# ANALYTICAL AND EXPERIMENTAL INVESTIGATION ON THE FREE VIBRATION OF SUBMERGED STIFFENED STEEL PLATE

(DOI No: 10.3940/rina.ijme.2018.a2.468)

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## SUMMARY

The approach developed in this paper applies to vibration analysis of rectangular stiffened plate coupled with fluid. It is obvious that the natural frequencies of a submerged structure are less than those of in vacuum and these are due to the effect of added mass of water to the structure. This paper focuses on the experimental, analytical and numerical solution of natural frequencies of submerged stiffened plate. The analytical solution based on the deflection equation of submerged orthotropic plate, Laplace's equation and Rayleigh's method in vibration analysis. By used the FEM software the numerical results for natural frequencies are derived. The natural frequencies of the stiffened plate are obtained practically by using Fast Fourier Transformation functions (FFT) in experimental analysis. Experimental results demonstrate the validity of analytical and numerical solution and results.


## NOMENCLATURE

| | |
|---|---|
| $E$ | Young's modulus (N m$^{-2}$ ) |
| G | Shear Young's modulus (N m$^{-2}$ ) |
| $\rho$ | Density of plate (kg m$^{-3}$) |
| $\rho_f$ | Fluid Density (kg m$^{-3}$) |
| $\phi(x,y,z,t)$ | Velocity Potential |
| $\mu$ | Plane Wave Number |
| $m^*$ | Added Mass (kg) |
| $m$ | Plate Mass (kg) |
| $\omega_{fluid}$ | Wet Natural Frequency (Hz) |
| $P$ | Dynamic Pressures (N m$^{-2}$) |
| [K] | Stiffness Matrix |
| [M] | Mass Matrix |

## 1. INTRODUCTION

The plates are used in a wide range of engineering applications such as modern construction engineering, aerospace and aeronautical industries, aircraft construction, shipbuilding, and the components of nuclear power plants. The effect of the surrounding medium on the vibration of plates and shells is of primary interest to scientists and engineers working in aerospace, marine and reactor technology (Kerboua et al, 2008). It is therefore very important that the static and dynamic behavior of plates be clearly understood when subjected to different loading conditions so that they may be safely used in these industrial applications.

It is well known that the natural frequencies of structures in contact with fluid are different from those in vacuum. In general, the effect of the fluid force on the structure is represented as added mass, which lowers the natural frequency of the structure compared to what that would be measured in vacuum. This decrease in the natural frequency of the fluid-structure system is caused by increasing the kinetic energy of the coupled system without corresponding increase in strain energy (Kerboua et al, 2008). Many numerical methods have been used in fluid–structure interaction problems, including added mass formulation, finite element method (FEM), doubly asymptotic approximation (DAA), mixed boundary element and finite element method (BEM/FEM), and arbitrary Lagrangian–Eulerian formulation (ALE). The above numerical methods have also been developed into commercial codes, such as NASTRAN, ABAQUS, ANSYS, and USA (Kundu, 1990).

An enormous amount of effort has been carried out on problems involving dynamic interaction between an elastic structure and a surrounding fluid medium. Fluid–structure interaction problems have received extensive attention since 1965 (Kundu, 1990). An analytical solution studied for dynamic behavior of a rectangular reservoir partially filled with fluid using the Rayleigh–Ritz method (Kim et al, 1996). Hydro-elastic analysis investigated on a rectangular tank completely filled with liquid using a NASTRAN program and compared results with analytical solutions (Kim and Lee, 1997). Vibration modes numerically computed of an elastic thin structure in contact with a compressible fluid (Hernandez, 2006). Methods for modeling fluid effects are also available in some commercial finite element analysis (FEA) codes (Herting, 1997 and Hibbitt et al, 2001). Some aspects discussed of incorporating heavy fluid loading effects into SEA (Creighton, 1989). Based on Kwak's approach the problem of the axe symmetric vibration of circular and annular plates in contact with fluid studied (Liang et al, 1999). The natural frequencies were calculated for clamped, simply supported and free plates. Moreover, the results were compared with the experimental data. An energy finite element analysis (EFEA) formulation for computing the high frequency behavior of plate structures in contact with a dense fluid derived (Zhang et al, 2003).

The heavy fluid loading effect is incorporated in the derivation of the EFEA governing differential equations and in the computation of the power transfer coefficients between plate members. An energy finite element analysis (EFEA) formulation for high frequency vibration analysis of stiffened plates under heavy fluid loading derived (Zhang et al, 2005). G. Aksu used from a method based on the variational principles in conjunction with the finite difference technique to determine the dynamic characteristic of eccentrically stiffened plates (Aksu, 1982). Ömer Civalek developed the discrete singular convolution (DSC) method, for static analysis of thick symmetric cross-ply laminated composite plates based on the first-order shear deformation theory of Whitney and Pagano (Civalek, 2008). T. Holopainen, proposed a new finite element model for free vibration analysis of eccentrically stiffened plates (Holopainen, 1995). Murat Gürses, Ömer Civalek et al (Gürses, 2009) investigated discrete singular convolution (DSC) method for numerical solution of vibration problems. To overcome the complexities in the modal analysis of the fluid–structure interaction, the Mindlin plate theory and the potential flow theory are applied; the velocity potential is also expressed using double finite Fourier transforms (Li et al, 2011). In the paper, experimental, analytical and numerical analyses for submerged stiffened plate are studied. The numerical and analytical results are verified comparing to experimental results.

## 2. THEORETICAL ANALYSIS

In this section, theoretical analysis of stiffened plate is studied in the vacuum and fluid. In the analytical study, a stiffened plate undergoing a flexural bending vibration in a body of homogeneous, uncompressible and invisid fluid whose motion is irrotational, is considered. The governing equation for the surface displacement of the plate- fluid system is derived. The boundary conditions for the plate are considered as fixed on all sides. All of geometrical and physical properties of the stiffened plate as shown in Figure 1 are summarized in Table 1.

Table 1: Geometrical and Physical Properties of Stiffened Plate

| $s(m)$ | $a(m)$ | $b(m)$ | $c(m)$ | $h(m)$ | $l(m)$ | $E(Gp$ | $G(Gpa)$ | $\rho(\frac{kg}{m^3})$ |
|---|---|---|---|---|---|---|---|---|
| **0.05** | **0.3** | **0.3** | **0.004** | **0.004** | **0.02** | **208** | **70** | 7800 |

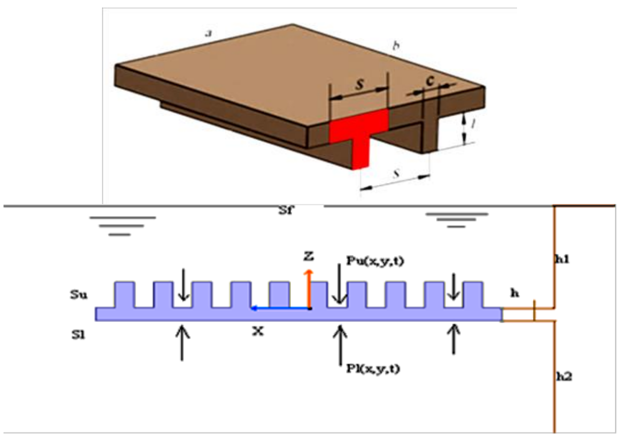


Figure 1: Stiffened plate schematics underwater

Where $E$ is Young's modulus and $G$ is shear Young's modulus. Consider a stiffened plate in which its both sides are exposed to still fluid as shown in Figure 1. The plate undergoes small amplitude free bending vibration. The fluid motion due to the vibration of the plate produces dynamic pressures $P_L(x,y,t)$ and $P_U(x,y,t)$ on the lower and upper fluid-plate interfaces $S_L$ and $S_U$ respectively. Governing equation of submerged deflection of stiffened plate is (Ventsel, 2001)

$$D_x \frac{\partial^4 W(x,y,t)}{\partial x^4} + 2H \frac{\partial^4 W(x,y,t)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W(x,y,t)}{\partial y^4} + \rho \frac{\partial^2 W}{\partial t^2} = P_L(x,y,t) - P_U(x,y,t) \quad (1)$$

Where, $W(x,y,t)$ is the upward displacement of the plate measured from its static equilibrium position and $\rho$ is density of the plate. $D_x = \frac{EI}{s}$ and $D_y = \frac{Eh^3}{12(1-\frac{c}{s}+\frac{ch^3}{s(h+l)^3})}$ are the flexural rigidities of an orthotropic plate. I is the moment of inertia of an inverse T-shaped section corresponding to one spacing of the rib location. Where $H = D_{xy} + 2D_s$ , $D_s = \frac{Gh^3}{12}$ and $D_{xy}$ is torsional rigidities and for stiffener with rectangular cross section is zero (Ventsel, 2001). The density of fluid $\rho_f$ , is assumed to be homogeneous, incompressible, in viscid and its motion irrotational. Therefore, the velocity potential $\phi(x,y,z,t)$ satisfies Laplace's equation given by (Esmailzadeh, 2007):

$$\nabla^2 \phi(x,y,z,t) = 0 \quad (2)$$

Since the fluid motion is irrotational, the unsteady Bernoulli equation

$$\rho_f \frac{\partial \phi}{\partial t} + P + \frac{1}{2}\rho_f(\phi_x^2 + \phi_y^2 + \phi_z^2) + \rho_f gz = 0 \quad (3)$$

Where x, y and z indices denote to partial derivative variables. Neglecting the non-linear term $(\phi_x^2 + \phi_y^2 + \phi_z^2)$ for small amplitude waves, equation (3) can be replaced by its linearized form as:

$$\rho_f \frac{\partial \phi}{\partial t} + P + \rho_f gz = 0 \quad (4)$$

Since waves are assumed to be small, the free surface boundary condition

$$\frac{\partial \phi}{\partial z}\Big|_{z=0} - \frac{\omega^2}{g}\phi\Big|_{z=0} = 0 \quad (5)$$

 

Can be applied on $S_F$, where $\omega$ is the angular frequency of the wave motion caused by the vibration of the plate. On the bottom of the tank, the normal component of velocity is zero, i.e.

$$\frac{\partial \phi}{\partial z}\Big|_{z=-(h_1+h+h_2)} = 0 \qquad (6)$$

The kinematics boundary conditions at plate-fluid interfaces $S_L$ and $S_U$ are given by

$$\frac{\partial \phi}{\partial z}\Big|_{z=-(h_1+h)} = \frac{\partial \phi}{\partial z}\Big|_{z=-h_1} = \frac{\partial W}{\partial t} \qquad (7)$$

Using equation (4) we have:

On $S_U$

$$\rho_f \frac{\partial \phi}{\partial t}\Big|_{z=-h_1} + P_U - \rho_f g h_1 = 0 \qquad (8)$$

And on $S_L$

$$\rho_f \frac{\partial \phi}{\partial t}\Big|_{z=-(h_1+h)} + P_L - \rho_f g(h_1+h) = 0 \qquad (9)$$

Substitution of equation (8) and (9) in to equation (3) gives the following governing equation

$$\rho\frac{\partial^2 W}{\partial t^2} + \rho_f\left[\frac{\partial \phi}{\partial t}\Big|_{z=-(h_1+h)} - \frac{\partial \phi}{\partial t}\Big|_{z=-h_1}\right] + D_x\frac{\partial^4 W(x,y,t)}{\partial x^4} + 2H\frac{\partial^4 W(x,y,t)}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 W(x,y,t)}{\partial y^4} - \rho_f g h = 0 \qquad (10)$$

Based on separation variable method, the displacement of the small amplitude bending vibration of the plate can be assumed as

$$W(x,y,t) = \overline{W}(x,y)T(t) \qquad (11)$$

Let's also assume that the velocity potential $\phi(x,y,z,t)$ can be considered in the form of

$$\phi(x,y,z,t) = G(x,y)F(z)S(t) \qquad (12)$$

Substituting Equation (12) into Equation (2) gives

$$F(z)\nabla^2 G(x,y) + G(x,y)\frac{d^2F(z)}{dz^2} = 0 \qquad (13)$$

$G(x,y)$, is a function of x and y, and F(z) is a function of z, so the following equation holds, viz.

$$\frac{\nabla^2 G}{G} = -\frac{\frac{d^2F}{dz^2}}{F} = -\mu^2 \qquad (14)$$

This can be written as

$$\nabla^2 G + \mu^2 G = 0 \qquad (15)$$

$$\frac{d^2F}{dz^2} - \mu^2 F = 0 \qquad (16)$$

Where $\mu^2$ is a real constant, and $\mu$ is the plane wave number, which is determined by the vibrating frequency of the submerged plate and fluid boundary conditions in the x-y plane. Substituting Equation (11) and (12) in to Equation (7) gives

$$G(x,y)S(t) = \overline{W}(x,y)\frac{T'}{F'\big|_{z=-(h_1+h)}} \qquad (17)$$

Where, $T' = \frac{dT}{dt}$ and $F'(z) = \frac{dF}{dz}$

Applying Equation (17) to Equation (12) gives

$$\phi(x,y,z,t) = F(z)\overline{W}(x,y)\frac{T'(t)}{F'\big|_{z=-(h_1+h)}} \qquad (18)$$

We also have

$$\phi(x,y,z,t) = F(z)\overline{W}(x,y)\frac{T'(t)}{F'\big|_{z=-h_1}} \qquad (19)$$

The combination of Equations (11), (18), (19) and Equation (10) gives

$$T'' + \omega_{fluid}^2 T = 0 \qquad (20)$$

$$\nabla^4\overline{W} - \beta^4\overline{W} = 0 \qquad (21)$$

Applying Equation (11) gives

$$\frac{\partial^2 W}{\partial t^2} = \overline{W}\frac{d^2T}{dt^2} = \overline{W}T'' \qquad (22)$$

Applying Equation (18) and Equation (19) gives

$$\frac{\partial \phi}{\partial t}\Big|_{z=-(h_1+h)} - \frac{\partial \phi}{\partial t}\Big|_{z=-h_1} = \overline{W}T''\left[\frac{F}{F'}\Big|_{z=-(h_1+h)} - \frac{F}{F'}\Big|_{z=-h_1}\right] \qquad (23)$$

$$\nabla^4 = T\nabla^4\overline{W} \qquad (24)$$

Applying Equations (22), (23) and (24) to Equation (10) gives

$$\rho\overline{W}T'' + \rho_f\overline{W}T''\left[\frac{F}{F'}\bigg|_{z=-(h_1+h)} - \frac{F}{F'}\bigg|_{z=-h_1}\right] + D_x\frac{\partial^4 W(x,y,t)}{\partial x^4} + 2H\frac{\partial^4 W(x,y,t)}{\partial x^2\partial y^2} + D_y\frac{\partial^4 W(x,y,t)}{\partial y^4} - \rho_f gh = 0 \tag{25}$$

Where the term $\rho_f gh$ is a static load. It has effect only on the equilibrium position, but not on the dynamical response. So Equation (25) can be solved without considering this static term. Consider the equation

$$\rho\overline{W}T'' + \rho_f\overline{W}T''\left[\frac{F}{F'}\bigg|_{z=-(h_1+h)} - \frac{F}{F'}\bigg|_{z=-h_1}\right] + D_x\frac{\partial^4 \overline{W}(x,y)}{\partial x^4}T(t) + 2H\frac{\partial^4 \overline{W}(x,y)}{\partial x^2\partial y^2}T(t) + D_y\frac{\partial^4 \overline{W}(x,y)}{\partial y^4}T(t) = 0 \tag{26}$$

Where $\rho^* = \rho_f\left[\frac{F}{F'}\bigg|_{z=-(h_1+h)} - \frac{F}{F'}\bigg|_{z=-h_1}\right]$ is the added mass due to the fluid- loading effect. That is

$$(\rho+\rho^*)\overline{W}T'' + D_x\frac{\partial^4 W(x,y,t)}{\partial x^4} + 2H\frac{\partial^4 W(x,y,t)}{\partial x^2\partial y^2} + D_y\frac{\partial^4 W(x,y,t)}{\partial y^4} = 0 \tag{27}$$

The combination of Equation 11, 18, 19 and Equation 10 gives

$$T'' + \omega_{fluid}^2 T = 0 \tag{28}$$

$$\nabla^4\overline{W} - \beta^4\overline{W} = 0 \tag{29}$$

$$\beta^4 = (\rho+\rho^*)\omega_{fluid}^2 \tag{30}$$

Where, $\omega_{fluid}$ is the natural frequency of the plate underwater; $\beta$ is a constant determined by the plate boundary conditions. If the plate vibrates in air, the response equations corresponding to Equations 28, 29 can be obtained

$$T'' + \omega_{air}^2 T = 0 \tag{31}$$

$$D_x\frac{\partial^4 W(x,y,t)}{\partial x^4} + 2H\frac{\partial^4 W(x,y,t)}{\partial x^2\partial y^2} + D_y\frac{\partial^4 W(x,y,t)}{\partial y^4} - \beta'^4\overline{W} = 0 \tag{32}$$

Here

$$\beta'^4 = \omega_{air}^2\rho \tag{33}$$

The constant $\beta'$ is determined by plate boundary conditions, which are the same whenever the plate is submerged in fluid or in air. Thus one has

$$\beta = \beta' \tag{34}$$

In this case we have the following basic equation

$$\omega_{fluid} = \omega_{air}\frac{1}{\sqrt{1+\frac{m^*}{m}}} \tag{35}$$

Equation (35) displays the relationship between natural frequency of the plate in fluid and that in air.

## 2.1 THE NATURAL FREQUENCY FOR DRY STIFFENED PLATE

In this section, Rayleigh's method is used to compute the natural frequencies in vacuum environment. The Rayleigh's method is based on following relation (Ventsel, 2001).

$$U_{\max} = K_{\max} \tag{36}$$

Where

$$U_{1\max} = \frac{1}{2}\int_0^a\int_0^b\left[D_x\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + D_y\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2D_{xy}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4D_s\left(\frac{\partial^2 W}{\partial y\partial x}\right)^2\right]dxdy$$

$$D_x = \frac{EI}{t},\quad D_y = \frac{Eh^3}{12(1-\frac{b}{t}+\frac{bh^3}{tH_1^3})},\quad D_{xy} = 0,\quad D_s = \frac{Gh^3}{12}$$

$$H = D_{xy} + 2D_s$$

$$K_{\max} = K_{\max\,plate} + K_{\max\ stiffeners} = \frac{\omega^2}{2}\left(\int_0^a\int_0^b \rho_{plate}h\overline{W}(x,y)\,dxdy + n\int_0^c\int_0^b \rho_{stiff}\,l\,\overline{W}(x,y)\,dxdy\right)$$

Where n is the number of stiffeners. By satisfying the boundary conditions of fixed four sides (Equation(37)), the deflection of plate can be obtained as follow and this leads to achieving natural frequency as shown in Equation (38).

$$\overline{W} = A_{mn}(1-\cos\frac{2m\pi x}{a})(1-\cos\frac{2n\pi y}{b}) \qquad (37)$$

$$\omega_{air} = \frac{22.79}{a^2}\sqrt{\frac{1}{\rho}\left[D_x + D_y(\frac{a}{b})^4 + \frac{2}{3}H(\frac{a}{b})^2\right]} \qquad (38)$$

By using differential equation (14) and applying the Rayleigh's method**,** the natural frequency of submerged reinforced plate is obtained. When Rayleigh's method is written to find natural frequency under the water, the frequency is proportional to the ratio of the maximum potential energy to the sum of kinetic energy of reinforced plate $T_p^*$ and fluid $T_F^*$. The values of $V_p$ and $T_p^*$ in fluid and vacuum are constant, so the natural frequency of reinforced plate in vacuum and fluid are related to each other as shown in Equation(35). Using Rayleigh's method for submerged stiffened plate leads to the following natural frequency.

$$\omega_{fluid} = \frac{22.79}{a^2}\sqrt{\frac{m}{\rho(m+m_a)}\left[D_x + D_y(\frac{a}{b})^4 + \frac{2}{3}H(\frac{a}{b})^2\right]} \qquad (39)$$

There are different ways to calculate the added mass ($m_a$) for example Strip or Greenspon method (Goninan 2001 and Liu 1995).

## 3. NUMERICAL SIMULATIONS

In order to validate the present formulation, the natural frequency obtained by the present method under clamped boundary conditions is compared with finite element whose results was validated through natural frequency of the modal testing under free boundary conditions in section 4**.** The reason for using this method validation, is that clamped boundary conditions applied in practice and experimental setup is very difficult and inaccurate, therefore, FEM software is used as an intermediary to compare the results of the analytical and experimental procedure. In the modeling of the submerged plate, the shell elements are used to design the plate and stiffeners. To achieve the natural frequencies of the model, free vibration analysis is used without any external excitations. For the free vibration analysis the numerical solution is reduced to solving the problem (X.H. Wang, 2006).

$$[K]\times(u) = \omega\times[M]\times(u) \qquad (40)$$

Where [K] and [M] are, respectively, the stiffness and mass matrix while ($u$) and $\omega$ are the modal vector and the frequency parameter. Numerical results of first two mode shapes of the dry stiffened plate are shown in Figure 2. As shown in Figure 2 (c), first mode shape of the dry stiffened plate is in twisted form but in it can be seen in Figure 2 (d) the second mode shape is in bended form. It is expected that the same mode shapes could be seen in the fluid with different frequency values.

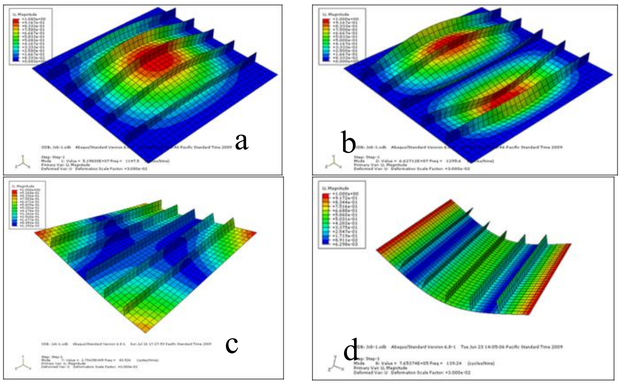


Figure 2: dry stiffened plate frequency; (a), (b) first and second natural frequency (1147 and 1295.6 Hz) in clamped boundary conditions; (c), (d) first and second natural frequency (83.5 and 139.2 Hz) in free boundary conditions

It should be noted that in the submerged case in order to improve the mode shape view, the fluid of upper side of the plate is hidden in the software results. Numerical results of mode shapes of the wet stiffened plate are shown in Figure 3.

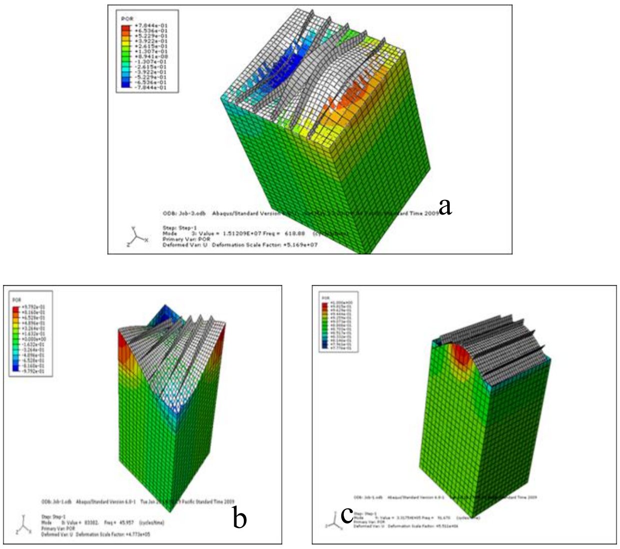


Figure 3:wet stiffened plate frequency; (a) first natural frequency (618.8 Hz) in clamped boundary conditions; (b) , (c) first and second natural frequency (45.95 and 91.6 Hz) in free boundary conditions

As shown in Figure 3 (b), first mode shape of the plate in the water environment is in twisted form but in Figure 3 (c) it can be seen that the second mode shape is in bended form. As expected previously, the obtained mode shapes in the vacuum and fluid are the same. Obviously, as mentioned in the literature the natural frequencies of structures in the fluid are less than of in the vacuum. This is due to the added mass of the fluid in the structure. Numerical results of the study verified the above claim. In all of simulations the density of water and its bulk modulus are considered as 1000 $({}^{kg}/_{m^3})$ and $2.7 \times 10^9 (pa)$ respectively.

## 4. EXPERIMENTAL ANALYSIS

The natural frequencies of the stiffened plate are obtained using modal analysis of frequency response functions (FRF) practically. The dynamic response of a mechanical structure while either in a development phase or an actual use environment can readily be determined by impulse force testing. These tests are performed by B&K modal testing equipment. Using an FFT (Fast Fourier Transformation) analyzer, the transfer function of the structure can be determined from a force pulse generated by the impact of a hammer and the response signal measured with an accelerometer. The impact force yields extensive information about the frequency and attenuation behavior of the system under test. By using FFT and phase analysis the natural frequencies can be easily obtained. The boundary conditions of stiffened plate are considered free on all sides. Thickness of stiffeners and plate are considered 4*mm*. The test vessel, stiffened plate and Portable Vibration Analysis Toolbox (PVAT) system are shown in Figures 4.

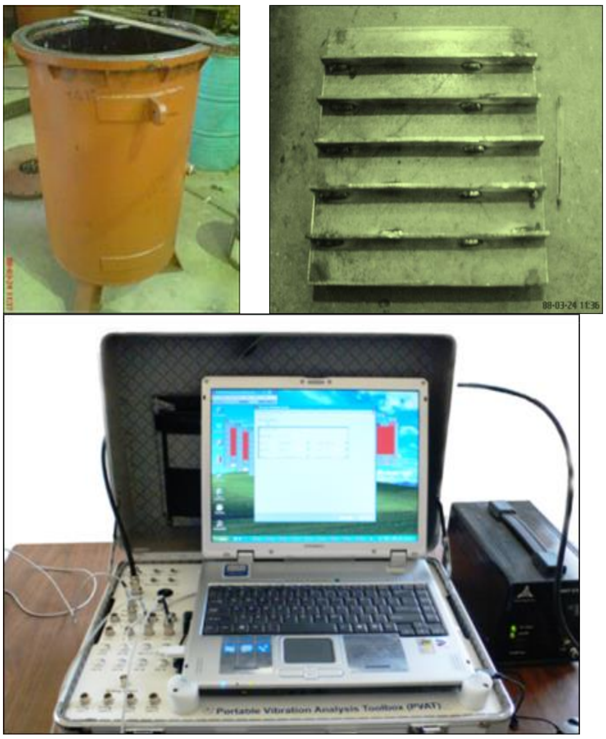

Figure 4: Modal test equipment

### 4.1 MODAL TEST RESULTS

Test specimens flouted in the air and submerged in the water are shown in Figures 5 (a) and 5 (b) respectively (The stiffeners could not be seen in the pictures because that was welded on the lower side of stiffened plate).

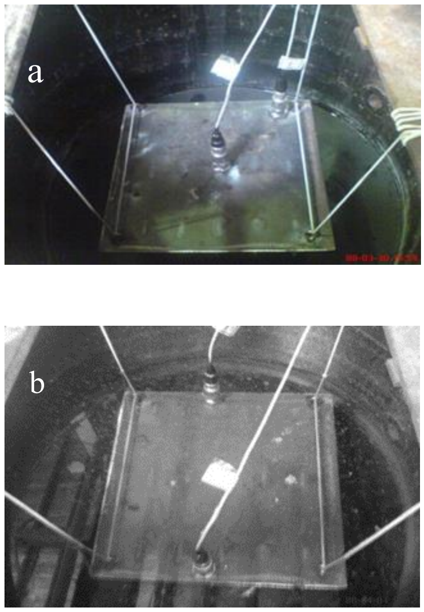


Figure 5: stiffened plate in air (a) and underwater (b).

In order to analyze stiffened plate in the air, each of accelerometers is connected to separate channels of the PVAT system analyzer. Some tests were done under different conditions. For example, two accelerometers are attached to plate can be seen in Figure 5. Hammer impacts are applied to specified points on the plate. The locations of attachment of accelerometers and points of hammer exciting impact are shown in Figure 6.

All points of the plate attached to the accelerometers are selected according to the obtained results from mode shapes in numerical simulations. Indeed, the accelerometer has to be attached to the point showing the maximum displacement in the particular mode. FFT responds are then obtained based on velocity and acceleration criterion. The FFT graphs in air are shown in Figure 6 (X values show the frequency).

The submerged sensors in the water, are isolated using water proof adhesive. Height of water on the upper side of stiffened plate in the vessel is 25*cm*. All of the tests in the air environment are the same as in the fluid. The FFT graphs of underwater case are shown in Figure 7.

 

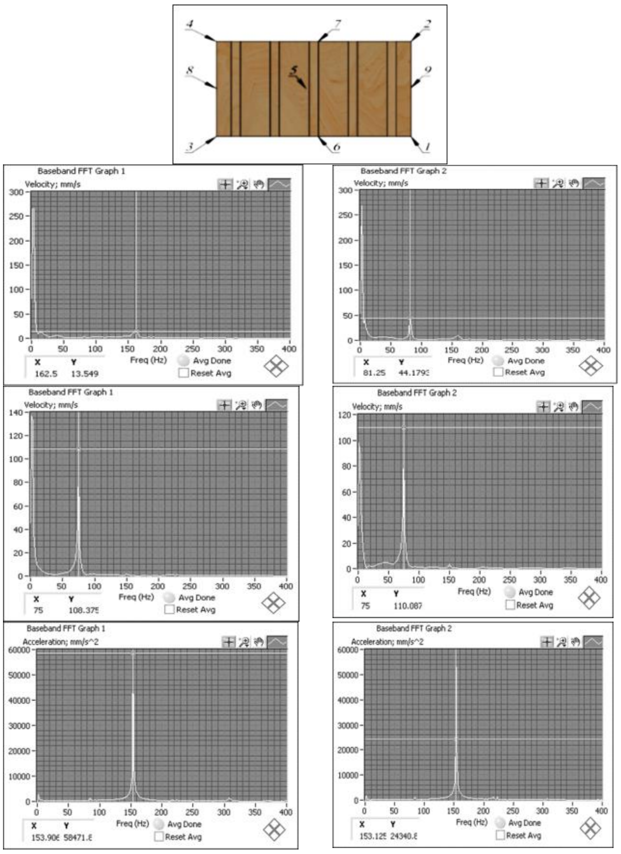

Figure 6: the FFT graphs of dry stiffened plate

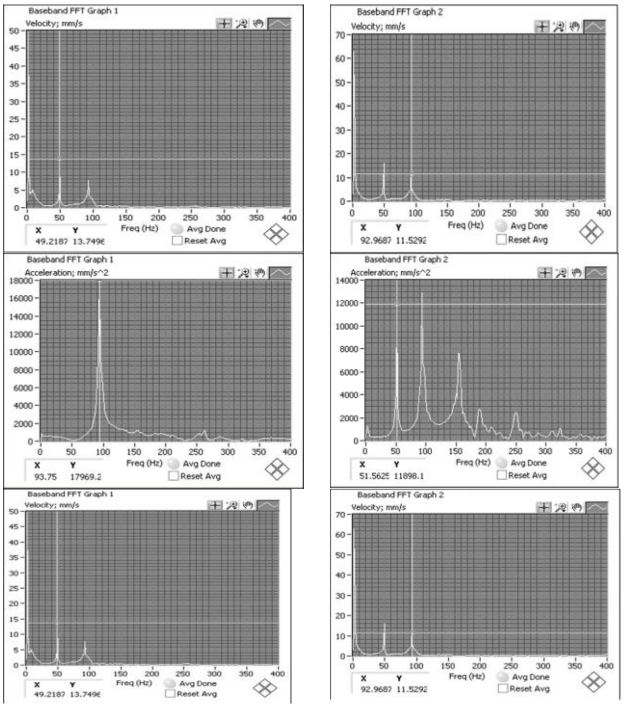

Figure 7: the FFT graphs of wet stiffened plate

## 5. RESULTS

To evaluate the coupled acoustic analysis in numerical solution the final solution was compared to the experimental results. The results and the relative error of both finite element solutions to the experiment are shown in Table 2 with free boundary conditions on all sides.

Table 2 Comparison of test and the numerical results for first two modes shapes frequency

| | Numerical result | Test result | Error % |
|---|---|---|---|
| Wet frequency (Hz) | 45.957 | 49.2 | 6.5 |
| | 91.670 | 93.7 | 2 |
| Dry frequency (Hz) | 83.526 | 78 | 7 |
| | 139.24 | 155 | 10.1 |

The previous calculations show that it was possible to simulate the vibration of the stiffened plate in the water tank. In building three models with an increasing number of elements, it was ensured that the solution be converged. It is therefore justified to use the numerical solution in the design process of vessels that should be used underwater which are subject to vibration. Comparison of the analytical and the numerical results for first mode shapes frequency are shown in Table 3.

Table 3 Comparison of the analytical and the numerical results for first mode shapes frequency

| | Numerical result | Analytical result | Error % |
|---|---|---|---|
| Wet frequency (Hz) | 618.88 | 649.026 | 4.6 |
| Dry frequency (Hz) | 1147.5 | 1295 | 11.3 |

Table 2 and 3 show the accuracy of numerical and analytical result. Strings as free boundary conditions for all sides of plate, make error in results. Also improper hammer impact produces an error.

## 6. CONCLUSIONS

In this study, the free vibration of a stiffened plate in contact with bounded water is investigated both theoretically and experimentally. The kinetic and potential energy for stiffened plate with the kinetic energy of the bounded water are obtained and used in Rayleigh's method to extract the natural frequency. The effect of contact with water on the vibration of stiffened plate is therefore appeared as an added mass in vertical displacement of the plate. Free-free boundary conditions are arranged for the stiffened plate in a hammer-accelerometer modal testing. The shape of the Fast Fourier Transform function versus frequency graph for the dry and wet stiffened plates are observed to be similar but shifted along the frequency axis with a factor of about 0.5 for frequency.

FEM is used as an intermediary to verify the present analytical results for a clamped stiffened plate. However, since the application of clamped boundary conditions is difficult in practice, free-free boundary conditions are set up for the plate experimentally. The results of FEM are on the other hand verified by the modal testing for free-free boundary conditions. Finally, the natural frequency obtained by the present method is compared and validated with FEM under clamped boundary conditions. So, with a good approximation, the Equation (39) can be used to calculate the natural frequency of stiffened plates in contact with any fluid.

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