DEVELOPMENT OF A MATHEMATICAL MODEL FOR PERFORMANCE PREDICTION OF PLANING CATAMARAN IN CALM WATER

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SUMMARY

In this paper, an attempt has been made to predict the performance of a planing catamaran using a mathematical model. Catamarans subjected to a common hydrodynamic lift, have an extra lift between the two asymmetric half bodies. In order to develop a mathematical model for performance prediction of planing catamarans, existing formulas for hydrodynamic lift calculation must be modified. Existing empirical and semi-empirical equations in the literature have been implemented and compared against available experimental data. Evaluation of lift in comparison with experimental data has been documented. Parameters influencing the interaction between demi-hulls and separation effects have been analyzed. The mathematical model for planing catamarans has been developed based on Savitsky's method and results have been compared against experimental data. Finally, the effects of variation in hull geometry such as deadrise angle and distance between two half bodies on equilibrium trim angle, resistance and wetted surface have been examined.

NOMECLATURE

B	Molded beam (m)
B_1	Half body width (m)
m	Ship's weight (kg)
T	Thrust (N)
R_f	Frictional resistance (N)
a	Horizontal distance between frictional resistance and Center of the mass, CoM (m)
ε	Angle of Thrust line to keel (deg)
c	Perpendicular distance of the thrust line to the CoM (m)
N	Lift force (N)
τ	Trim (deg)
C_v	Froude number
V	Ship's Velocity (m/s)
λ	Wetted length to width ratio
L_c	Wetted chine length (m)
L_k	Wetted keel length (m)
$Β$	Deadrise angle (deg)
C_L	Lift Coefficient
V_m	Relative velocity (m/s)
S	Wetted surface (m^2)
Δ	Ship Displacement (t)
LCG	Longitudinal Center of gravity (m)
C_p	Center of Pressure
VCG	Vertical Center of gravity (m)
A	Interference coefficient
R_p	Pressure resistance (N)

1. INTRODUCTION

Multihull craft has been successfully deployed and their usefulness has been proved over the last few decades. The catamaran is a twin hull vessel featuring two parallel hulls of equal size. High-speed, low power engine, high stability, wide beam etc., are the main features of these vessels. Compared to mono-hull vessels, the catamarans have major capabilities in damping the effect of induced transverse waves. Because of their capabilities and features, catamarans have got the desired attention for various applications since 1980 (Allan, 1996). In catamaran planing boats, hydrodynamic lift acting on the hull results in consequent reduction in wetted surface area (Morabito, 2011). Since the demi-hulls are prismatic in nature, it is quite appropriate to state that separation and interference effects between the two half bodies are quite possible. These effects have been shown in Figure 1.

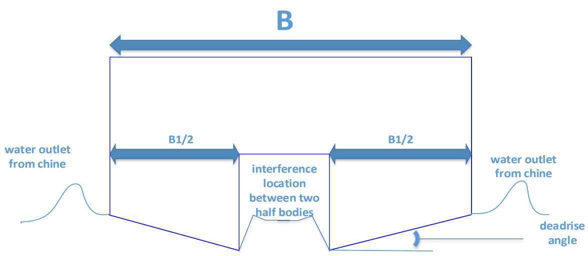

Figure 1: Interference location and effects between two half bodies

So far, some researchers have been working toward the development of Savitsky method (Savitsky, 1964) to achieve appropriate algorithm leading to performance prediction of catamaran planing boats. Earlier work on performance prediction of mono-hulls by Savitsky (1964) has been utilized to calculate the hydrodynamic forces. Furthermore, the mathematical model that was proposed by Savitsky (1964) could provide useful guidelines on the trim, resistance and wetted surface are of the mono-hulls (1964).

As discussed by Faltinsen (Faltinsen, 2005), if the divergent waves generated by one hull impinge on and become reflected by the other hull, then the wave field generated by a multihull vessel cannot be a simple superposition of wave fields produced by each hull. This happens if the hulls are sufficiently close to each other, and as a consequence, the complex wave pattern in the central region will have a strong influence on the hull hydrodynamics. Therefore, Savitsky and Dingee (Savitsky & Dingee, 1954) worked in this context. Their study on two flat parallel plates concluded that the lift force obtained from these two separated plates was about 40 percent more than a single plate of equal area. However, this increased in the percentage of lift force was strongly influenced on the separation between hulls. Thus, according to Savitsky and Dingee's (1954) study, it can be concluded that Savitsky's algorithm (Savitsky, 1964) needs some transformation to be applicable for catamaran vessels. Liu and Wang (Liu & Wang, 1979) introduced two important parameters of separation effects, *r*, and the interaction factor between two bodies, *A*, to the primary relationship for parallel plates and it was considered as the foundation characteristics of Thong-See's study (Thong-See, 1982).

Thong-See (1982) considered the separation effects as a ratio of the two half bodies width to overall width and interaction factor was linked to distances between two hulls and speed of the vessel. Based on experiment carried out by Thong-See (1982), it was determined that the interaction factor was in the range of 1 to $\sqrt{2}$. Korvin-Kroukovsky, Savitsky and Lehman (Kroukovsky, Savitsky & Lehman, 1949) have also offered a relationship to calculate the catamaran lift factor, similar to Liu and Wang (1979) which was different between the dynamic and buoyancy coefficient that was proposed earlier. On the other hand, Shuford (Shuford, 1957) had conducted two tests to evaluate the hydrodynamic characteristics of each half-hulls. In his work, he had used flat plate in horizontal and vertical configuration and observed that the hydrodynamic lift had a greater role in overall lift force. Shuford (1957) proposed an equation to predict and calculate the flat plate's lift force. Shuford's (1957) equations had been revised by Brown (Brown, 1971) and he also proposed a relationship to calculate the sum of dynamic lift and buoyancy of the half body at low-speed.

Morabito (Morabito, 2011) employed Shuford's (1957) and Brown's (1971) equations to calculate the lift coefficient of the half body. Shuford's (1957) and Brown's (1971) achievements were compared against experimental data and a little difference were observed. According to Morabito (2011), Shuford's (1957) & Brown's (1971) relationships were used to develop the mathematical model for calculating of catamaran's performance. However, the relationships did not include modifications to consider the effects of separation and interference, and thus, makes it difficult to use Shuford's (1957) and Brown's (1971) equations. Also, Brown's (1971) equations were limited to low-speed operation, which curtailed its use in the planing phase. However, based on Morabito's (2011) and Savitsky's equations, Ghadimi et al. (Ghadimi et al, 2014) utilized Morabito's mathematical model (Morabito, 2011) to calculate the lift force of a planing hull. Morabito's relation (2011) was used by researchers previously (Ghadimi et al, 2013) in prediction of total pressure distribution on hulls and to calculate the lift force on a planing hull. The results were compared against the Savitsky's formulation (1964) which showed fairly good agreement. However, Bari and Matveev (Bari & Matveev, 2016) used potential flow method for acquiring planing catamarans hydrodynamics for lift coefficient and centre of pressure for a variety of geometrical parameters. In this work, only hard chine prismatic symmetric hulls were considered and it is to be noted that both hydrodynamic and hydrostatic lift are important as in mono-hull planing boats (Matveev, 2015). Parametric transformations were carried out for symmetric hulls at variant speed regimes with different spacing, hull aspect ratio, and deadrise angles. The lift coefficient was found to increase with reduced spacing, higher aspect ratios at moderate and high Froude numbers.

In this paper, various relationships in calculating dynamic lift are assessed to develop Morabito (2011) and Savitsky (1964), Prowse and Lueders (Savitsky, Prowse & Lueders, 1958). The results were compared against experimental values and the best relations were extracted. In addition, corrections regarding the effects of interference and separation have been considered. By modifying the required relationships, an appropriate computer program for calculating catamaran performance in calm water has been presented. Seif and Amini (Seif, Amini, 2004) laboratory models were used for validation of the results of the mathematical model.

2. PROBLEM DEFINITION

Due to the nature of catamaran hull features, additional hydrodynamic forces are induced between hulls. As shown in Figure 2, the shaded part of the surface is wet, but there are additional wetted surface between the demi-hulls. The main factor in additional hydrodynamic lift force can be attributed to the location of flow separation. Parameters such as separation and interaction effects are required to calculate the additional force and the relationship have been proposed in the following sections.

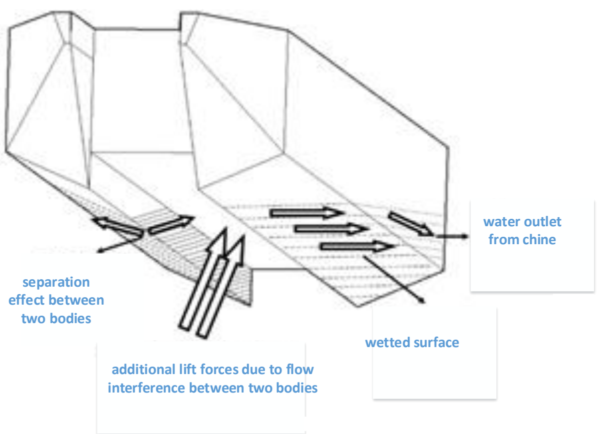

Figure 2: Additional parts of the resistance in catamaran

It should be mentioned that in previous experiments, only the demi-hull was considered and the effect of interaction and separation were not considered. It is clearly known that the effects of two bodies on lift coefficient and hydrodynamic performance of the catamaran vessels are undeniable and also considerable. Thus, the accuracy of the proposed equation is assessed in two steps. Firstly, existing relationships regardless of the interaction effects are compared with their experimental data and in the next step the effects of interaction and separation effects applied. The proposed mathematical model will be evaluated against experimental results for a planing catamaran.

3. MATHEMATICAL MODEL

High-speed boat is subjected to torque due to horizontal and vertical forces. This torque is zero when the vessel moves in balanced trim. Figure 3 illustrates forces and their locations during forward speed. It should be noted that the force diagram for mono-hull and multihull planing vessel are same and only parameters such as separation effects and interaction will be incorporated with the lift coefficient.

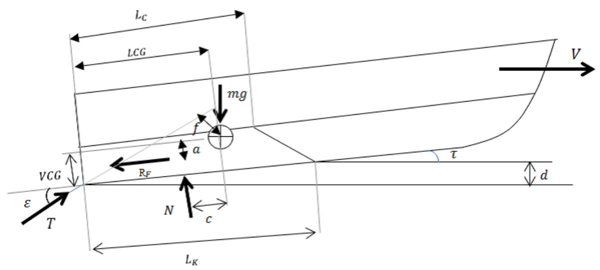

Figure 3: Effective forces on planing vessels

In general, all planing vessels equilibrium equation such as mono-hull or catamaran in horizontal, vertical and also pitch torque, is written as the equation (1)

$$\begin{aligned} \uparrow &: N\cos\tau + T\sin(\tau+\varepsilon) - mg - R_f\sin\tau = 0 \\ \rightarrow &: T\cos(\tau+\varepsilon) - N\sin\tau - R_f\cos\tau = 0 \\ CG &: Nc + R_f a - Tf = 0 \end{aligned} \quad (1)$$

If equilibrium forces are considered along the keel line, then equation (1) can be written as

$$T\cos\varepsilon = mg\sin\tau + R_f \quad (2)$$

The molded beam has been used to calculate the speed coefficient or Froude number.

$$Cv = \frac{V}{\sqrt{g.B}} \quad (3)$$

Also, the ratio of wetted length to the width of the ship is calculated as

$$\lambda = \frac{(L_k - L_c)}{B_1} \quad (4)$$

where, L_c is the wetted chine length and, L_k is the wetted length of the keel, B and B_1 are illustrated in Figure 1.

The hydrodynamic lift should be as large as possible to support the weight of the vessel. So, the lift coefficient with the deadrise angle β can be calculated by equation (5).

$$C_{L\beta} = \frac{mg}{0.5\rho V^2 B^2} \quad (5)$$

Using equation (5), the lift coefficient of a flat plate with a zero deadrise angle is calculated by equation (6)

$$C_{L\beta} = C_{L0} - 0.0065\beta C_{L0}^{0.6} \quad (6)$$

Obtaining the lift coefficient of the vessel, the ratio of the wetted keel length to the wetted beam of the ship can be determined.

It should be noted that in catamaran, an extra lift force is created due to interference between two half frames. The additional lift forces due to the separation effect and interference factor cause the changes in resistance, trim and wetted surface. In this regard, Liu and Wang (1979) used two effective parameters in correcting Savitsky's relationship (1964), separation effect parameter and interference parameter.

3.1 LIFT COEFFICENT CALCULATION

The equations offered by Kroukovsky, Savitsky and Lehman (1949), Liu and Wang (1979), Shuford (1957) and, Brown (1971) have been used to obtain catamaran half body lift coefficient. The first two are similar with respect to calculating lift coefficient. Firstly, trim, speed factor and the ratio of wetted length to half hull width of the body are placed in Kroukovsky (equation (7)) and Wang (equation (8)) which is the lift coefficients of the half body at zero deadrise angle, is obtained.

$$C_{L0} = \tau_{deg}^{1.1}\left(0.012\lambda^{0.5} + 0.0095\frac{\lambda^2}{Cv^2}\right) \quad (7)$$

$$C_{L0} = \tau_{deg}^{1.1}\left(0.012\lambda^{0.5} + 0.005\frac{\lambda^{2.5}}{Cv^2}\right) \quad (8)$$

After the lift coefficient calculated at zero deadrise angle, the half body deadrise angle is placed instead in equation (6) and the lift coefficients are obtained by considering the effect of deadrise angle for half body. Finally, the obtained values can be compared with experimental results.

However, Shuford (1957) and Brown (1971) empirical equations can also be used to calculate lift coefficient due to the ratio of the wetted length to deadrise angle, velocity coefficient and trim, and the effect of the deadrise angle can taking into account. This equation contains three parts as follow:

1. Linear part of lift coefficient which is obtained from

$$C_{L,L} = \frac{0.5\pi \sin\tau \cos^2\tau}{1+1/\lambda}(1-\sin\beta) \tag{9}$$

2. The lift caused by the intersection of the fluid flow, which is the force generated by the velocity of the fluid on keel

$$C_{L,x} = \frac{4\lambda \sin^2\tau \cos^3\tau \cos\beta}{3} \tag{10}$$

3. And the hydrostatic lift part, which is usually about 50% of the buoyancy force (1971).

$$C_{L,s} = \tau 0.0109 \frac{\lambda^2}{Cv^2} \tag{11}$$

Equations (9), (10) and (11) together give the lift coefficient of the hydrostatic term.

$$C_{Lift} = C_{L,L} + C_{L,x} + C_{L,s} \tag{12}$$

Finally, to determine the proper relationship to calculate the half body lift, lift coefficient results obtained from analytical and empirical models are compared with experimental results of Savitsky, Prowse and Lueders (1958) and, Morabito (2011). Savitsky, Prowse and Lueders (1958) twined flat plate in the laboratory and drove lift coefficient in high speed operation. In order to determine the impact of deadrise angle, tests were conducted at different heel angles. In Morabito's (2011) model, different angles were done on the model and the test was carried out at low-speeds. Lift coefficient mentioned before in two different modes was extracted for different deadrise angles, trim and ratio of wetted length to vessel's width in a towing tank. According to the experimental conditions mentioned above, the existing analytical and experimental formulas used to calculate the lift coefficient and, the results of calculations are compared with experimental data.

By calculating the percentage of the coefficient error obtained from Kroukovsky, Savitsky and Lehman relations (1949), Lui and Wang (1979) and the empirical relation of Shuford (1957) and Brown (1971) and then Compared to laboratory results, the proper hydrodynamic model to determine the half body lift coefficient obtained. Figure 4 is demonstrated the error obtain from Savistsky, Prowse and Lueders (1958) experimental test with analytical data and, errors obtained from Morabito (2011) experiments compared to analytical results are shown in figure 5.

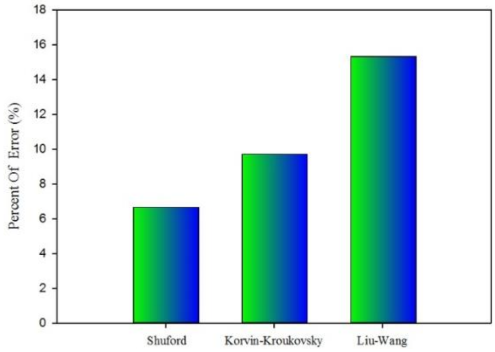

Figure 4: Obtained error from Savitsky, Prowse and Lueders experimental data compare to analytical results (1958)

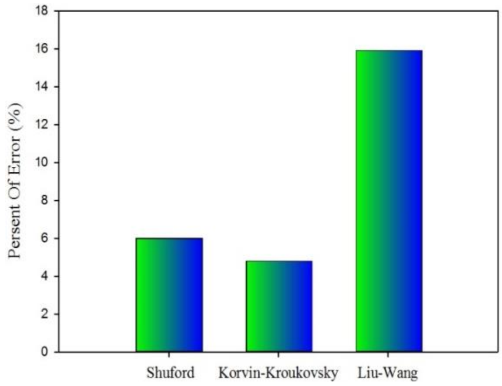

Figure 5: Obtained error from Brown experimental data compare to analytical results (Morabito, 2011)

From Figure 4 and 5, it is clear that Shuford (1957), Brown (1971) and, Kroukovsky, Savitsky and Lehman (1949) margin of error is less than Liu and Wang's (1979). Also, in Savitsky, Prowse and Lueders experiment (1958), Shuford (1957) and Brown (1971) had better performance than Kroukovsky, Savitsky and Lehman (1949). However, in the Morabito's work (2011), accuracy is better than Shuford (1957) and Brown (1971).

It should be noted that Shuford's (1957) and Brown's (1971) relations are suitable for analysis of the hydrodynamic model of half body. To calculate the hydrodynamic lift of two asymmetric half bodies, modified parameters should be added to the relations, which requires further investigation and is currently not considered. In this regards, only the Liu and Wang (1979), Kroukovsky, Savitsky and Lehman (1949) relations are appropriate for multi hull vessels and have the ability to add modifier parameters. In this context, existing relationships would apply to the mathematical model of catamaran vessels and separation and interference parameters which are offered in references are introduced. Verification of mathematical model against experimental data to calculate the appropriate relationship for catamaran lift force will be extracted.

3.2 SEPARATION PARAMETER

The relationship for the separation effect is obtained by dividing the width of half body by the total width of the ship as shown in equation (13)

$$r = B_1 / B \tag{13}$$

To enter the separation effect parameter into the lift coefficient equations, two relationships had developed separately by Kroukovsky, Savitsky and Lehman (1949) and Liu and Wang (1979). These equations are used to calculate the wetted length and the other parameters in wetted area. Kroukovsky, Savitsky and Lehman (1949) defined r as separation effect parameter and used it as follows

$$C_{L0} = r^{3/2}.\tau_{deg}^{1.1}\left(0.012\lambda^{0.5} + 0.0095\frac{\lambda^2}{Cv^2}\right) \tag{14}$$

however, in Liu and Wang (1979), the effect parameter was a little different as shown in equation (15)

$$C_{L0} = r^{3/2}\tau_{deg}^{1.1}\left(0.012\lambda^{0.5} + 0.005\frac{\lambda^{2.5}}{Cv^2}\right) \tag{15}$$

consequently, this difference could make considerable changes in lift coefficient of the flat plate.

From equation (14) and (15), the flat plate lift coefficient consists of two parts: dynamic and static. The difference in relations (14) and (15) are in the static section that the Kroukovsky, Savitsky and Lehman (1949) relationship added the hydrostatic effect as equation (16) to the lift coefficient while have used a different coefficient, 0.005, instead.

$$\tau_{deg}^{1.1} 0.0095\frac{\lambda^2}{Cv^2} \tag{16}$$

Of course, the coefficients were obtained from regression analysis of experimental data results. The experimental measurements indicate that the wetted length of the catamaran is not small, and Kroukovsky, Savitsky and Lehman (1949) relationship is better for predicting the values of λ. Nevertheless, in this paper, both relationships (14) and (15) were used and evaluated. The interference effects in equation (14) and (15) have not been implemented.

3.3 INTERFERENCE EFFECT

Interference effect parameter, A, is related to the distance between the two half bodies and the Froude number, obtained from experimental results shown in Table 1.

These charts shown by Liu and Wang (1979) where the coefficient of interference could be obtained empirically. However, in 1982, Thong-See (1982) concluded that the coefficient of interference is dependent on the velocity coefficient and the ratio of the wetted length to the total width. The important thing is that, Thong-See's (1982) relationship unlike previous relationships and formulations which were limited to the Froude Number or velocity, is not limited to the Froude Number. Therefore, a range of velocity can be used in this equation.

Previously, Insel and Molland (Insel & Molland, 1991) have worked on the resistance components of high speed displacement and semi-displacement catamarans. In this work, interference effects for both the wave pattern and the viscous resistance components are derived. The effects of the hull separation and beam ratio at different speed are discussed and the effect of the interference effect on wave and viscose resistances are described.

However, in order to investigate the degree of accuracy of different relationships, both Thong-See (1982) reforms and Liu and Wang amendments (1979) have been used to apply interference effects. Noticing that, equations (14) and (15) consider the separation effects to apply the effects of interference in them, the values of Liu and Wang (1979) and also Thong-See (1982) can be used. In this situation, four relationships will be obtained:

a) Kroukovsky- Thong-See

$$C_{L0} = r^{3/2}\tau_{deg}^{1.1}\left(0.0012\lambda_w^{0.5} + 0.0095\frac{\lambda_w^2}{C_v^2 r}\right) \tag{17}$$

b) Kroukovsky-Liu and Wang

$$C_{L0} = r^{3/2}r_{deg}^{1.1}\left(0.0141\frac{\lambda_w^{0.5}}{A} + 0.0095\frac{\lambda_w^2 A}{Cv^2 r}\right) \tag{18}$$

c) Liu & Wang-Thong-See

$$C_{L0} = r^{3/2}\tau_{deg}^{1.1}\left(0.012\lambda_w^{0.5}A + 0.005\frac{\lambda_w^{2.5}}{Cv^2 r}\right) \tag{19}$$

d) Liu & Wang-Liu & Wang

$$C_{L0} = r^{3/2}\tau_{deg}^{1.1}\left(0.012\frac{\lambda_w^{0.5}}{A} + 0.005\frac{\lambda_w^{2.5}A}{Cv^2 r}\right) \tag{20}$$

In equations (17) and (19), the effects of interference with the Thong-See's (1982) relationship was applied and in equations (18) and (20) Liu and Wang (1979) relationship was used as the effect of the interferences. Relationships (17) through (20) have different values for wetted length and characteristic of wetted surface, therefore different resistance values and trim will be achieved.

Table:1 Interference parameter between two bodies

Distance Between Hulls. No of (B1/2) beams	A: Cv=2	Cv=4	Liu & Wang [6] 2≤Cv≤3.5	Cv=6	Savitsky & Dingee [5] Cv=7.5	Cv=8.0	Cv=10.0
0	1.1	1.21	1.4	1.35	1.45	1.52	1.75
0.5	1.05625	1.13	1.38	1.27	1.31	1.45	1.63
1	1.025	1.09	1.14	1.2	1.22	1.32	1.53
1.5	1	1.05	1.07	1.13	1.16	1.23	1.42
2	1	1.02	1.02	1.08	1.12	1.17	1.33
2.5	1	1.001	1.001	1.05	1.08	1.12	1.25
3	1	1	1	1.02	1.06	1.086	1.31
3.5	1	1	1	1	1.03	1.04	1.12
4	1	1	1	1	1.02	1.01	1.08
4.5	1	1	1	1	1.012	1	1.03
5	1	1	1	1	1.01	1	1.01
5.5	1	1	1	1	1	1	1

The purpose of combining different relationships and modifications is to determine which relationship is capable of better performance at predicting the lift and the performance of catamaran more accurately. Liu and Wang's relationships (1979) along with Thong-See (1982), have been used to correct the centre of pressure. Also, due to the friction between the wetted surface and the fluid passing through it, the relative velocity used in calculating the frictional resistance differs from the speed of the vessel. The relative velocity is obtained from equation (21) (Savitsky, 1964)

$$V_m = V\sqrt{1-\frac{0.012\tau^{1.1}}{\sqrt{\lambda}\cos\tau}} \quad (21)$$

and then, the frictional effect, C_F, be taken into account by considering the effect of surface roughness. Due to the presence of roughness on the wetted surface, a part of the coefficients of friction is obtained from the ITTC-57 ship model correlation line. This relationship depends on the Reynolds number (Harvald, 1983). Finally, the frictional resistance value is calculated as follows

$$R_F = \frac{\rho V_m^2 B^2 C_F}{2\cos\beta\cos\tau} \quad (22)$$

Catamaran wetted surface is equal to the sum of the inner wetted surface of the bodies and wetted surface of the bottom which states as follows (Lui & Wang, 1979)

$$S = B^2[(\frac{r}{\cos\beta}) + 2\lambda.\tan\tau] \quad (23)$$

The compressive resistance, resulting from the displacement of the vessel has been calculated as follows

$$R_p = \Delta\tan\tau \quad (24)$$

Finally, the total resistance is obtained from the viscosity and compressive resistance.

On the other hand, the vessel is stable when the net torque obtained from all forces about the centre of pressure be zero. Now, with the lift and resistance force and the centre of pressure, only horizontal and vertical distance of the forces to the centre of the pressure is required to be calculated to determine the torque applied to the catamaran hull. The horizontal distance to the centre of pressure is equal to

$$c = LCG - C_p\lambda B \quad (25)$$

and the vertical distance to the centre of pressure is equal to

$$a = VCG - (\frac{B}{4})\tan\beta \quad (26)$$

To determine the equilibrium trim, a duplicate process must be performed. In the next section, the computational algorithm is presented.

4. COMPUTATIONAL ALGORITHM

The various steps in the computational process are presented in Figure 6, and explained as follows:

1. The input field, the geometric characteristics of the catamaran must be defined. As in figure 6, the letters represent the specification. Vessel Specifications are written in Table (2).
2. At this stage, the combination of speed and lift coefficient is calculated.
3. A random value for the lift coefficient at zero deadrise angle (C_{l0}) is chosen, after that using equation (6), $C_{l\beta}$ is obtained. Then the value obtained from equation (5) compare with the results from equation (6), and the same process continues to increase the error or reaches the constant values. First, the lift coefficient (equation (6)) is calculated on the vessel are far from the mass centre. Therefore, in order to estimate the total torque of these forces, the vertical arm of these forces from the mass centre must be calculated which these forces act in both and then having this parameter, the ratio of wetted length to width of the vessel will be obtained based on trial and error method.
4. The forces act in horizontal and vertical directions.
5. Equilibrium trim is obtained when the torque generated by the forces equates to zero. This step is defined as a loop.
6. If the trim relationship does not satisfy the torque relationship, its value will change until the torque can be satisfied.

At the end of the process, the catamaran hydrodynamic parameters including the equilibrium trim, resistance and wetted surface at different Froude numbers will be obtained.

Table 2: Catamaran Parameters

Parameters	values
Ship Weight (kg)	18700
Breadth (m)	4.35
Half bodies breadth(m)	2.58
Deadrise angle (deg)	13
LCG (m)	5.07
VCG (m)	0.98
Shaft angle (deg)	10
Thrust line to CG (m)	0.05

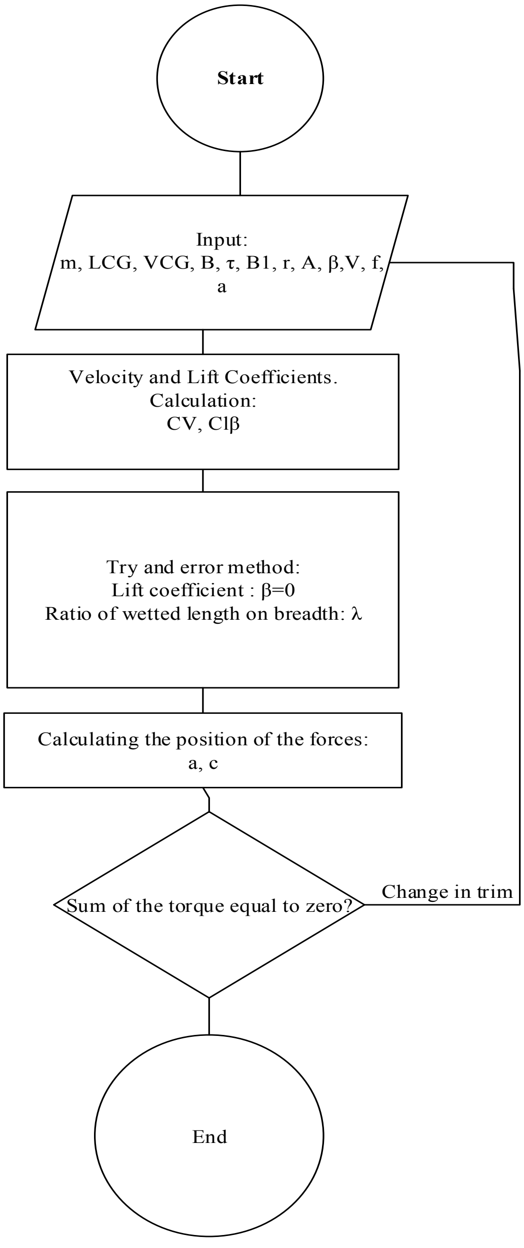

Figure 6: Computational procedure

5. VALIDATION

By comparing the results from the mathematical model against the experimental data, the hydrodynamic performance of the catamaran vessel in calm water has been obtained. The geometrical parameters of the model have been shown in the Table (2). This model is provided by Seif and Amini (2004). Consequently, using the results of this experimental model, the mathematical model, the equilibrium equation, wetted surface and total resistance of the catamaran have been validated. Finally, the average error is calculated for each relationship and the proper relationship with the lowest mean error has been determined.

Figure (7) shows that the mathematical model and existing relationships are not able to accurately calculate the dynamic trim of the vessel for Froude number less than 4. However, at higher speeds, they can compare different relationships with each other and sometimes have an error of less than 10%.

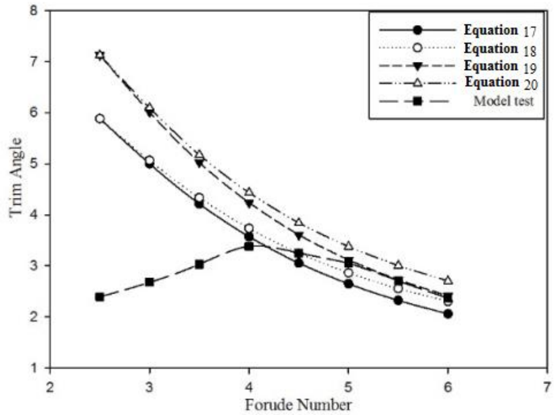

Figure 7: Trim from mathematical model with modified lift by experimental data

The total resistance of the vessel is obtained from the sum of compressive and frictional forces. In Figure 8, changes in the ratio of total resistance to ship weight at different speed are shown. Increased in ship velocity caused increases overall resistance which by studying the effect of separation and interference and increasing the wet surface, as a result of increased frictional resistance, can be justified. To the precision of the relationship, we will discuss it briefly.

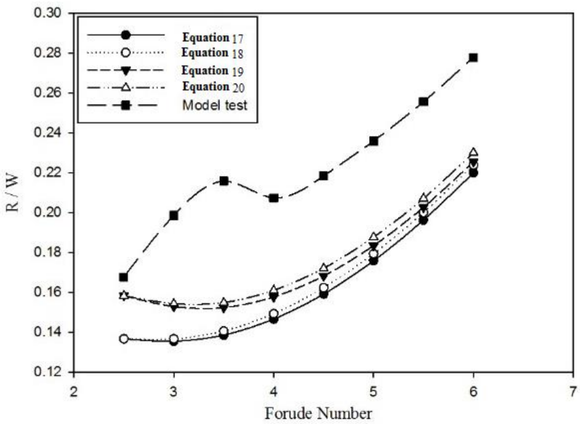

Figure 8: validation of non-dimensional values of resistance with experimental data

On the other hand, it is expected to rise in the wetted surface due to decreasing in trim. Nevertheless, it is considered in the speed range, with decreasing ship velocity, the wetted surface decrease which is conforming to Laboratory results (Figure 9).

In Figure (10), each equation has its own accuracy. In order to use a general equation for catamaran lift calculations, the average mean error of all values of trim, resistance, and wet surface is calculated, simultaneously. According to Figure (10), it can be concluded that equation (18) has the lowest relative error in three modes: the trim, the wet surface, and the resistance. Although the equation (18) has the least error, however, this error is greater than 25%. Therefore, it is obvious that there is a serious need to do more tests on the Catamaran hull forms and extracting more accurate experimental relations. However, in the current situation, only this relationship is available and can be used.

Uncertainty is one of the main causes of the error in using the mathematical model. The formulation used to obtain the lift force, calculating wetted surface regardless of body spray, the mathematical relations used to calculate the interference coefficient and separation effects, are the main reasons of the uncertainty and errors in the results of the proposed computational algorithm.

The mathematical model used in this algorithm is taken from the previous experimental and theoretical works done on planing catamarans. The results of this paper have led to more work on how to calculate the lifting force, separation and interferences effects, and to do more on planing catamaran vessels. It can also be a start point to use relationships like 2D + T which is proposed to be used in the later work in the fields of planing catamaran.

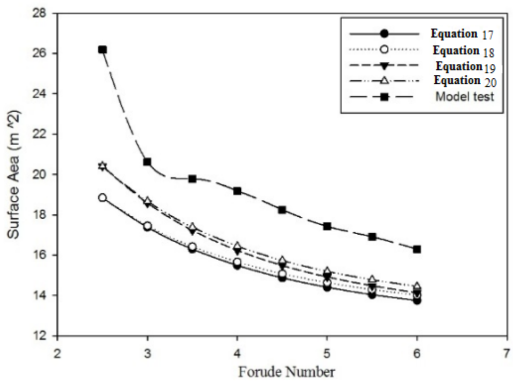

Figure 9: Wetted surface comparison against experimental data

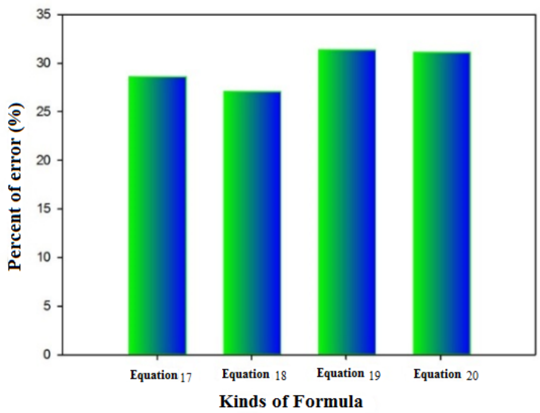

Figure 10: average error of four equations of modified relationship

The second formula of the lift factor is actually the second correction of Korvin Kroukovsky's equation (17) that is different in two parts of the static and hydrodynamic relationship with Liu and Wang (1979). In the hydrostatic, because the ratio of the wetted length to the width of the catamaran is greater than the mono-hull ship, equation (27) is used and in the dynamic section due to the deadrise angle of the catamaran hull, equation (28) is the most suitable option.

$$(\tau_{\deg}^{1.1} 0.0095 \frac{\lambda_w^2}{Cv^2}) \quad (27)$$

$$(\tau^{1.1} 0.0141\sqrt{\lambda}) \quad (28)$$

6. PARAMETRIC STUDY

It is clear that if the geometric characteristics of the catamaran changes, the balance trim and other hydrodynamic parameters, such as resistance and wetted surface will change. Here, the goal is to check and modify the effect of catamaran hull geometry changes and the deadrise angle as well as the distance between the two half body on the hydrodynamic characteristics.

7. EFFECT OF DEADRISE ANGLE

To investigate the effects of the deadrise angle on the hydrodynamic characteristics of the body of the model with an angle of 13 degrees, three angles 10, 20, and 30 are considered. In Figure (11), it is observed that with the rise of the deadrise angle, the trim will dynamically increase. Additionally, the resistance and the wetted surface are increased, accordingly, (Figures (12) and (13)).

The connection of these three parameters together is an important issue, and care should be taken in this regard. In fact, the design of a planing catamaran is the study of the mentioned parameters. As the results show, increasing the deadrise angle increases the value of trim. Therefore, this is a matter of achieving higher speed because trim decreases with increasing speed and resistance will be increased. Now increasing in the deadrise angle consequently increased the resistance. If the trim decreases as the velocity increases, the vessel penetrates into the water and will withstand extreme resistance. So, to cross the range of high speeds, a compromise between the trim angle and the amount of resistance will be required.

So, to achieve an ideal design in the graphs provided, the deadrise angle of 20 degree is the most appropriate option. The use of this angle increases the trim and does not increase the resistance too much. The wetted surface level will also increase to a reasonable level, which is quite expected. The results are completely in agreement with the recently vessel and this issue clarifies the validity of the mathematical model developed in this paper for conceptual design applications.

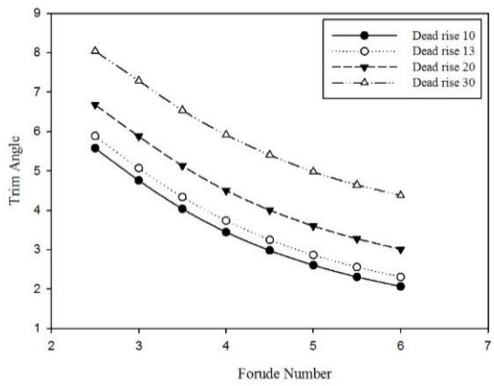

Figure 11: changes in trim angle at different deadrise angles

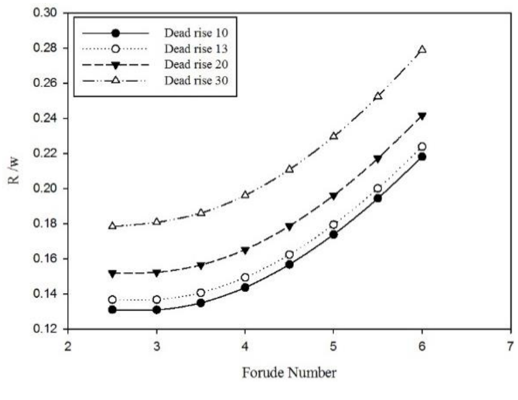

Figure 12: change in resistance at different deadrise angles

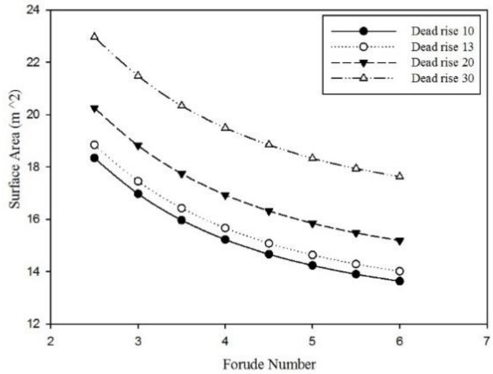

Figure 13: change in wetted surface at different deadrise angles

8. EFFECT OF DISTANCE BETWEEN TWO BODIES

If the distance between the two bodies changes, the two lift factor modifier parameters will change accordingly. Coefficient of interference, *A* will tend to 1 by increasing the half body distance. In general, this coefficient is within range $[1, \sqrt{2}]$ On the other hand, the coefficient of separation has a direct correlation with the half body distance. As the coefficient of interference increased, the lift coefficient increased and as the result, the vessel resistance (Figure (15)) increased and will reduce the trim (Figure (14)). As the trim decreased, the wetted surface increased (Figure (16)). If the distance between two bodies increases, it will reduce the effect of interference and increase the separation effect. The conditions of a maximum distance between the two bodies, Catamaran vessel need the lower equilibrium trim compared to other ships (Figure (14)).

The distance between the two half bodies is accompanied by a reduction in the effect of interference, but the separation effect, which is directly related to the distance, makes the vessel meet more resistance than a vessel with a minimum distance of the bodies (Figure (15)).

The ratio of the length to the wetted surface of the vessel is multiplied by the factor of the separation ratio. As the distance between the two half bodies increases, the separation ratio increased accordingly. Finally, the ratio of the wetted length to the width increased. On the other hand, the wetted surface, which has a direct relationship with them, increased. In Figure (16), it is observed that by increasing the velocity, the wetted surface trends downward.

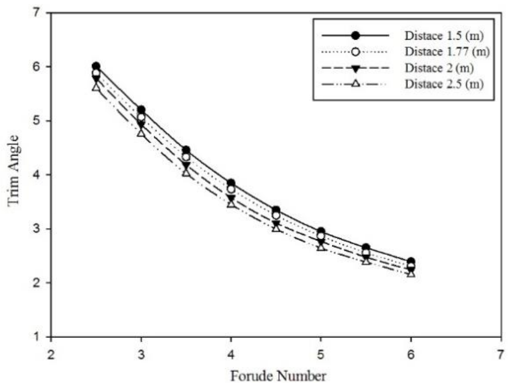

Figure 14: changes in trim angles at different distances

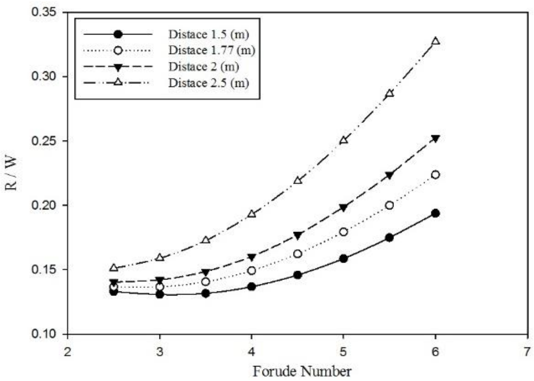

Figure 15: changes in resistance at different deadrise angles

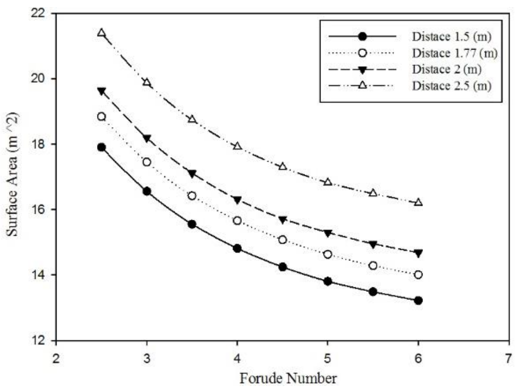

Figure 16: change in wetted surface at different distances

9. CONCLUSION

This paper focuses on an additional lift force of the catamarans due to the arch between two half hulls. Regard to the phenomena created because of this arch, two parameters involved in the creation of extra lift force is added to the hydrodynamic numerical relations at the appropriate position. The position of these two parameters has been investigated in two modes, which the most appropriate mode is when the minimum difference with the experimental results obtained. The corrected coefficient of Korvin-Kroukovsky (equation (20)), due to the high wetted surface and the deadrise angle, is suitable for catamaran vessels. Also, the following results are obtained from this assessment:

1. Quantitative correction of the effects of separation and interference can result in closer consequence to the laboratory data.
2. The geometric changes of the catamaran affecting the equilibrium trim, with the changes in the deadrise angle and the distance between the two half bodies, can be optimized equilibrium trim, total resistance, and wetted surface.
3. In order to estimate the hydrodynamic forces, a half body model is used for the numerical relations of Shuford (1957) and Brown (1971) and Korvin-Kroukovsky, Savitsky and Lehman (1949) with respect to the speed range.
4. The Shuford (1957) and Brown (1971) relationship can be evaluated by adding corrective parameters for two-body vessels. In the condition that all the required data in the Tables 1 and 2 should be provided to calculate lift coefficient.
5. The advantage of Korvin-Kroukovsky, Savitsky, and Lehman (1949) relation to Shuford (1957) and Brown (1971) is that it requires less initial data to calculate the catamaran lift.

The mathematical models presented in this paper can be useful tools in the preliminary design stage. Given that these relations have been developed with the results from previous experimental and theoretical works, can provide a desirable design estimate in the first stage, which will provide the best design along with laboratory tests. Also, because of the different uncertainties like the formula which is used to calculate the wetted surface, separation and interference coefficient and also the relations used to obtain the lift forces, there is an error showed in the final results. This work demonstrates that the fields of catamaran need to be considered more and proposed that using relations like 2D+t could be taken into consideration.

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