DISCUSSION

VISCOSITY IN SEAKEEPING

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COMMENT

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The paper addresses the issue of actuality in ship hydrodynamics: the estimation of ship's linear and angular oscillations with respect to the state of equilibrium. The prediction of seakeeping properties raises a question about a relative importance of viscous and free-surface effects (Quérard et al. 2009), yet this question remains of more general importance in fluid mechanics, since it is related to the dynamic characteristics of objects/bodies immersed in a liquid. From a theoretical standpoint, the problem refers to flows with moving boundaries. It can also be considered in terms of fluid-structure interaction (FSI), however, not necessarily linked with the computation of the body deformation and stresses due to the flow. As the Author correctly notices, the computational solution to this problem in its full setup reveals to be extremely costly due to the 3D and unsteady nature of the fluid motion under turbulent flow conditions at nominally high Reynolds numbers (Re~10^9, as stated by the Author in Tab. 1) in presence of the free surface. For this reason, the full solution, or direct numerical simulation (DNS), of the governing Navier-Stokes (N-S) equations at these Re will remain unfeasible in the foreseeable future; see, e.g., Pozorski (2017) for an estimation of the DNS capability in simple wall-bounded turbulent flows. The situation gets even worse in ship hydrodynamics when a DNS of fluid flow would need to be coupled to the dynamics of the rigid body (of complex geometry, usually).

When the statistical approach to turbulent flow computation is adopted in terms of the Reynolds-averaged Navier-Stokes (RANS) closure, the mean hydrodynamic fields of velocity and pressure remain fully three-dimensional and time-dependent. Therefore, even a RANS computation of flow past a 3D hull is still quite costly, see Quérard et al. (2009) and references therein. As pointed out by those Authors, simpler methods based on the so-called strip theory are still of interest. In this theory, the 2D cross-sections (strips) of the hull are considered w.r.t. the ship's longitudinal axis. To describe the dynamics of such a section, the degrees of freedom correspond to rotational oscillations (roll) and to translational ones, both vertical (heave) and horizontal (sway). According to the strip theory, the governing equations to predict the seakeeping response in waves rely on a suitable estimation of the sectional added mass and viscous damping, see Eqs. (16). These two-dimensional hydrodynamic coefficients have often been determined using the assumption of potential flow. Quérard et al. (2009) adopted the RANS approach and a CFD commercial solver to obtain the added mass and damping coefficients in turbulent flow conditions.

Still staying on the grounds of strip theory, the IJME paper by M. Pawłowski discusses an important question about the role of viscosity when it comes to the estimation of the sectional coefficients for the added mass and damping. In the first part of the work, some concepts of turbulent flow modelling are revisited and discussed. For the sake of clarity and correct statement of these concepts, a number of comments to this part are due. First, the N-S equations are formulated for a compressible flow of the Newtonian fluid, with the νgrad(div**V**)/3 term present on the RHS of Eq. (2); there, ν is the kinematic viscosity coefficient. Such a general form is fine, provided that one keeps in mind a natural consequence of the compressibility assumption, which is the variability of the fluid density ρ and the presence of density fluctuations ρ' in turbulent flows. Therefore, a rigorous derivation of the Reynolds-averaged momentum equation will give rise to a number of additional terms involving correlations of **V**' and p' with ρ'. Alternatively, the density-weighted (or Favre) averaging is most often applied, leading to a simpler form of the averaged equations. Since the problem considered in the paper basically refers to an incompressible flow, then the simplest way to keep the equations to their intended physical meaning would be to write Eq. (2) without the grad(div**V**) right from the outset term for the sake of consistency. Another point that might come out unclear from the reading of the paper is the notion of linear equations. As far as I understand, the linearity is important when constructing the system of (and fitting the coefficients to) equations of sectional dynamics, Eqs. (16). Yet, the linearised N-S equation as written in the paper, Eq.(5), means that the Stokes flow dynamics is considered and the inertia forces are negligible with respect to the viscous forces. In other words, the viscous time scale L^2/ν is much shorter than the convective time scale L/V_0 and the Reynolds number (which can be expressed as the ratio of the two scales) is very small. Under such conditions, the flow will not become turbulent.

Another point that needs to be clarified here refers to the nature of the turbulent (Reynolds) stress tensor. As the Author recalls, this symmetric, second-rank tensor consists of six independent components and it can be represented by three diagonal elements, see the matrix expressions below Eq. (8), in the coordinate system determined by the eigenvectors of this tensor. Yet, to do so, a general transformation needs to be found at each point of the flow. It is given by the rotation matrix with three independent angles of rotation (the Euler angles).

Consequently, to fully describe the Reynolds stress tensor **R**, six quantities (and not just three) are needed anyway. The Author puts forward an assumption that the Reynolds stress tensor and the mean strain rate tensor **S** are aligned. Rigorously, on the grounds of the Boussinesq hypothesis, which makes the basis of the eddy-viscosity type closures in RANS, such an assumption should hold for the stress anisotropy tensor $\mathbf{a}=\mathbf{R}-(2k/3)\mathbf{I}$. In other words, $\mathbf{a}=-2\nu_t\mathbf{S}$, see for example Pope (2000), where ν_t is the turbulent (eddy) viscosity coefficient. Yet, an important caveat is in order. It is well known that the Boussinesq hypothesis is not true in general, in particular in complex flows, in the separation regions, etc. (BTW: the Prandtl mixing length hypothesis does not hold there, either). So, the assumption about the main axes of **a** and **S** being identical is also flawed for the same reason.

One more remark seems appropriate at this point. The turbulent viscosity ν_t used in RANS may be assessed *a priori* when complete information on the turbulent stresses and the mean velocity gradients is available. This is possible in DNS; a comprehensive study of the turbulent boundary layer (TBL), relevant for the present paper, was performed in a seminal paper by Spalart (1988). Based on this result, the profile of ν_t across the TBL can be determined, see Figure. 7.30 in the monograph by Pope (2000). This corresponds to the Author's intuition, with zero value of turbulent viscosity at the wall, a maximum somewhere in the BL, and then again a zero asymptotic value outside of the layer. Yet, in my opinion the statement about the computation of ν_t proportional to k^2/ε as being "clearly ill conditioned" has to be taken with caution. First, depending on the inflow (or free-stream) levels of the turbulence kinetic energy k and its dissipation rate ε, their values, albeit small, may not be strictly zero outside the TBL; then, ε does not vanish at the wall either. Second, outside the boundary layer, the mean velocity gradients are usually much smaller than the mean shear within the TBL and so will be the stresses **R** computed with the Boussinesq hypothesis. As the last remark to this part, it is appropriate to note that unsteady flows may also be dealt with in the statistical approach, called unsteady RANS (URANS). It reveals to be useful in some situations, in particular when the non-stationarity in the mean flow field is due to a regular, large-scale process, such as (quasi-periodic) vortex shedding in flow past a bluff body, or when an external forcing is present, as in the so-called synthetic jets or in the unsteady TBL around the hull due to incoming surface waves (e.g., in the head sea conditions).

Next, the paper discusses a number of analytical solutions of viscous flow problems, such as the oscillating plane, the oscillating and rotating cylinder, and a general rigid body. Thanks to the assumptions about the flow kinematics, true at small enough Re, these problems become linear and essentially 1D (the relevant spatial coordinate is wall-normal and goes across the BL-type region), allowing for an analytical, time-dependent solution. The respective solutions yield the estimate of the so-called penetration depth δ which is, basically, the viscous length scale. The Author nicely identifies the unsteady boundary layer thickness in the roll motion of the hull with δ to have an estimation of the damping coefficient for individual sections. He estimates the boundary layer thickness in oscillating flows (or forming on oscillating bodies) as considerably smaller than the BL forming on a respective body (on a ship hull) in a steady forward motion. The presented analyses are certainly of relevance for the roll motion. Yet, is it justified to separate the two cases (when the seakeeping features are to be studied at the cruise speed, for instance)? Also, as argued before, the conclusion about the N-S solvers to be used without any turbulent stresses does not seem to be general or well substantiated, at least for sufficiently large Re that may occur in the finite-amplitude oscillations. It seems that the account for the Reynolds stresses will make no harm, as they will appear as negligible in the laminar flow regions, after all. The computational overhead due to the RANS solver (rather than the one for unsteady 3D laminar flow) does not seem to be excessive. Finally, the discussion offered by the Author shows the main mechanisms how the viscosity acts (and where it should be accounted for) while estimating the hydrodynamic coefficients in seekeeping analyses.

AUTHORS' RESPONSE

The Author is grateful to **Professor Jacek Pozorski** for detailed discussion of the paper. It provides a good opportunity for some clarifications and elucidation. The subject of turbulence is not easy, and often misunderstood.

Firstly, it is worth recalling the reasons for turbulence. The sufficient condition for this phenomenon is vorticity, which occurs in the vicinity of the wall, whereas the necessary condition is that flow is fast enough, i.e. the Reynolds number for given problem is higher than the critical one.

Hence, the idea of turbulence does not apply to inviscid flows, as they are irrotational by nature. Fluid particles in such flows perform a translational motion, without rotation. Such flows have a potential ϕ that defines the velocity field $\boldsymbol{v} = \mathrm{grad}\phi$. The potential fulfils the Laplace equation $\Delta\phi = 0$ along with boundary conditions. As this problem is linear, it leaves no room for turbulence. Apart from that, its solutions are unique, which follows directly from the properties of the Neumann problem, well known in theoretical physics, in which the values of the normal derivatives of the potential (normal components of velocity) are specified at the boundaries of the domain. There is no such a theorem for viscous flows. Solutions of the N–S equations can be twofold: laminar or turbulent.

It is worth noting, however that viscous flows can also be potential (irrotational) provided that there are no solid walls, as in the case of surface waves, e.g. ocean waves, or other sources of vorticity, as in the case of baroclinic flows. In other words, no walls, no vorticity in barotropic flows, which means potential flows, free of turbulence, see equation (1) in the paper. In the case of atmospheric winds, turbulence occurs, the so-called free turbulence, when masses of the air have different velocities, rubbing against each other, and/or they are baroclinic. The latter comes from works of Vilhelm Bjerknes (1862–1951), a founder of modern meteorology and weather forecasting.

Hence, turbulence is associated with vorticity. By the way, when Newton established his formulation for the shear stress $\tau = \mu\partial \boldsymbol{v}/\partial y$, he did not realise that flow near the wall was rotational – this notion did not exist that time. Even nowadays, hardly anyone links the slipping of fluid films against each other with vorticity. Everybody links this phenomenon with the shear stress, but not with vorticity.

When the Reynolds number exceeds the critical one, the regular vorticity lines (rings – in flow through a pipeline or horseshoe lines – in flow past a flat plate) are no longer flat. They become irregular in shape, creating 3-D velocity pulsations, in otherwise stationary flow, that are next converted into the additional apparent stress tensor $-\rho\boldsymbol{R}$, see equation (11) in the paper. For this reason, turbulence is always 3-D. In other words, 2-D turbulence does not exist. The link between vorticity and velocity pulsations opens room for theoretical analysis of turbulence.

Going back to the Navier–Stokes equation – equation (2) in the paper – the Laplacian of velocity $\Delta\boldsymbol{v}$ is the source of velocity pulsations $\boldsymbol{v}'$ that are converted to the tensor of Reynolds stresses $-\rho\boldsymbol{R}$ by the non-linear convective acceleration $(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}$. For potential flows the source of velocity pulsations vanishes ($\Delta\boldsymbol{v} \equiv 0$), therefore the convective term $(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}$ has nothing to convert into the turbulent stress tensor ($-\rho\boldsymbol{R} \equiv 0$). Potential flows are therefore never turbulent.

For the sake of generality, equation (2) in the paper is written for a compressible flow. Professor Pozorski is right – in such a case the said equation should by complemented by a number of additional terms, providing correlations between $\boldsymbol{v}'$, p' and ρ'. Bearing in mind that the paper refers basically to incompressible flows (div$\boldsymbol{v}$ = 0), the simplest way to keep the equations to their intended meaning is to neglect all the terms containing div$\,\boldsymbol{v}$, as e.g. graddiv$\boldsymbol{v}$.

Another point worth discussing is the notion of linearity of the equations of ship motion in waves. If the said equations are linear, the response of the ship in realistic sea conditions, subjected to irregular waves, can be obtained with the help of superposition and spectral analysis. In particular, equations (16) are linear, if the hydrodynamic coefficients a_{ii}, b_{ii}, c_{ii} (for i = 3, 4, and 5) are independent of the amplitude of *forced* oscillations. This is the case, if equations for fluid motion are linear. Though roll is prone to viscosity and non-linear effects, evidence shows (Salvesen *et al*., 1970) that the prediction of ship motions based on potential flow, with the omission of viscosity, provides satisfactory results.

Professor Pozorski is right saying that the Boussinesq hypothesis does not always hold, in particular in complex flows, in separation regions, for flows over curved surfaces, etc. Notwithstanding the above, the Boussinesq hypothesis is widely applied, without any criticism, as if it were universal. Nonetheless, it is worth knowing that in cases where it does hold, the net Reynolds stress tensor ($-\rho\boldsymbol{R} + p_t\boldsymbol{I}$) and net mean strain rate tensor of deformation ($\boldsymbol{S}_d - ⅓\boldsymbol{I}\mathrm{div}\bar{\boldsymbol{v}}$) are aligned, and related to each other by equation (13) in the paper. The above statement is difficult to find in literature. In such a case, turbulence is entirely described by one quantity – the coefficient of turbulent viscosity ν_t, found with the help of any turbulent model.

It would be a good practice that papers dealing with turbulent boundary layers show the run of turbulent viscosity ν_t across the boundary layer. This quantity should vanish on the wall and on the outer surface of the layer. Regrettably, it is difficult to find information in literature on this topic. Figure 1, taken from the monograph of Pope (2000), one of the few shows how turbulent viscosity ν_t varies across a boundary layer. As can be seen, it does not vanish on the outer edge of the layer, as it should be expected.

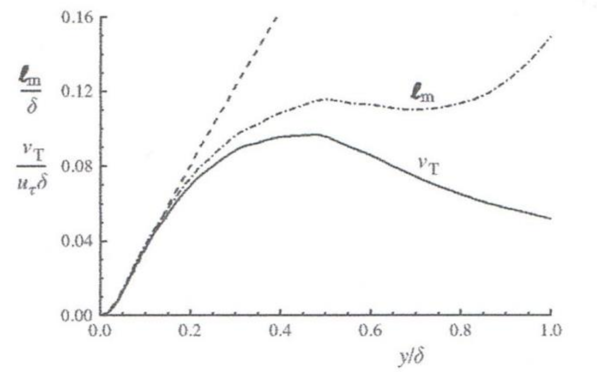

Figure 1. Turbulent viscosity and mixing length across a turbulent boundary layer (Spalart, 1988)

A much better profile of turbulent viscosity across the boundary layer on a flat plate can be found in the paper of Li *at al*. (2016). As seen in Figure 2, taken from this reference, the Reynolds shear stress <u'v'> clearly vanishes at the extremities of the boundary layer. The resulting turbulent viscosity ν_t is shown in Figure 3. It vanishes at the extremities of the boundary layer, reaching a maximum value inside the layer. It is worth noting that this quantity is much larger than the kinematic coefficient of viscosity ν.

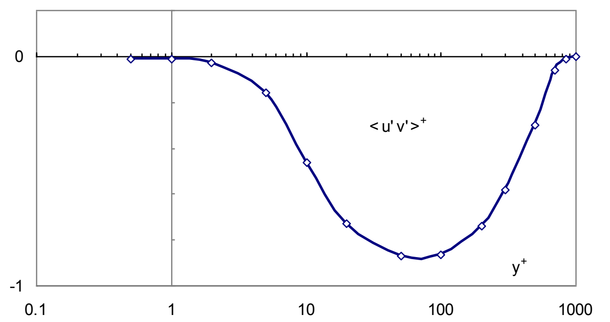

Figure 2. Turbulent shear stress $<u'v'>^+$ versus distance from wall y^+ in flow past a flat plate (Li *at al*., 2016)

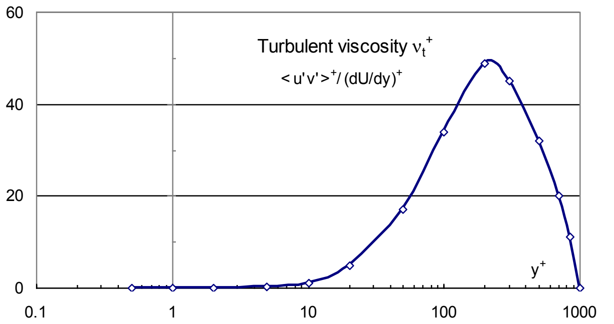

Figure 3. Turbulent viscosity across a turbulent boundary layer on a flat plate (Li *at al*., 2016)

The Author's paper is mainly concerned with hydrodynamic forces acting on oscillating bodies. There is no problem to calculate them for potential flows. However, many researchers are concerned with the omission of viscosity, which means for them the omission of turbulence. The Author's opinion is that turbulence has little chance to develop on oscillating bodies. One of the symptoms proving this statement is a much thinner boundary layer, by about one order, than for stationary flows. Many researches do apply turbulence models, developed for steady flows, for unsteady flows, which is unreasonable. One could think that a neat remedy in this situation is the application of the unsteady RANS (URANS), but this can be tricky, if the time for averaging is improperly suited to oscillations. As discussed by McDonough (2007), the unsteady RANS methods are problematic.

We have to tell loudly that a good prediction of turbulence is not an easy task. To be not ungrounded, consider a well-known case of flow through a pipeline. In such a case, only one Reynolds stress component $R_{rz} \equiv \tau_t$ has to be estimated with the help of the mixing-length hypothesis:

$$-\rho<v_r'v_z'> = \rho l_m^2 (du/dy)^2,$$

where du/dy is the derivative of the smoothed velocity profile with respect to the distance $y = R - r$ from the inner surface of the pipeline. According to Prandtl, the mixing length is given by the equation:

$$l_m/R = 0.14 - 0.08(r/R)^2 - 0.06(r/R)^4,$$

dependent on the relative radius r/R. On the other hand, the turbulent shear stress is given by the equation: $\tau_t = \mu_t(du/dy)$. Equating the two equations for the shear stress yields the equation for the kinematic turbulent viscosity: $\nu_t = l_m^2(du/dy)$. Assuming for the velocity profile $u = u_0 \cdot (y/R)^{1/7}$, the gradient of velocity equals:

$$du/dy = {}^1/_7(u_0/R)(y/R)^{-6/7},$$

where u_0 is the maximum velocity at the axis of a pipeline. A graph of turbulent viscosity ν_t is shown in Figure 4, while for turbulent shear stress – in Figure 5.

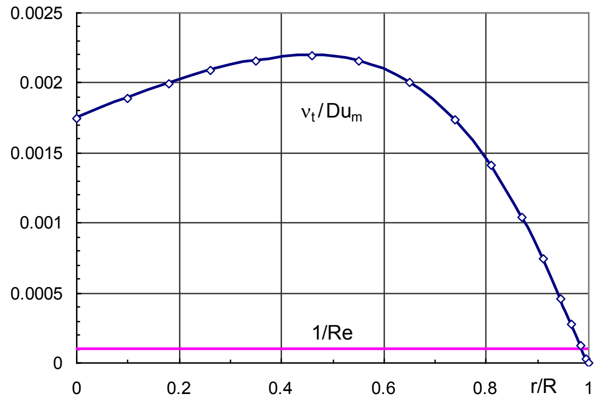

Figure 4. Run of turbulent viscosity ν_t in pipeline

The quantity $\nu_t/u_m D$ in Figure 4 plays the role of the inverse of a turbulent Reynolds number Re_t, where $u_m \equiv \bar{u}$ is the mean velocity of flow, and D is the diameter of a pipe. For instance, for $\mathrm{Re} = 10^4$, $1/\mathrm{Re} = 10^{-4}$, it is easy to deduce from Figure 4 that the turbulent viscosity ν_t is about 20 times larger than the molecular viscosity ν. Regarding the run of the turbulent viscosity ν_t it seems to be faulty, as it should vanish at the axis of a pipeline, for $r = 0$, where the vorticity and gradient of velocity vanish.

Of the key meaning for the analysis of flow in a boundary layer is the knowledge of the shear stress on the wall τ_w. For flow through a pipe this quantity results from the equation: $\Delta p ¼\pi D^2 = \pi D l \tau_w$, where $\Delta p = \lambda(l/D)½\rho\bar{u}^2$ is the drop of pressure along the pipeline of length l, and λ is the friction factor, dependent on the Reynolds number. The equation yields $\tau_w = ⅛\lambda\rho\bar{u}^2$. Substituting for $\tau_w = \rho u_\tau^2$, where u_τ is the friction velocity, useful in applications, we get $u_\tau = (⅛\lambda)^{1/2}\bar{u}$.

The maximum flow velocity $u_{\square}$ is given by the equation: $u_0 = 8.74 u_\tau \mathrm{Re}_\tau^{1/7}$, where $\mathrm{Re}_\tau = u_\tau R/\nu = ¼(½\lambda)^{1/2}\mathrm{Re}$, and $\mathrm{Re} = \bar{u}D/\nu$ is the Reynolds number. Substituting for u_τ, the following is obtained: $u_0 = \bar{u}\, 8.74(½)^{13/7}\lambda^{4/7}\mathrm{Re}^{1/7}$. Substituting now the Blasius formulation for the friction factor $\lambda = 0.3164/\mathrm{Re}^{1/4}$, we get eventually that $u_0 = 1.25\bar{u}$, independent of the Reynolds number.

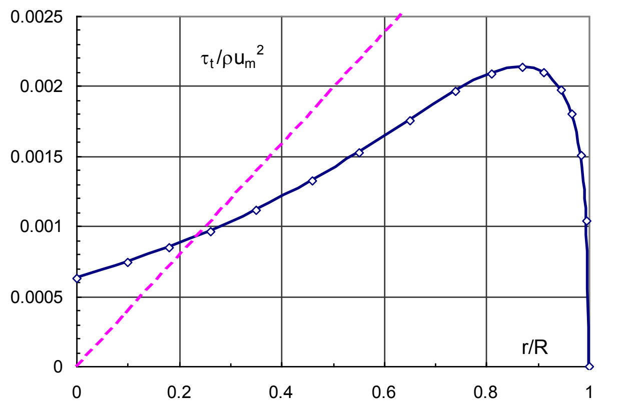

Figure 5. Turbulent sheer stress τ_t in pipeline – solid line, dashed line – total

Now, the nondimensional shear stress on the wall equals: $\tau_w/\rho u_0^2 = ⅛\lambda(\bar{u}/u_0)^2 = 0.08\lambda$, or $\tau_w/\rho\bar{u}^2 = 0.125\lambda$. Inside the pipe, the shear stress varies linearly: $\tau = \tau_w r/R$, as seen in Figure 5 (dashed line for Re = 10^4, with $\tau_w/\rho\bar{u}^2$ = 0.00396). The shear stress τ varies linearly inside a pipeline both in laminar and turbulent flows.

As can be seen in Figure 5, there is amazingly large discrepancy between the real and calculated values of turbulent shear stresses inside the pipeline. The former is dependent on the Reynolds number, while the latter – not. Such a situation indicates a conceptual error. The mixing length l_m should depend simply on the Reynolds number for any viscous flow, not only for flow through a pipe.

Regarding the application of CFD, it is equally well applicable to laminar and turbulent flows. Calculations for turbulent flows are, however, much more complex and time consuming, as there is need to solve more differential equations, needed for definition of the tensor of Reynolds stresses.

Figure 1 in the paper shows a comparison between experimental results of the hydrodynamic coefficients for roll with inviscid (potential) theory. The differences are modest even for extreme cases, such as a rectangular section. If we apply the N–S equations or RANS equations the improvement can be limited by the nature of things. The problem is that nearly everybody applies RANS equations instead of the N–S ones, because of the widely available commercial codes. Meanwhile, results obtained by the RANS equations are not sensational and dubious, which can be seen in literature, e.g. in the two references: Quérard *et al*. (2009) and Salui *et al*. (2000), cited in the paper. It should be obvious that applying steady turbulent models to unsteady cases cannot guarantee reliable results. But even if we apply the same turbulent models to the same case of steady flow as other researches it does not guarantee that we get similar results. As shown by McDonough (2007) (page 93, Figure 2.3), results obtained from various commercial flow codes are amazingly different. For potential flows such a situation is impossible – results obtained from different sources are practically the same.

The three analytical solutions for oscillating bodies, discussed in the paper, could be used as reference for validation of the commercial codes, assuming no Reynolds stresses.

The main purpose of the paper is to show that in the case of oscillating bodies resorting to turbulent models is unnecessary and conceptually wrong, due to the thin boundary layer and the lack of well developed turbulent models for unsteady flows. To some extent the situation resembles the flow around a wing –aerodynamic forces are found without resorting to turbulence. Note that in section 5.4 reference is made to Landau (2009), a Nobel Prize winner in physics for 1962.

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