1} \left[[N_u]^T[N_u] + [N_v]^T[N_v] + [N_w]^T[N_w] + \frac{h^2}{12}\left[N_{\theta_x}\right]^T\left[N_{\theta_x}\right] + \frac{h^2}{12}\left[N_{\theta_y}\right]^T\left[N_{\theta_y}\right] \right] |J| d\xi d\eta \tag{17}$$

where,

$$\begin{aligned} [N_u] &= \left[[N_r][N_0][N_0][N_0][N_0]\right] \\ [N_v] &= \left[[N_0][N_r][N_0][N_0][N_0]\right] \\ [N_w] &= \left[[N_0][N_0][N_r][N_0][N_0]\right] \\ \left[N_{\theta_x}\right] &= \left[[N_0][N_0][N_0][N_r][N_0]\right] \\ \left[N_{\theta_y}\right] &= \left[[N_0][N_0][N_0][N_0][N_r]\right] \end{aligned}$$

where, [N0] = null matrix of the order 1×9, ρ is the density of the material and h is the thickness of the laminate.

In equation (17) the first two terms of the mass matrix are associated with in-plane movements of mass and the third term indicates transverse movement of mass (which is usually found to contribute the major inertia) whereas the last two terms are associated with rotary inertia and their contribution becomes significant only in a plate having higher thickness.

The element stiffness matrix and mass matrix having an order of forty-five are evaluated for all the elements and they are assembled together to form the overall stiffness matrix $[K_0]$ and mass matrix $[M_0]$. Once $[K_0]$ and $[M_0]$ are obtained the equations of motion of the plate may be expressed as

$$[K_0]\{\delta\} = \omega^2[M_0]\{\delta\} \quad (18)$$

After incorporating the boundary conditions in equation (18), it is solved by the simultaneous iterative technique following Corr and Jennings (Corr & Jennings, 1976) to obtain the natural frequency ω.

The boundary conditions are defined as,

Simply supported condition (denoted by S):

$w = \theta_x = 0$, at boundary line parallel to x-axis
$w = \theta_y = 0$, at boundary line parallel to y-axis

Clamped condition (denoted by C):

$w = \theta_x = \theta_y = 0$

Free boundary condition (denoted by F):

$w \neq 0, \theta_x \neq 0, \theta_y \neq 0$

The authors have previously shown this formulation to be able to yield very accurate results (Kalita & Haldar, 2017) (Kalita & Haldar, 2016) (Kalita & Haldar, 2018) (Kalita & Haldar, 2015) (Kalita, et al., 2018a) (Kalita, et al., 2016b) (Kalita, et al., 2015) (Kalita, et al., 2016c) (Kalita, et al., 2016a) (Kalita, et al., 2018b).

5. RESULTS AND DISCUSSION

Numerical examples of skew isotropic and composite plates with skew cut-outs are solved by the present finite element formulation. The finite element approach used in this research is first established by comparing with the published results. Subsequently using the current high-fidelity finite element analysis a few new results are reported as benchmark results for future studies.

5.1 VALIDATION STUDY

***Example 1**: Isotropic skew plate*

A simply supported isotropic skew plate as shown in Figure. 1a is analysed for different skew angles (30° and 45°). The necessary transformation for the inclined edges is done. The present results are reported in Table 2 along with those of Liew and Lam (Liew & Lam, 1990). The results show convergence at 16 × 16 mesh.

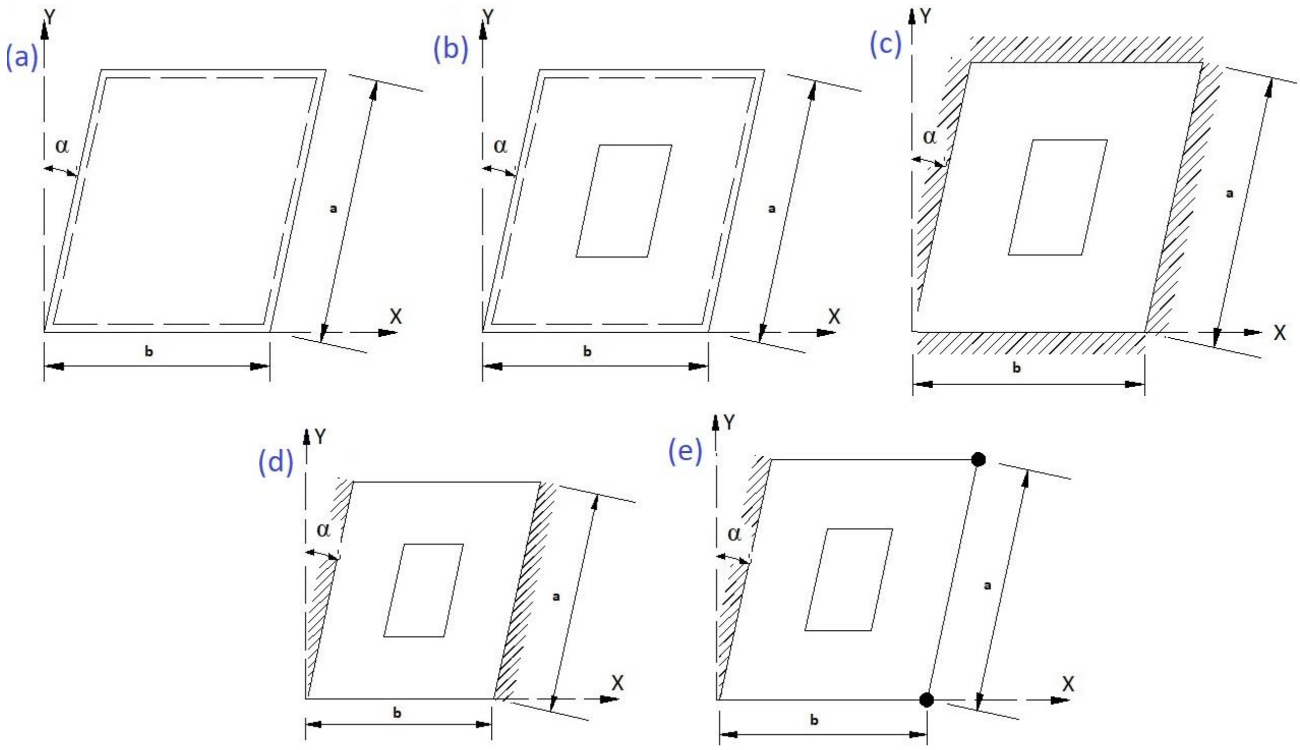


Figure.1. Configuration of plates considered in the study.

***Example 2**: Composite square plate with central square cut-out*

A simply supported, cross-ply (0/90), square laminate with thickness ratio (h/a=0.01) having square cut-outs at the center is considered (Figure. 1b). The study is made for different cut-out sizes where the edges of the cut-out are taken parallel to the edges of the plate. A mesh converge is carried out (not shown here) and in case of composite laminates, convergence is seen at 20x20. Thus, henceforth in this study, the same mesh is used unless otherwise stated. The present results are reported in Table 3 along with those of Sheikh et al. (Sheikh, et al., 2004). Material properties are considered as $E_1=25E_2$, $G_{12}= G_{13}= 0.5E_2$, $G_{23}=0.2E_2$ and $\nu_{12}=0.25$.

From the above two examples, it is seen that the current finite element formulation is capable of producing highly accurate results. Thus, the same formulation is used for analysis of composite skew plate having a skew cut-out at the center of the plate under different situations.

5.2 NUMERICAL RESULTS

***Example 3:** Perforated composite skew plates with and without rotary inertia.*

A simply supported skew-symmetric cross-ply (0/90/0), having a skew cut-out (0.2a × 0.2b) at the plate center is considered (Figure. 1b). The plate is analysed with different thickness ratios (h/a=0.01, 0.1 and 0.2). Both types of mass lumping (MLORI and MLWRI) schemes are used. From Table 4, it is seen that for thin plate there is no effect of rotary inertia. As thickness increases, the effect of rotary inertia also increases. Percentage change of results of both the lumping schemes is also been presented in Table 4. Since the lumping scheme MLWRI is useful for both thick and thin plates, the subsequent examples have been studied for mass lumping scheme MLWRI.

Table 2. Frequencies $\lambda = \omega a^2\sqrt{h\rho/D}$ of a simply supported isotropic skew plate (*h/a*=0.01)

Skew angle (α)	Source	First five natural frequencies				
		1	2	3	4	5
30°	Present (16 × 16)	24.89	52.59	71.62	83.71	122.56
	Liew and Lam (Liew & Lam, 1990)	25.07	52.90	72.34	84.78	-
45°	Present (16 × 16)	34.77	66.20	100.09	106.83	140.46
	Liew and Lam (Liew & Lam, 1990)	34.94	66.42	100.87	107.78	-

Table 3. Frequencies $\lambda = \omega a^2\sqrt{\rho/E_2}\,/h$ of a square laminate with square cut-out at plate center (*h/a*=0.01, *a*=*b*)

Cut-out size	Source	First five natural frequencies				
		1	2	3	4	5
0.2a × 0.2a	Present (20 × 20)	9.11	25.41	25.41	38.00	53.99
	Sheikh et al. (Sheikh, et al., 2004)	9.12	25.50	25.51	38.04	54.03
0.4a × 0.4b	Present (20 × 20)	9.09	20.41	20.43	35.48	44.60
	Sheikh et al. (Sheikh, et al., 2004)	9.09	20.30	20.30	35.46	44.28
0.6a × 0.6b	Present (20 × 20)	11.14	18.51	18.51	32.71	34.34
	Sheikh et al. (Sheikh, et al., 2004)	11.11	18.54	18.55	32.94	34.27

Table 4. Frequencies $\lambda = \omega a^2\sqrt{\rho/E_2}\,/h$ of a simply supported, cross-ply (0/90/0) skew laminate having skew cut-out (0.2a × 0.2b) at the plate center (*a*=*b*, *α* = 30°)

Mass lumping	h/a	First five natural frequencies				
		1	2	3	4	5
MLORI*	0.01	15.45	27.88	50.31	50.71	65.91
MLWRI&		15.45	27.88	50.29	50.70	65.88
% variation		0	0	0	0	0
MLORI (20 × 20)	0.1	12.70	21.45	25.31	27.45	35.18
MLWRI (20 × 20)		12.58	21.23	25.31	27.22	34.69
% variation		0.95	1.036	0	0.845	1.41
MLORI (20 × 20)	0.2	9.60	12.65	14,78	16.55	18.41
MLWRI (20 × 20)		9.43	12.65	14.55	16.02	18.04
% variation		1.8	0	1.58	3.31	2.051

*Mass lumping without rotary inertia / &Mass lumping with rotary inertia

Example 4: *Perforated composite skew plates at different skew angles.*

In the next example, a simply supported skew laminate (0/90/0) with a skew cut-out (0.2a × 0.2b) at the center is analysed considering thickness ratio h/a=0.1. An analysis is performed considering various skew angles (α=15°, 30°, 45°, 60° and 75°) as shown in Figure. 1b. The results are presented in Table 5. As expected, as the skew angle increases, frequency also increases since the mass of the plate decreases.

Example 5: *Perforated composite skew plates with different cut-out sizes.*

Cross-ply (0/90) skew laminate having simply supported (Figure. 1b) and fixed supported (Figure. 1c) along all the four edges with skew laminate (α=30°), thickness ratio h/a=0.01 and different cut-out sizes at the plate center are analysed. The results are reported in Table 6. It is seen that as cut out size increases, frequency decreases due to decrease of the stiffness of the plate in case of simply supported but it is reversed in case of the fixed supported plate.

Example 6: *Perforated composite skew plates with different aspect ratios.*

Next, a four-layer anti-symmetric (0/90/0/90) skew laminate with two inclined edges fixed and other two straight edges are free having central skew cut out (0.2a × 0.2b) is investigated (Figure. 1d). The analysis is performed considering different aspect ratio (a/b=1.0, 1.5, 2.0, 2.5, 3.0). Results are presented in Table 7. It is seen that as the aspect ratio increases frequency decreases, since the mass of the plate increases.

Table 5. Frequencies $\lambda = \omega a^2\sqrt{\rho/E_2}\,/h$ of a simply supported, cross-ply (0/90/0) skew laminate having skew cut-out (0.2a × 0.2b) at the plate center (*a*=*b*, *h*/*a*=0.1)

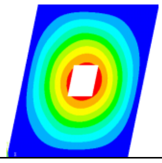
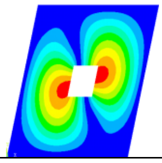
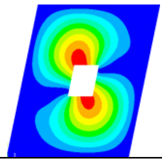
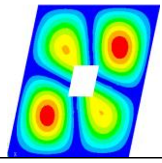
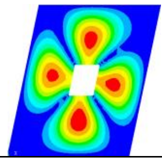
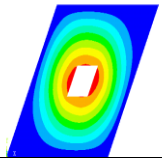
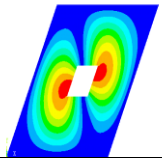
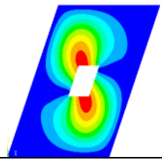
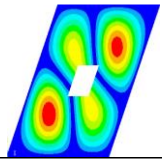
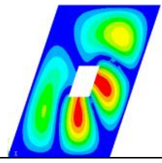
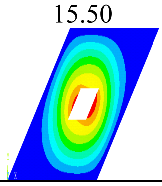
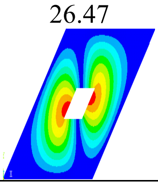
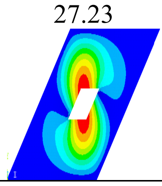
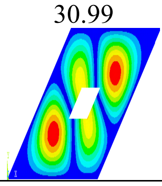
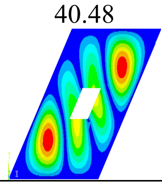
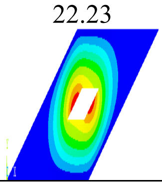
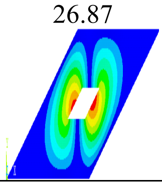
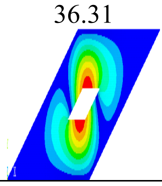
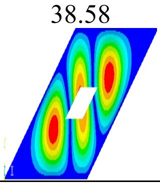
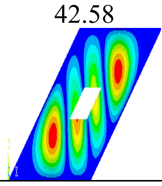
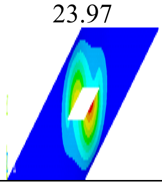
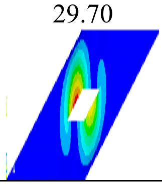
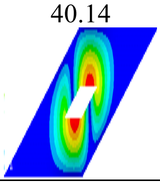
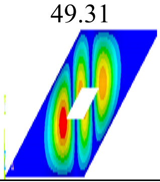
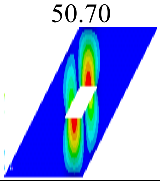
Skew angle	First five natural frequencies				
	1	2	3	4	5
15°	11.41 	18.67 	22.88 	25.57 	27.94 
30°	12.59 	21.23 	25.31 	27.22 	34.69 
45°	15.50 	26.47 	27.23 	30.99 	40.48 
60°	22.23 	26.87 	36.31 	38.58 	42.58 
75°	23.97 	29.70 	40.14 	49.31 	50.70 

Table 6. Frequencies $\lambda = \omega a^2 \sqrt{\rho/E_2}\,/h$ of a simply supported and clamped cross-ply (0/90) skew laminate having skew cutout at the plate center (a=b, α = 30°).

Cut-out size	First five natural frequencies				
	1	2	3	4	5
			Simply supported		
0.1a × 0.1b	13.14	29.43	38.65	46.19	66.84
0.2a × 0.2b	12.75	28.67	35.03	46.17	65.52
0.3a ×.3b	12.74	27.16	29.08	45.38	59.85
0.4a × 0.4b	13.30	24.55	26.12	44.44	50.78
0.6a × 0.6b	12.74	22.44	27.71	40.28	45.99
			Clamped		
0.1a × 0.1b	25.42	45.32	57.38	65.67	89.57
0.2a × 0.2b	26.11	43.97	50.17	65.70	86.31
0.3a × 0.3b	28.69	42.03	46.18	64.64	77.08
0.4a × 0.4b	35.78	42.82	49.47	64.85	68.03
0.6a × 0.6b	70.62	71.48	77.86	82.09	89.27

Table 7. Frequencies $\lambda = \omega a^2 \sqrt{\rho/E_2}\,/h$ of a cross-ply (0/90/0/90) skew laminate having skew cut-out (0.2a × 0.2b) at the plate center (h/a = 0.1, α = 30°)

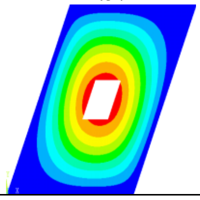
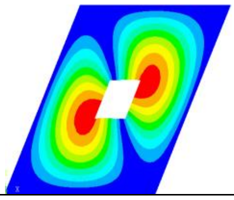
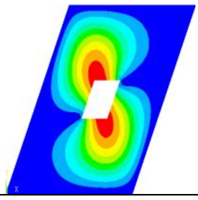
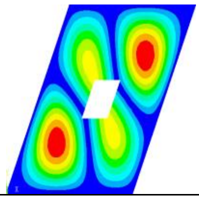
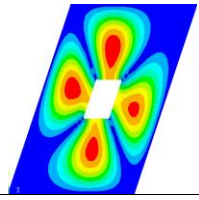
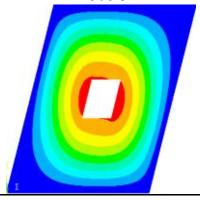
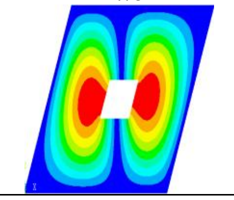
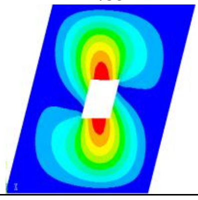
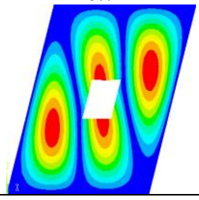
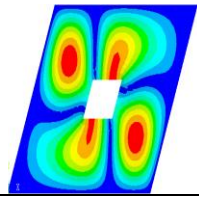
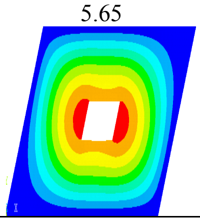
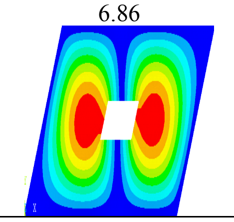
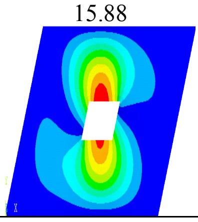
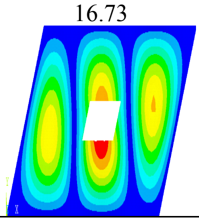
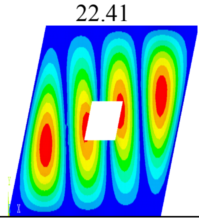
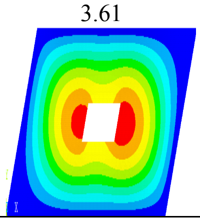
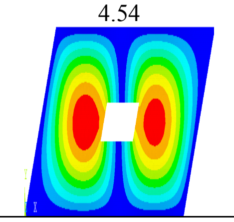
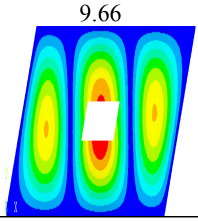
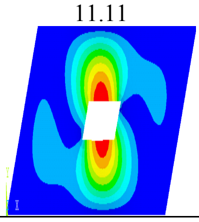
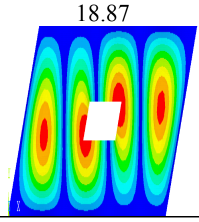
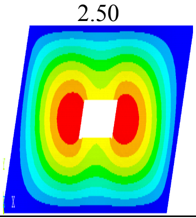
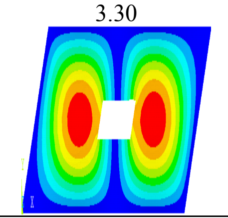
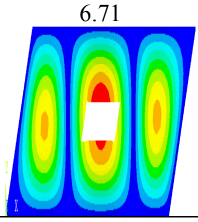
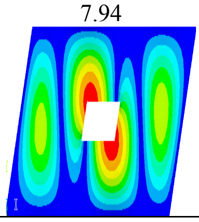
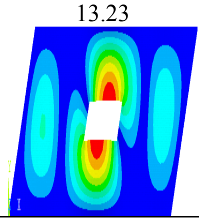
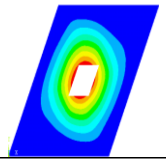
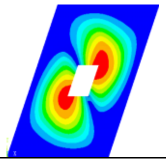
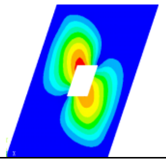
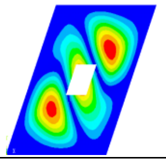
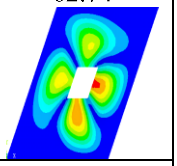
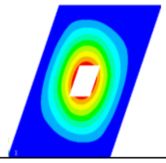
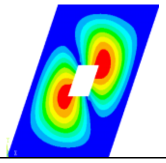
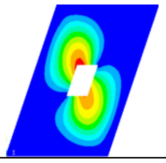
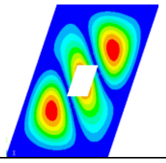
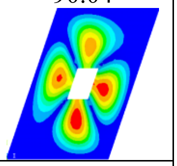
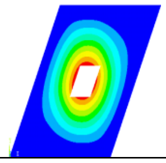
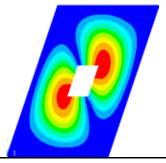
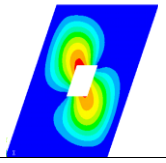
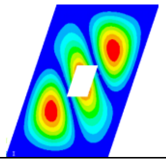
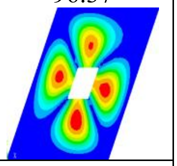
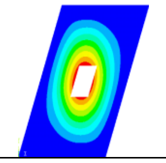
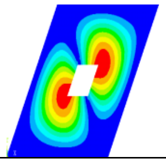
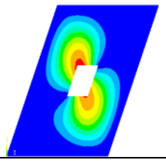
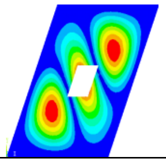
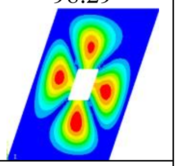
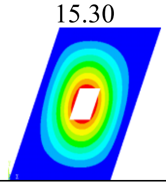
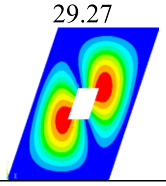
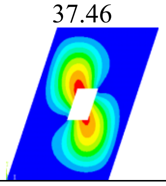
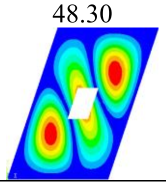
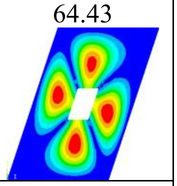
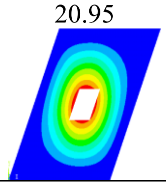
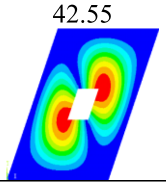
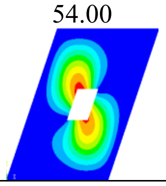
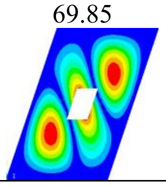
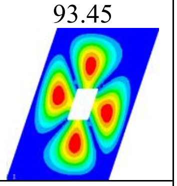
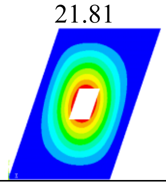
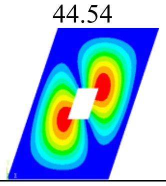
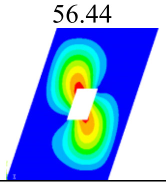
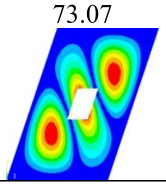
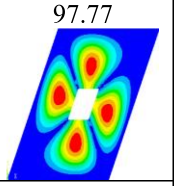
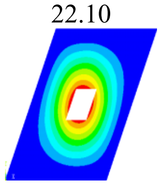
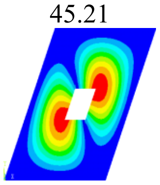
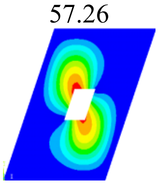
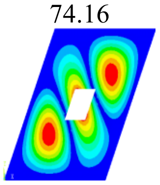
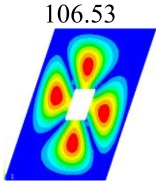
a/b	First five natural frequencies				
	1	2	3	4	5
1	22.57 	24.90 	37.07 	58.81 	63.20 
1.5	10.06 	11.75 	24.85 	26.71 	29.86 
2.0	5.65 	6.86 	15.88 	16.73 	22.41 
2.5	3.61 	4.54 	9.66 	11.11 	18.87 
3.0	2.50 	3.30 	6.71 	7.94 	13.23 

Table 8. Frequencies $\lambda = \omega a^2 \sqrt{\rho/E_2}\,/h$ of simply supported angle-ply skew laminate having skew cutout (0.2a × 0.2b) at the plate center (a=b, α = 30°)

Ply orientations	First five natural frequencies				
	1	2	3	4	5
For symmetric laminate					
45/-45/45	15.77 	32.30 	43.55 	59.65 	62.74 
45/-45/45/-45/45	19.85 	43.34 	50.25 	73.12 	90.04 
45/-45/45/-45/45/-45/45	20.93 	45.17 	52.99 	75.22 	96.37 
45/-45/45/-45/45/-45/45/-45/45	21.40 	45.72 	54.42 	75.86 	98.29 
For anti-symmetric laminate					
45/-45	15.30 	29.27 	37.46 	48.30 	64.43 
45/-45/45/-45	20.95 	42.55 	54.00 	69.85 	93.45 
45/-45/45/-45/45/-45	21.81 	44.54 	56.44 	73.07 	97.77 
45/-45/45/-45/45/-45/45/-45	22.10 	45.21 	57.26 	74.16 	106.53 

Example 7: *Perforated composite skew plates with different number of layers.*

In this example, a simply supported angle ply skew laminate with skew cut out (0.2a × 0.2b) at the center is analysed (Figure. 1b). In this analysis different number of layers is considered and the results are presented in Table 8. Both symmetric and anti-symmetric plates are analysed. As the number of layer increases, stiffness of the laminate increases and thus, frequencies increase.

Example 8: *Perforated composite skew plates with corner point constraints and having different cut-out sizes.*

In the last example an angle-ply skew laminate (30/-30/30) with skew cut out having different sizes at the center is analysed (Figure. 1e). The skew laminate is fixed along the left edge and the opposite two corner points A and B are also restrained with all the five degrees of freedom. The results are presented in Table 9. Here it is seen that as the cut-out size increases the frequency decreases.

Table 9. Frequencies $\lambda = \omega a^2 \sqrt{\rho / E_2}\, / h$ of an angle-ply skew laminate (30/-30/30) having skew cutout at the plate center (a=b, h/a=0.01, α = 45°)

Cut out size	First five natural frequencies				
	1	2	3	4	5
0.1a × 0.1a	10.01	17.12	17.62	34.71	40.43
0.2a × 0.2a	9.89	16.89	17.32	34.05	39.68
0.3a × 0.3a	9.80	16.46	16.84	33.94	38.21

6. CONCLUSION

In this research, the equivalent single layer theories are critically discussed. A brief cross-section of the literature on equivalent single layer theories is reviewed. An exhaustive survey on works related to the analysis of skew composite laminates is also presented. A finite element analysis on free vibration behavior of skew laminates is then carried out. The shear deformation across the thickness is included by considering a first-order shear deformation theory. The rotary inertia effects are also included. Certain numerical examples are solved using the formulation which will serve as benchmark results for future studies.

7. REFERENCES

1. ADINEH, M. & KADKHODAYAN, M., 2017. *Three-dimensional thermo-elastic analysis and dynamic response of a multi-directional functionally graded skew plate on elastic foundation*. Composites Part B: Engineering, Volume 125, pp. 227-240.
2. ALAVI, S. H. & EIPAKCHI, H., 2018. *An analytical approach for free vibrations analysis of viscoelastic circular and annular plates using FSDT*. Mechanics of Advanced Materials and Structures, pp. 1-15.
3. AMBARTSUMIAN, S. A., 1960. *On the theory of bending of anisotropic plates and shallow shells*. Journal of Applied Mathematics and Mechanics, Volume 24, pp. 500-514.
4. AMBARTSUMIAN, S. A., 1969. *Theory of Anisotropic Plates*, translated from Russian by T. Cheron and JE, Ashton, ed., Tech Pub Co.
5. AMBARTSUMIAN, S. A., 1970. *Theory of anisotropic plates: strength, stability, vibration*. s.l.:Technomic Publishing Company.
6. ANLAS, G. & GÖKER, G., 2001. *Vibration analysis of skew fiber-reinforced composite laminated plates*. Journal of sound and vibration, Volume 242, pp. 265-276.
7. ARDESTANI, M. M., ZHANG, L. W. & LIEW, K. M., 2017. *Isogeometric analysis of the effect of CNT orientation on the static and vibration behaviors of CNT-reinforced skew composite plates*. Computer Methods in Applied Mechanics and Engineering, Volume 317, pp. 341-379.
8. ARKHIPOV, V. N., 1968. *On modeling the deformed state of layered plates in flexure*. Soviet Applied Mechanics, Volume 4, pp. 29-32.
9. ASEMI, K., SALAMI, S. J., SALEHI, M. & SADIGHI, M., 2014. *Dynamic and static analysis of FGM skew plates with 3D elasticity based graded finite element modeling*. Latin American Journal of Solids and Structures, Volume 11, pp. 504-533.
10. ASHOUR, A. S., 2009. *The free vibration of symmetrically angle-ply laminated fully clamped skew plates*. Journal of Sound and Vibration, Volume 323, pp. 444-450.
11. ASHTON, J. E. & WHITNEY, J. M., 1970. *Theory of laminated plates*. s.l.:CRC Press.
12. AURICCHIO, F. & TAYLOR, R. L., 1995. *A triangular thick plate finite element with an exact thin limit*. Finite Elements in Analysis and Design, Volume 19, pp. 57-68.
13. BARDELL, N. S., 1992. *The free vibration of skew plates using the hierarchical finite element method*. Computers & structures, Volume 45, pp. 841-874.
14. BARKER, R. M. & PRYOR JR, C. W., 1971. *A finite-element analysis including transverse shear effects for applications to laminated plates*. AIAA journal, Volume 9, pp. 912-917.
15. BEAKOU, A. & TOURATIER, M., 1993. *A rectangular finite element for analysing composite multilayered shallow shells in statics, vibration and buckling.* International Journal for Numerical Methods in Engineering, Volume 36, pp. 627-653.

16. BECKER, W., 1993. *Closed-form solution for the free-edge effect in cross-ply laminates*. Composite Structures, Volume 26, pp. 39-45.
17. BECKER, W., 1994. *Closed-form analysis of the free edge effect in angle-ply laminates*. Journal of applied mechanics, Volume 61, pp. 209-211.
18. BERT, C. W. & MALIK, M., 1996. *The differential quadrature method for irregular domains and application to plate vibration*. International Journal of Mechanical Sciences, Volume 38, pp. 589-606.
19. BHASKAR, K. & VARADAN, T. K., 1993. *Interlaminar stresses in composite cylindrical shells under transient loads*. Journal of sound and vibration, Volume 168, pp. 469-477.
20. BHIMARADDI, A. & STEVENS, L. K., 1984. *A higher order theory for free vibration of orthotropic, homogeneous, and laminated rectangular plates*. Journal of applied mechanics, Volume 51, pp. 195-198.
21. BHIMARADDI, A. & STEVENS, L. K., 1986. *'On the Higher Order Theories in Plates and Shells*. Int. J. Struct, Volume 6, pp. 35-50.
22. BOLLE, L., 1947. *Contribution au probleme lineaire de flexion d'une plaque elastique*. s.l.:Ed. de la Societe du Bulletin technique de la Suisse romande.
23. BYUN, C. & KAPANIA, R. K., 1992. *Prediction of interlaminar stresses in laminated plates using globalorthogonal interpolation polynomials*. AIAA journal, Volume 30, pp. 2740-2749.
24. CARRERA, E., 2007. *On the use of transverse shear stress homogeneous and non-homogeneous conditions in third-order orthotropic plate theory*. Composite structures, Volume 77, pp. 341-352.
25. CHALAK, H. D., CHAKRABARTI, A., SHEIKH, A. H. & IQBAL, M. A., 2014. *C0 FE model based on HOZT for the analysis of laminated soft core skew sandwich plates: Bending and vibration*. Applied Mathematical Modelling, Volume 38, pp. 1211-1223.
26. CHATTERJEE, S. N. & KULKARNI, S. V., 1979. *Shear correction factors for laminated plates*. AIAA Journal, Volume 17, pp. 498-499.
27. CHEUNG, Y. K., THAM, L. G. & LI, W. Y., 1988. *Free vibration and static analysis of general plate by spline finite strip*. Computational mechanics, Volume 3, pp. 187-197.
28. CHOW, T. S., 1971. On the propagation of flexural waves in an orthotropic laminated plate and its response to an impulsive load. Journal of Composite Materials, Volume 5, pp. 306-319.
29. CHOW, T. S., 1975. *Theory of unsymmetric laminated plates*. Journal of Applied Physics, Volume 46, pp. 219-221.
30. CIVALEK, Ö., 2017. *Free vibration of carbon nanotubes reinforced (CNTR) and functionally graded shells and plates based on FSDT via discrete singular convolution method*. Composites Part B: Engineering, Volume 111, pp. 45-59.
31. COOK, R. D. & OTHERS, 2007. *Concepts and applications of finite element analysi*s. s.l.:John Wiley & Sons.
32. COSENTINO, E. & WEAVER, P., 2010. *An enhanced single-layer variational formulation for the effect of transverse shear on laminated orthotropic plates*. European Journal of Mechanics-A/Solids, Volume 29, pp. 567-590.
33. DEY, P. & SINGHA, M. K., 2006. *Dynamic stability analysis of composite skew plates subjected to periodic in-plane load*. Thin-walled structures, Volume 44, pp. 937-942.
34. DOBYNS, A. L., 1981. *Analysis of simply-supported orthotropic plates subject to static and dynamic loads*. AiAA Journal, Volume 19, pp. 642-650.
35. DONG, S. B., 1962. *On the theory of laminated anisotropic shells and plates*. Journal of the Aerospace Sciences, Volume 29, pp. 969-975.
36. DYM, C. L., SHAMES, I. H. & OTHERS, 1973. *Solid mechanics*. s.l.:Springer.
37. EFTEKHARI, S. A. & JAFARI, A. A., 2012. *High accuracy mixed finite element-Ritz formulation for free vibration analysis of plates with general boundary conditions*. Applied Mathematics and Computation, Volume 219, pp. 1312-1344.
38. EFTEKHARI, S. A. & JAFARI, A. A., 2013. *Modified mixed Ritz-DQ formulation for free vibration of thick rectangular and skew plates with general boundary conditions*. Applied Mathematical Modelling, Volume 37, pp. 7398-7426.
39. FALLAH, A., KARGARNOVIN, M. H. & AGHDAM, M. M., 2011. *Free vibration analysis of symmetrically laminated fully clamped skew plates using extended Kantorovich method*. s.l., s.n., pp. 739-744.
40. FALLAH, N. & DELZENDEH, M., 2018. *Free vibration analysis of laminated composite plates using meshless finite volume method*. Engineering Analysis with Boundary Elements, Volume 88, pp. 132-144.
41. FERREIRA, A. J. M., ROQUE, C. M. C. & JORGE, R. M. N., 2005. *Free vibration analysis of symmetric laminated composite plates by FSDT and radial basis functions*. Computer Methods in Applied Mechanics and Engineering, Volume 194, pp. 4265-4278.
42. GANAPATHI, M. & PRAKASH, T., 2006. *Thermal buckling of simply supported functionally graded skew plates*. Composite Structures, Volume 74, pp. 247-250.
43. GARCÍA-MACÍAS, E. et al., 2016. *Static and free vibration analysis of functionally graded carbon nanotube reinforced skew plates*. Composite Structures, Volume 140, pp. 473-490.

44. GARG, A. K., KHARE, R. K. & KANT, T., 2006. *Free vibration of skew fiber-reinforced composite and sandwich laminates using a shear deformable finite element model*. Journal of Sandwich Structures & Materials, Volume 8, pp. 33-53.
45. GRUTTMANN, F. & WAGNER, W., 2017. *Shear correction factors for layered plates and shells*. Computational Mechanics, Volume 59, pp. 129-146.
46. GÜRSES, M., CIVALEK, Ö., KORKMAZ, A. K. & ERSOY, H., 2009. *Free vibration analysis of symmetric laminated skew plates by discrete singular convolution technique based on first-order shear deformation theory*. International journal for numerical methods in engineering, Volume 79, pp. 290-313.
47. HA, K. H., 1990. *Finite element analysis of sandwich plates: an overview*. Computers & Structures, Volume 37, pp. 397-403.
48. HE, D., YANG, W. & CHEN, W., 2017. *A size-dependent composite laminated skew plate model based on a new modified couple stress theory*. Acta Mechanica Solida Sinica, Volume 30, pp. 75-86.
49. HENCKY, H., 1947. *Uber die Berucksichtigung der Schubverzerrung in ebenen Platten*. ingenieur-archiv, Volume 16, pp. 72-76.
50. IDLBI, A., KARAMA, M. & TOURATIER, M., 1997. *Comparison of various laminated plate theories*. Composite Structures, Volume 37, pp. 173-184.
51. JABERZADEH, E., AZHARI, M. & BOROOMAND, B., 2013. *Inelastic buckling of skew and rhombic thin thickness-tapered plates with and without intermediate supports using the element-free Galerkin method*. Applied Mathematical Modelling, Volume 37, pp. 6838-6854.
52. JEGLEY, D. C., 1988. *An analytical study of the effects of transverse shear deformation and anisotropy on natural vibration frequencies of laminated cylinders*.
53. JEMIELITA, G., 1975. *Technical theory of plates with moderate thickness.* Rozprawy Ink, Volume 23, pp. 483-499.
54. JIN, G., SU, Z., SHI, S., YE, T., & GAO, S., 2014a. *Three-dimensional exact solution for the free vibration of arbitrarily thick functionally graded rectangular plates with general boundary condition*s. Composite Structures, Volume 108, pp. 565-577.
55. JIN, G., SU, Z., YE, T. & JIA, X., 2014b. *Three-dimensional vibration analysis of isotropic and orthotropic conical shells with elastic boundary restraints.* International Journal of Mechanical Sciences, Volume 89, pp. 207-221.
56. JIN, C. & WANG, X., 2015. *Weak form quadrature element method for accurate free vibration analysis of thin skew plates.* Computers & Mathematics with Applications, Volume 70, pp. 2074-2086.
57. JIN, G., SHI, S., SU, Z., LI, S., & LIU, Z., 2015a. *A modified Fourier--Ritz approach for free vibration analysis of laminated functionally graded shallow shells with general boundary conditions.* International Journal of Mechanical Sciences, Volume 93, pp. 256-269.
58. JIN, G., SU, Z., YE, T. & GAO, S., 2015b. *Three-dimensional free vibration analysis of functionally graded annular sector plates with general boundary conditions.* Composites Part B: Engineering, Volume 83, pp. 352-366.
59. JIN, G., YE, T. & SHI, S., 2015c. *Three-Dimensional Vibration Analysis of Isotropic and Orthotropic Open Shells and Plates with Arbitrary Boundary Conditions*. Shock and Vibration, Article ID 896204.
60. JIN, G., YE, T., WANG, X. & MIAO, X., 2016c. *A unified solution for the vibration analysis of FGM doubly-curved shells of revolution with arbitrary boundary conditions*. Composites Part B 89, 230-252
61. JONES, R. M., 1975. *Mechanics of composite materials*. s.l.:Scripta Book Company Washington, DC.
62. KABIR, H. R. H., 1996. *A novel approach to the solution of shear flexible rectangular plates with arbitrary laminations.* Composites Part B: Engineering, Volume 27, pp. 95-104.
63. KALITA, K. & HALDAR, S., 2015. *Parametric study on thick plate vibration using FSDT.* Mechanics and Mechanical Engineering, Volume 19, pp. 81-90.
64. KALITA, K., SHINDE, D. & HALDAR, S., 2015. *Analysis on Transverse Bending of Rectangular Plate*. Materials Today: Proceedings, Volume 2, pp. 2146-2154.
65. KALITA, K. & HALDAR, S., 2016. *Free vibration analysis of rectangular plates with central cutout*. Cogent Engineering, Volume 3, p. 1163781.
66. KALITA, K. RAMACHANDRAN, M., RAICHURKAR, P., MOKAL, S. D., & HALDAR, S., 2016a. *Free vibration analysis of laminated composites by a nine node iso-parametric plate bending element*. Advanced Composites Letters, Volume 25, pp. 108-116.
67. KALITA, K., SHIVAKOTI, I., GHADAI, R. K. & HALDAR, S., 2016b. *Rotary Inertia Effect in Isotropic Plates Part I: Uniform Thickness*. Romanian Journal of Acoustics and Vibration, Volume 13, pp. 68-74.
68. KALITA, K., SHIVAKOTI, I., GHADAI, R. K. & HALDAR, S., 2016c. *Rotary Inertia Effect in Isotropic Plates Part II: Taper Thickness*. Romanian Journal of Acoustics and Vibration, Volume 13, pp. 75-80.
69. KALITA, K. & HALDAR, S., 2017. *Eigenfrequencies of simply supported taper*

plates with cut-outs. Structural Engineering and Mechanics, Volume 63, pp. 103-113.
70. KALITA, K. & HALDAR, S., 2018. *Natural Frequencies of Rectangular Plate With-and Without-Rotary Inertia*. Journal of The Institution of Engineers (India): Series C, Volume 99, pp. 539-555.
71. KALITA, K., DEY, P. & HALDAR, S., 2018. *Robust genetically optimized skew laminates*. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, p. 0954406218756943.
72. KALITA, K., NASRE, P., DEY, P. & HALDAR, S., 2018. *Metamodel based multi-objective design optimization of laminated composite plates*. Structural Engineering And Mechanics, Volume 67, pp. 301-310.
73. KASSAPOGLOU, C. & LAGACE, P. A., 1986. *An efficient method for the calculation of interlaminar stresses in composite materials*. Journal of Applied Mechanics, Volume 53, pp. 744-750.
74. KASSAPOGLOU, C. & LAGACE, P. A., 1987. *Closed form solutions for the interlaminar stress field in angle-ply and cross-ply laminates*. Journal of Composite Materials, Volume 21, pp. 292-308.
75. KASSAPOGLOU, C., 1990. *Determination of interlaminar stresses in composite laminates under combined loads*. Journal of reinforced plastics and composites, Volume 9, pp. 33-58.
76. KENNEDY, J. A. H. M., 1964. *Series solution of skew stiffened plates*. ASCE Journal of Engineering Mechanics, 90(1), pp. 1--22.
77. KENNEDY, J. B. & TAMBERG, K. G., 1969. *Problems of skew in concrete bridge design*.
78. KHANDAN, R., NOROOZI, S., SEWELL, P. & VINNEY, J., 2012. *The development of laminated composite plate theories: a review*. Journal of Materials Science, Volume 47, pp. 5901-5910.
79. KIANI, Y., 2016. *Free vibration of FG-CNT reinforced composite skew plates*. Aerospace Science and Technology, Volume 58, pp. 178-188.
80. KIANI, Y., DIMITRI, R. & TORNABENE, F., 2018. *Free vibration of FG-CNT reinforced composite skew cylindrical shells using the Chebyshev-Ritz formulation*. Composites Part B: Engineering, Volume 147, pp. 169-177.
81. KIANI, Y. & MIRZAEI, M., 2018. *Rectangular and skew shear buckling of FG-CNT reinforced composite skew plates using Ritz method*. Aerospace Science and Technology, Volume 77, pp. 388-398.
82. KIL'CHEVSKIY, N. A., 1965. *Fundamentals of the analytical mechanics of shells*, s.l.: s.n.
83. KIM, C. K. & HWANG, M.-H., 2012. *Non-linear analysis of skew thin plate by finite difference method.* Journal of mechanical science and technology, Volume 26, pp. 1127-1132.
84. KIRCHHOFF, G., 1850. *On the equilibrium and deflection of an elastic plate German*. J. Reine Angew. Math, Volume 40, pp. 51-88.
85. KIRCHOFF, G., 1850. *Uber das Gleichgewicht und die Bewegung einer elastischen Scheibe*. Journal fur die reine und angewandte Mathematik (Crelle's Journal), Volume 40, pp. 51-88.
86. KITIPORNCHAI, S., XIANG, Y., WANG, C. M. & LIEW, K. M., 1993. *Buckling of thick skew plates*. International journal for numerical methods in engineering, Volume 36, pp. 1299-1310.
87. KNIGHT JR, N. F. & QI, Y., 1997a. *On a consistent first-order shear-deformation theory for laminated plates*. Composites Part B: Engineering, Volume 28, pp. 397-405.
88. KNIGHT JR, N. F. & QI, Y., 1997b. *Restatement of first-order shear-deformation theory for laminated plates*. International journal of solids and structures, Volume 34, pp. 481-492.
89. KONIECZNY, S. & WOZNIAK, C., 1994. *Corrected 2D-theories for composite plates*. Acta mechanica, Volume 103, pp. 145-155.
90. KRISHNA MURTY, A. V., 1987. *Theoretical modelling of laminated composite plates*. Sadhana, Volume 11, pp. 357-365.
91. KRISHNAN, A. & DESHPANDE, J. V., 1992. *Vibration of skew laminates*. Journal of Sound Vibration, Volume 153, pp. 351-358.
92. KUMAR, R., KUMAR, A. & PANDA, S. K., 2015. *Parametric resonance of composite skew plate under non-uniform in-plane loading*. Structural Engineering and Mechanics, Volume 55, pp. 435-459.
93. KUMAR, R., MONDAL, S., GUCHHAIT, S. & JAMATIA, R., 2017. *Analytical approach for dynamic instability analysis of functionally graded skew plate under periodic axial compression*. International Journal of Mechanical Sciences, Volume 130, pp. 41-51.
94. LAI, S. K., ZHOU, L., ZHANG, Y. Y. & XIANG, Y., 2011. *Application of the DSC-Element method to flexural vibration of skew plates with continuous and discontinuous boundaries*. Thin-Walled Structures, Volume 49, pp. 1080-1090.
95. LEE, S. Y., 2018. *Dynamic Instability Assessment of Carbon Nanotube/Fiber/Polymer Multiscale Composite Skew Plates with Delamination Based on HSDT*. Composite Structures, Volume 200, pp. 757-770.
96. LEKHNITSKII, S. G., 1968. *Anisotropic Plates (translated from Russian by SW Tsai and T. Cheron*). Gordon and Breach Science Publishers, New York, pp. 57-62.
97. LEVINSON, M., 1980. *An accurate, simple theory of the statics and dynamics of elastic*

plates. Mechanics Research Communications, Volume 7, pp. 343-350.

98. LEVY, M., 1877. *Memoire sur la theorie des plaques elastiques planes*.

99. LIBRESCU, L., 1967. *On the theory of anisotropic elastic shells and plates*. International Journal of Solids and Structures, Volume 3, pp. 53-68.

100. LIEW, K. M., HUNG, K. C. & LIM, M. K., 1995. *Vibration characteristics of simply supported thick skew plates in three-dimensional setting*. Journal of applied mechanics, Volume 62, pp. 880-886.

101. LIEW, K. M. & LAM, K. Y., 1990. *Application of two-dimensional orthogonal plate function to flexural vibration of skew plates*. Journal of Sound and Vibration, Volume 139, pp. 241-252.

102. LIEW, K. M., WANG, J., NG, T. Y. & TAN, M. J., 2004. *Free vibration and buckling analyses of shear-deformable plates based on FSDT meshfree method*. Journal of Sound and Vibration, Volume 276, pp. 997-1017.

103. LIEW, K. M., XIANG, Y., KITIPORNCHAI, S. & WANG, C. M., 1993. *Vibration of thick skew plates based on Mindlin shear deformation plate theory*. Journal of Sound and Vibration, Volume 168, pp. 39-69.

104. LOVE, A. E. H., 2013. *A treatise on the mathematical theory of elasticity*. s.l.:Cambridge university press.

105. LU, X. & LIU, D., 1992. *Interlayer shear slip theory for cross-ply laminates with nonrigid interfaces*. AIAA journal, Volume 30, pp. 1063-1073.

106. MAKHECHA, D. P., PATEL, B. P. & GANAPATHI, M., 2001. *Transient dynamics of thick skew sandwich laminates under thermal/mechanical loads*. Journal of reinforced plastics and composites, Volume 20, pp. 1524-1545.

107. MALEKZADEH, P., 2007. *A differential quadrature nonlinear free vibration analysis of laminated composite skew thin plates*. Thin-walled structures, Volume 45, pp. 237-250.

108. MALEKZADEH, P., 2008. *Differential quadrature large amplitude free vibration analysis of laminated skew plates based on FSDT*. Composite structures, Volume 83, pp. 189-200.

109. MALEKZADEH, P. & FIOUZ, A. R., 2007. *Large deformation analysis of orthotropic skew plates with nonlinear rotationally restrained edges using DQM*. Composite Structures, Volume 80, pp. 196-206.

110. MALEKZADEH, P. & KARAMI, G., 2005. *Polynomial and harmonic differential quadrature methods for free vibration of variable thickness thick skew plates*. Engineering Structures, Volume 27, pp. 1563-1574.

111. MALEKZADEH, P. & KARAMI, G., 2006. *Differential quadrature nonlinear analysis of skew composite plates based on FSDT*. Engineering Structures, Volume 28, pp. 1307-1318.

112. MALEKZADEH, P., MAHARLOEI, H. M. & VOSOUGHI, A. R., 2014. *A three-dimensional layerwise-differential quadrature free vibration of thick skew laminated composite plates*. Mechanics of Advanced Materials and Structures, Volume 21, pp. 792-801.

113. MALEKZADEH, P. & ZAREI, A. R., 2014. *Free vibration of quadrilateral laminated plates with carbon nanotube reinforced composite layers*. Thin-Walled Structures, Volume 82, pp. 221-232.

114. MANTARI, J. L., OKTEM, A. S. & SOARES, C. G., 2012. *A new trigonometric shear deformation theory for isotropic, laminated composite and sandwich plates*. International Journal of Solids and Structures, Volume 49, pp. 43-53.

115. MCGEE, O. G. & BUTALIA, T. S., 1994. *Natural vibrations of shear deformable cantilevered skew thick plates*. Journal of sound and vibration, Volume 176, pp. 351-376.

116. MINDLIN, R. D., 1951. *Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates.* Journal of Applied Mechanics, Volume 18, pp. 31-38.

117. MISHRA, P. K., PRADHAN, A. K. & PANDIT, M. K., 2016. *Delamination propagation analyses of spar wingskin joints made with curved laminated FRP composite panels*. Journal of adhesion science and Technology, Volume 30, pp. 708-728.

118. MIZUSAWA, T. & KAJITA, T., 1987. *Vibration of skew plates resting on point supports*. Journal of Sound and Vibration, Volume 115, pp. 243-251.

119. MIZUSAWA, T., KAJITA, T. & NARUOKA, M., 1979. *Vibration of skew plates by using B-spline functions*. Journal of Sound and Vibration, Volume 62, pp. 301-308.

120. MIZUSAWA, T., KAJITA, T. & NARUOKA, M., 1980. *Analysis of skew plate problems with various constraints*. Journal of Sound and Vibration, Volume 73, pp. 575-584.

121. MULLER, P. & TOURATIER, M., 1995. *On the so-called variational consistency of plate models, I. Indefinite plates: evaluation of dispersive behaviour.* Journal of sound and vibration, Volume 188, pp. 515-527.

122. MURTHY, M. V. V., 1981. *An improved transverse shear deformation theory for laminated antisotropic plates*.

123. MURTY, A. V. K. & VELLAICHAMY, S., 1988. *High-order theory of homogeneous plate flexure*. AIAA journal, Volume 26, pp. 719-725.

124. NAGHSH, A. & AZHARI, M., 2015. *Non-linear free vibration analysis of point supported laminated composite skew plates.* International Journal of Non-Linear Mechanics, Volume 76, pp. 64-76.
125. NGUYEN-VAN, H., MAI-DUY, N. & TRAN-CONG, T., 2008. *Free vibration analysis of laminated plate/shell structures based on FSDT with a stabilized nodal-integrated quadrilateral element.* Journal of Sound and Vibration, Volume 313, pp. 205-223.
126. P. SHIMPI, R., 2000. *A layerwise shear deformation theory for two-layered cross-ply laminated plates*. Mechanics of Composite Materials and Structures, Volume 7, pp. 331-353.
127. PADHI, A. & PANDIT, M. K., 2017. *Bending and free vibration response of sandwich laminate under hygrothermal load using improved zigzag theory*. The Journal of Strain Analysis for Engineering Design, Volume 52, pp. 288-297.
128. PAGANO, N. J., 1969. *Exact solutions for composite laminates in cylindrical bending*. Journal of composite materials, Volume 3, pp. 398-411.
129. PAGANO, N. J., 1970a. *Exact solutions for rectangular bidirectional composites and sandwich plates*. Journal of composite materials, Volume 4, pp. 20-34.
130. PAGANO, N. J., 1970b. *Influence of shear coupling in cylindrical. bending of anisotropic laminates*. Journal of composite materials, Volume 4, pp. 330-343.
131. PANC, V., 1975. *Theories of elastic plates*. s.l.:Springer Science & Business Media.
132. PANDIT, M. K., HALDAR, S. & MUKHOPADHYAY, M., 2007. *Free vibration analysis of laminated composite rectangular plate using finite element method.* Journal of Reinforced Plastics and Composites, Volume 26, pp. 69-80.
133. PANDIT, M. K., SHEIKH, A. H. & SINGH, B. N., 2010. *Analysis of laminated sandwich plates based on an improved higher order zigzag theory*. Journal of Sandwich Structures & Materials, Volume 12, pp. 307-326.
134. PANDYA, B. N. & KANT, T., 1988. *Flexural analysis of laminated composites using refined higher-order C° plate bending elements*.
135. PARK, T., LEE, S.-Y. & VOYIADJIS, G. Z., 2009. *Finite element vibration analysis of composite skew laminates containing delaminations around quadrilateral cutouts.* Composites Part B: Engineering, Volume 40, pp. 225-236.
136. PHAN, N. D. & REDDY, J. N., 1985. *Analysis of laminated composite plates using a higher-order shear deformation theory.* International Journal for Numerical Methods in Engineering, Volume 21, pp. 2201-2219.
137. QI, Y. & KNIGHT JR, N. F., 1996. *A refined first-order shear-deformation theory and its justification by plane strain bending problem of laminated plates*. International journal of solids and structures, Volume 33, pp. 49-64.
138. RANGO, R. F., NALLIM, L. G. & OLLER, S., 2015. *Formulation of enriched macro elements using trigonometric shear deformation theory for free vibration analysis of symmetric laminated composite plate assemblies*. Composite Structures, Volume 119, pp. 38-49.
139. REDDY, A. R. K. & PALANINATHAN, R., 1995. *Buckling of laminated skew plates.* Thin-walled structures, Volume 22, pp. 241-259.
140. REDDY, A. R. K. & PALANINATHAN, R., 1999. *Free vibration of skew laminates*. Computers & structures, Volume 70, pp. 415-423.
141. REDDY, J. N., 1984. *A refined nonlinear theory of plates with transverse shear deformation.* International Journal of solids and structures, Volume 20, pp. 881-896.
142. REDDY, J. N., 1984. *A simple higher-order theory for laminated composite plates.* Journal of applied mechanics, Volume 51, pp. 745-752.
143. REDDY, J. N., 2004. *Mechanics of laminated composite plates and shells: theory and analysis*. s.l.:CRC press.
144. REISSNER, E., 1945. *The effect of transverse shear deformation on the bending of elastic plates*. Journal of Applied Mechanics, pp. A69--A77.
145. REISSNER, E., 1947. *On bending of elastic plates*. Quarterly of Applied Mathematics, Volume 5, pp. 55-68.
146. REISSNER, E. & STAVSKY, Y., 1961. *Bending and stretching of certain types of heterogeneous aeolotropic elastic plates.* Journal of applied mechanics, Volume 28, pp. 402-408.
147. REN-HUAI, L. & LING-HUI, H., 1991. *A simple theory for non-linear bending of laminated composite rectangular plates including higher-order effects*. International journal of non-linear mechanics, Volume 26, pp. 537-545.
148. REZVANI, S. S. & KIASAT, M. S., 2018. *Analytical and experimental investigation on the free vibration of submerged stiffened steel plate*. International Journal of Maritime Engineering, Volume 160, pp. A165--A172.
149. ROBINSON, J., 1985. *An evaluation of skew sensitivity of thirty three plate bending elements in nineteen FEM systems*. Nuclear Engineering and Design, Volume 90, pp. 67-85.
150. ROUFAEIL, O. L. & TRAN-CONG, T., 2002. *Finite strip elements for laminated composite plates with transverse shear strain discontinuities*. Composite structures, Volume 56, pp. 249-258.

151. SCHMIDT, R., 1977. *A refined with transverse shear nonlinear theory of plate deformation.* Indust Math, Volume 27, pp. 23-38.
152. SHEIKH, A. H., HALDAR, S. & SENGUPTA, D., 2004. *Free flexural vibration of composite plates in different situations using a high precision triangular element.* Modal Analysis, Volume 10, pp. 371-386.
153. SHIRAKAWA, K., 1983. *Bending of plates based on improved theory.* Mechanics research communications, Volume 10, pp. 205-211.
154. SHOJAEE, M., SETOODEH, A. R. & MALEKZADEH, P., 2017. *Vibration of functionally graded CNTs-reinforced skewed cylindrical panels using a transformed differential quadrature method*. Acta Mechanica, Volume 228, pp. 2691-2711.
155. SINGHA, M. K. & DARIPA, R., 2007. *Nonlinear vibration of symmetrically laminated composite skew plates by finite element method*. International Journal of Non-Linear Mechanics, Volume 42, pp. 1144-1152.
156. SINGHA, M. K. & GANAPATHI, M., 2004. *Large amplitude free flexural vibrations of laminated composite skew plates*. International Journal of Non-Linear Mechanics, Volume 39, pp. 1709-1720.
157. SINGH, B. & CHAKRAVERTY, S., 1994. *Flexural vibration of skew plates using boundary characteristic orthogonal polynomials in two variables*. Journal of sound and vibration, Volume 173, pp. 157-178.
158. SINGH, S. N. L. G. & RAO, G. V., 1996. *Stability of laminated composite plates subjected to various types of in-plane loadings.* International journal of mechanical sciences, Volume 38, pp. 191-202.
159. SKY, Y. S., 1961. *Bending and stretching of laminated aeolotropic plates*. Journal of the Engineering Mechanics Division, Volume 87, pp. 31-56.
160. SOLDATOS, K. P., 1992. *A transverse shear deformation theory for homogeneous monoclinic plates.* Acta Mechanica, Volume 94, pp. 195-220.
161. SRINIVASA, C. V., SURESH, Y. J. & KUMAR, W. P. P., 2014. *Experimental and finite element studies on free vibration of skew plates*. International Journal of Advanced Structural Engineering (IJASE), Volume 6, p. 48.
162. STEIN, M., 1986. *Nonlinear theory for plates and shells including the effects of transverse shearing*. AIAA journal, Volume 24, pp. 1537-1544.
163. STEIN, M. & JEGLEY, D. C., 1987. *Effects of transverse shearing on cylindrical bending, vibration, and buckling of laminated plates*. AIAA journal, Volume 25, pp. 123-129.
164. STEIN, M., SYDOW, P. D. & LIBRESCU, L., 1990. *Postbuckling of long thick plates in compression including higher order transverse shearing effects*. In: Studies in Applied Mechanics. s.l.:Elsevier, pp. 63-86.
165. SUN, C.-T. & WHITNEY, J. M., 1973. *Theories for the dynamic response of laminated plates.* AIAA journal, Volume 11, pp. 178-183.
166. SUNDARARAJAN, N., PRAKASH, T. & GANAPATHI, M., 2005. *Nonlinear free flexural vibrations of functionally graded rectangular and skew plates under thermal environments.* Finite Elements in Analysis and Design, Volume 42, pp. 152-168.
167. SU, Z., JIN, G., SHI, S. & YE, T., 2014c. *A unified accurate solution for vibration analysis of arbitrary functionally graded spherical shell segments with general end restraints.* Composite Structures, Volume 111, pp. 271-284.
168. SU, Z. et al., 2014d. *A unified solution for vibration analysis of functionally graded cylindrical, conical shells and annular plates with general boundary conditions.* International Journal of Mechanical Sciences, Volume 80, pp. 62-80.
169. SU, Z., JIN, G. & WANG, X., 2015a. *Free vibration analysis of laminated composite and functionally graded sector plates with general boundary conditions*. Composite Structures, Volume 132, pp. 720-736.
170. SU, Z., JIN, G., WANG, X. & MIAO, X., 2015b. *Modified Fourier--Ritz approximation for the free vibration analysis of laminated functionally graded plates with elastic restraints*. International Journal of Applied Mechanics, Volume 7, p. 1550073.
171. SU, Z., JIN, G., WANG, Y. & YE, X., 2016b. *A general Fourier formulation for vibration analysis of functionally graded sandwich beams with arbitrary boundary condition and resting on elastic foundations*. Acta mechanica, Volume 227, pp. 1493-1514.
172. SU, Z., JIN, G. & YE, T., 2014a. *Free vibration analysis of moderately thick functionally graded open shells with general boundary conditions*. Composite Structures, Volume 117, pp. 169-186.
173. SU, Z., JIN, G. & YE, T., 2014b. *Three-dimensional vibration analysis of thick functionally graded conical, cylindrical shell and annular plate structures with arbitrary elastic restraints*. Composite Structures, Volume 118, pp. 432-447.
174. SU, Z., JIN, G. & YE, T., 2016a. *Vibration analysis and transient response of a functionally graded piezoelectric curved beam with general boundary conditions.* Smart Materials and Structures, Volume 25, p. 065003.
175. SZILARD, R. & NASH, W. A., 1974. *Theory and analysis of plates, Classical and Numberical Methods*. Journal of Applied Mechanics, Volume 41, p. 1149.

176. TAJ, M. N. A. G., CHAKRABARTI, A. & TALHA, M., 2014. *Bending analysis of functionally graded skew sandwich plates with through-the thickness displacement variations.* Journal of Sandwich Structures & Materials, Volume 16, pp. 210-248.
177. TAMUROV, N. G. & GRUD'EVA, G. A., 1974. *Theory of bending of three-layered plates taking into account the physical nonlinearity of materials*. Soviet Applied Mechanics, Volume 10, pp. 1300-1305.
178. THAI, H.T. & KIM, S.E., 2015. *A review of theories for the modeling and analysis of functionally graded plates and shells.* Composite Structures, Volume 128, pp. 70-86.
179. TIMOSHENKO, S. P. & GERE, J. M., 1961. *Theory of elastic stability*. s.l.:McGraw-Hill, New York.
180. TIMOSHENKO, S. P. & GOODIER, J. N., 1971. *Theory of Elasticity*, McGraw-Hill, New York, 1970. Fok-Ching Chong received the BS degree from the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, in.
181. TIMOSHENKO, S. P. & WOINOWSKY-KRIEGER, S., 1959. *Theory of plates and shells*. s.l.:McGraw-hill.
182. TOURATIER, M., 1991. *An efficient standard plate theory*. International journal of engineering science, Volume 29, pp. 901-916.
183. TOURATIER, M., 1992. *A refined theory of laminated shallow shells.* International Journal of Solids and Structures, Volume 29, pp. 1401-1415.
184. TOURATIER, M. & FAYE, J.P., 1995. *On a refined model in structural mechanics: finite element approximation and edge effect analysis for axisymmetric shells*. Computers & structures, Volume 54, pp. 897-920.
185. TURVEY, G. J., 1977. *Bending of laterally loaded, simply supported, moderately thick, antisymmetrically laminated rectangular plates*. Fibre Science and Technology, Volume 10, pp. 211-232.
186. UFLYAND, Y. S., 1948. *The propagation of waves in the transverse vibrations of bars and plates*. Akad. Nauk. SSSR, Prikl. Mat. Mech, Volume 12, p. 8.
187. UGURAL, A. C. & UGURAL, A. C., 1999. *Stresses in plates and shells*. s.l.:McGraw-Hill Boston.
188. UPADHYAY, A. K. & SHUKLA, K. K., 2013. *Non-linear static and dynamic analysis of skew sandwich plates*. Composite Structures, Volume 105, pp. 141-148.
189. VASIL'EV, V. V., 1992. *Theory of thin plates*. Rossijskaya Akademiya Nauk Izvestiya Mekhanika Tverdogo Tela, Volume 3, pp. 26-47.
190. VIMAL, J., SRIVASTAVA, R., BHATT, A. & SHARMA, A., 2014. *Free vibration analysis of moderately thick functionally graded skew plates*. Engineering Solid Mechanics, Volume 2, pp. 229-238.
191. VINSON, J. R. & CHOU, T.W., 1975. *Composite materials and their use in structures*.
192. VLACHOUTSIS, S., 1992. *Shear correction factors for plates and shells*. International Journal for Numerical Methods in Engineering, Volume 33, pp. 1537-1552.
193. VLASOV, B. F., 1957. *On the equations of bending of plates.* Dokla Ak Nauk Azerbeijanskoi-SSR, Volume 3, pp. 955-979.
194. VOLOKH, K. Y., 1994. *On the classical theory of plates.* Journal of Applied Mathematics and Mechanics, Volume 58, pp. 1101-1110.
195. VUKSANOVIC, D., 2000. *Linear analysis of laminated composite plates using single layer higher-order discrete models*. Composite Structures, Volume 48, pp. 205-211.
196. WANG, A. S. D. & CHOU, P. C., 1972. *A comparison of two laminated plate theories*. Journal of Applied Mechanics, Volume 39, pp. 611-613.
197. WANG, C. M., ANG, K. K., YANG, L. & WATANABE, E., 2000. *Free vibration of skew sandwich plates with laminated facings*. Journal of sound and vibration, Volume 235, pp. 317-340.
198. WANG, C. M., LIM, G. T., REDDY, J. N. & LEE, K. H., 2001. *Relationships between bending solutions of Reissner and Mindlin plate theories*. Engineering structures, Volume 23, pp. 838-849.
199. WANG, S., 1997. *Buckling analysis of skew fibre-reinforced composite laminates based on first-order shear deformation plate theory*. Composite Structures, Volume 37, pp. 5-19.
200. WANG, S., 1997. *Free vibration analysis of skew fibre-reinforced composite laminates based on first-order shear deformation plate theory*. Computers & structures, Volume 63, pp. 525-538.
201. WANG, X., WANG, Y. & YUAN, Z., 2014. *Accurate vibration analysis of skew plates by the new version of the differential quadrature method*. Applied Mathematical Modelling, Volume 38, pp. 926-937.
202. WANG, X. & WU, Z., 2013. *Differential quadrature analysis of free vibration of rhombic plates with free edges*. Applied Mathematics and Computation, Volume 225, pp. 171-183.
203. WANG, X. & YUAN, Z., 2018. *Buckling analysis of isotropic skew plates under general in-plane loads by the modified differential quadrature method*. Applied Mathematical Modelling, Volume 56, pp. 83-95.
204. WANG, Y. Y., LAM, K. Y., LIU, G. R., REDDY, J. N., & TANI, J., 1997. *A strip element method for bending analysis of orthotropic plates*. JSME International Journal Series A

Solid Mechanics and Material Engineering, Volume 40, pp. 398-406.

205. WATTS, G., PRADYUMNA, S. & SINGHA, M. K., 2018. *Free vibration analysis of non-rectangular plates in contact with bounded fluid using element free Galerkin method*. Ocean Engineering, Volume 160, pp. 438-448.

206. WEBBER, J. P. H. & MORTON, S. K., 1993. *An analytical solution for the thermal stresses at the free edges of laminated plates*. Composites science and technology, Volume 46, pp. 175-185.

207. WHITNEY, J. M., 1969a. *Bending-extensional coupling in laminated plates under transverse loading*. Journal of Composite Materials, Volume 3, pp. 20-28.

208. WHITNEY, J. M., 1969b. *Cylindrical bending of unsymmetrically laminated plates*. Journal of Composite Materials, Volume 3, pp. 715-719.

209. WHITNEY, J. M., 1969c. *The effect of transverse shear deformation on the bending of laminated plates*. Journal of Composite Materials, Volume 3, pp. 534-547.

210. WHITNEY, J. M., 1973. *Shear correction factors for orthotropic laminates under static load*. Journal of Applied Mechanics, Volume 40, pp. 302-304.

211. WHITNEY, J. M. & LEISSA, A. W., 1969. *Analysis of heterogeneous anisotropic plates.* Journal of Applied Mechanics, Volume 36, pp. 261-266.

212. WHITNEY, J. M. & PAGANO, N. J., 1970. *Shear deformation in heterogeneous anisotropic plates*. Journal of applied mechanics, Volume 37, pp. 1031-1036.

213. WOO, K. S., HONG, C. H., BASU, P. K. & SEO, C. G., 2003. *Free vibration of skew Mindlin plates by p-version of FEM*. Journal of Sound and Vibration, Volume 268, pp. 637-656.

214. XUE, Y., JIN, G., DING, H. & CHEN, M., 2018. *Free vibration analysis of in-plane functionally graded plates using a refined plate theory and isogeometric approach.* Composite Structures, Volume 192, pp. 193-205.

215. YADAV, D., SHARMA, A. & SHIVHARE, V., 2015. *Free vibration analysis of isotropic plate with stiffeners using finite element method*. Engineering Solid Mechanics, Volume 3, pp. 167-176.

216. YANG, P. C., NORRIS, C. H. & STAVSKY, Y., 1966. *Elastic wave propagation in heterogeneous plates.* International Journal of solids and structures, Volume 2, pp. 665-684.

217. YORK, C. B. & WILLIAMS, F. W., 1995. *Buckling analysis of skew plate assemblies: classical plate theory results incorporating Lagrangian multipliers*. Computers & structures, Volume 56, pp. 625-635.

218. YE, T., JIN, G. & SU, Z., 2014. *Three-dimensional vibration analysis of laminated functionally graded spherical shells with general boundary conditions*. Composite Structures 116, pp. 571–588

219. YE, T., JIN, G. & ZHANG, Y., 2015. *Vibrations of composite laminated doubly-curved shells of revolution with elastic restraints including shear deformation, rotary inertia and initial curvature*. Composite Structures 133, pp. 202–225

220. YE, T. & JIN, G., 2016. *Elasticity solution for vibration of generally laminated beams by a modified Fourier expansion-based sampling surface method.* Computers and Structures 167,115–130

221. YE, T., JIN, G. & SU, Z., 2016a. *Three-dimensional vibration analysis of functionally graded sandwich deep open spherical and cylindrical shells with general restraints*. Journal of Vibration and Control, Vol. 22(15) 3326–3354

222. YE, T., JIN, G. & SU, Z., 2016b. *A spectral-sampling surface method for the vibration of 2-D laminated curved beams with variable curvatures and general restraints*. International Journal of Mechanical Sciences 110,170–189

223. YE, T., JIN, G. & SU, Z., 2017. *Three-dimensional vibration analysis of sandwich and multilayered plates with general ply stacking sequences by a spectral-sampling surface method*. Composite Structures 176, 1124–1142.

224. YU, T. T., YIN, S., BUI, T. Q. & HIROSE, S., 2015. *A simple FSDT-based isogeometric analysis for geometrically nonlinear analysis of functionally graded plates.* Finite Elements in Analysis and Design, Volume 96, pp. 1-10.

225. YU, T., YIN, S., BUI, T. Q., XIA, S., TANAKA, S., & HIROSE, S., 2016. *NURBS-based isogeometric analysis of buckling and free vibration problems for laminated composites plates with complicated cutouts using a new simple FSDT theory and level set method*. Thin-Walled Structures, Volume 101, pp. 141-156.

226. ZAMANI, M., FALLAH, A. & AGHDAM, M. M., 2012. *Free vibration analysis of moderately thick trapezoidal symmetrically laminated plates with various combinations of boundary conditions*. European Journal of Mechanics-A/Solids, Volume 36, pp. 204-212.

227. ZENG, H. & BERT, C. W., 2001. *Free vibration analysis of discretely stiffened skew plates*. International Journal of Structural Stability and Dynamics, Volume 1, pp. 125-144.

228. ZGHAL, S., FRIKHA, A. & DAMMAK, F., 2018. *Free vibration analysis of carbon nanotube-reinforced functionally graded composite shell structures.* Applied Mathematical Modelling, Volume 53, pp. 132-155.

229. ZHANG, L. W., 2017. *On the study of the effect of in-plane forces on the frequency parameters of CNT-reinforced composite skew plates*. Composite Structures, Volume 160, pp. 824-837.

230. ZHANG, L. W., ARDESTANI, M. M. & LIEW, K. M., 2017. *Isogeometric approach for buckling analysis of CNT-reinforced composite skew plates under optimal CNT-orientation*. Composite Structures, Volume 163, pp. 365-384.
231. ZHANG, L. W., LEI, Z. X. & LIEW, K. M., 2015a. *Free vibration analysis of functionally graded carbon nanotube-reinforced composite triangular plates using the FSDT and element-free IMLS-Ritz method*. Composite Structures, Volume 120, pp. 189-199.
232. ZHANG, L. W., LEI, Z. X. & LIEW, K. M., 2015b. *Vibration characteristic of moderately thick functionally graded carbon nanotube reinforced composite skew plates*. Composite Structures, Volume 122, pp. 172-183.
233. ZHAO, X., LEE, Y. Y. & LIEW, K. M., 2009. *Free vibration analysis of functionally graded plates using the element-free kp-Ritz method*. Journal of sound and Vibration, Volume 319, pp. 918-939.
234. ZHOU, D., LO, S. H., AU, F. T. K., CHEUNG, Y. K., & LIU, W. Q., 2006. *3-D vibration analysis of skew thick plates using Chebyshev--Ritz method*. International journal of mechanical sciences, Volume 48, pp. 1481-1493.
235. ZHOU, L. & ZHENG, W. X., 2008. *Vibration of skew plates by the MLS-Ritz method*. International Journal of Mechanical Sciences, Volume 50, pp. 1133-1141.

